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Economic analysis of system reliability model under operation in changing weather

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Abstract
The main concentration of the present work is to evaluate and analyze economic nature of a system reliability model which is allowed to operate under varying weather conditions – normal and abnormal. In this context a system reliability model consisting of two non-identical units is developed in which initially main unit is operative and the duplicate unit is at standby. Single repairmen/server comes immediately to rectify the fault in normal weather whereas in abnormal weather the repair is not feasible. The techniques used for the analysis are SMP & RPT.

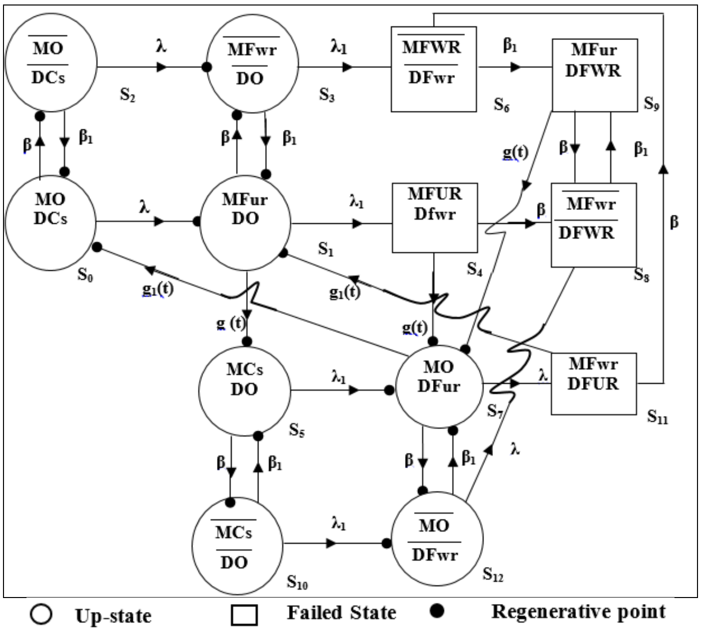
Keywords: Economic analysis, varying weather conditions, repairable system, SMP, RPT

1. Introduction
The weather environment has a significant impact on the performance of the operating systems. The physical stresses created by adverse weather conditions deteriorate performance and efficiency of these systems. Besides, scientists and engineers have tried a lot to develop systems which can work in varying environmental conditions. And, the performance of such systems has been improved up to a considerable level by adopting better repair policies and redundancy techniques. Therefore, in the earlier study, stochastic Models of a system of non-identical units have been analyzed under two weather conditions-normal and abnormal. However, in these Models operation and repair activities are not allowed in abnormal weather. But, sometimes we may have emergency situations in which operation of the system becomes necessary irrespective of weather conditions. S Chander (2005) [1] have determined reliability measures of a single unit system with Priority for Operation and Repair with Arrival Time of Server. But, redundant systems and impact of weather on identical or non-identical units have not been studied so far by considering operation of the system in abnormal weather. Malik, S.C. and Barak, M.S (2009) [2] analyzed Reliability and Economic Analysis of a System Operating under Different Weather Conditions. Later Malik. S and Deswal. S (2012 – 2015) [3, 5] performed Stochastic Analysis of a Repairable System of Non-identical Units with Priority for Operation and Repair Subject to Weather Conditions. Deswal. S (2019) [8] measured the Cost Benefit Analysis of Reliability Models under Diverse Climatic Surroundings
The purpose of the present study is not only to strengthen the existing literature on reliability but also to know the variations in reliability and economic measures of a system of non-identical units operating in different weather conditions. To meet out this objective a stochastic Model is developed under different set of assumptions on operation and repair policies. Initially, one original unit (called main unit) is operative and other substandard unit (called duplicate unit) is kept at spare in cold standby. Each unit has constant failure unit from normal mode. It is assumed that both units are capable of performing the same set of functions and activities but with different proficiencies. The system operates in two weather conditions - normal and abnormal. However, repair of the system is allowed only in normal weather by a server visits the system immediately as and when needed. The distribution of failure times of units and change of weather conditions are taken as negative exponential while that of repair times of the units follow arbitrary distributions. The units work as new after repair. All random variables are statistically independent. The semi-Markov and regenerative point technique are adopted to drive the expressions for the reliability measures such as availability, busy period of the server, expected number of visits by the server and profit function. The results are analyzed through graphs for particular values of various parameters and costs.

2. Notations

E	The set of regenerative states
MO/DO	Main/Duplicate unit is good and operative
$\overline{MO}$ / $\overline{DO}$	Main/Duplicate unit is good and operating in abnormal weather
MCs/DCs	Main/Duplicate unit is in cold standby mode
$\overline{MCs}$ / $\overline{DCs}$	Main/Duplicate unit is in cold standby mode in abnormal weather
λ/ λ_1	Constant failure rate of Main /Duplicate unit
β/ β_1	Constant rate of change of weather from normal to abnormal/abnormal to normal weather
MFur/DFur	Main/duplicate unit failed and under repair
MFUR/DFUR	Main/duplicate unit failed and under repair continuously from previous state
MFwr/DFwr	Main/duplicate unit failed and waiting for repair
MFWR/DFWR	Main/duplicate unit failed and waiting for repair continuously from previous state
$\overline{MFwr}$ / $\overline{DFwr}$	Main/Duplicate unit failed and waiting for repair due to abnormal weather
$\overline{MFWR}$ / $\overline{DFWR}$	Main/Duplicate unit failed and waiting for repair continuously from previous state due to abnormal weather
g(t)/G(t)	pdf/cdf of repair time of Main unit
$g_1(t)$/ $G_1(t)$	pdf/cdf of repair time of Duplicate unit
q_{ij} (t)/Q_{ij} (t)	pdf/cdf of passage time from regenerative state i to regenerative state j or to a failed state j without visiting any other regenerative state in (0,t]
$q_{ij.kr}$ (t)/ $Q_{ij.kr}$ (t)	pdf/cdf of direct transition time from regenerative state i to a regenerative state j or to a failed state j visiting state k,r once in (0,t]
$q_{ij.k,(r,s)}{}^n(t)$/$Q_{ij.k,(r,s)}{}^n(t)$	pdf/cdf of direct transition time from regenerative state i to a regenerative state j or to a failed state j visiting state k once and n times states r and s.
$M_i(t)$	Probability that the system is up initially in regenerative state Si at time t without visiting to any other regenerative state
$W_i(t)$	Probability that the server is busy in state Siupto time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states
m_{ij}	The conditional mean sojourn time in regenerative state Si when system is to make transition in to regenerative state Sj. Mathematically, it can be written as $m_{ij} = E(T_{ij}) = \int_0^\infty t\,d[Q_{ij}(t)] = -q_{ij}^{*\prime}(0)$, where T_{ij} is the transition time from state Si to Sj; Si, Sj ε E.
μ_i	The mean Sojourn time in state Si this is given by $\mu_i = E(T_i) = \int_0^\infty P(T_i > t)dt = \sum_j m_{ij}$, where T_i is the sojourn time in state Si.
Ⓢ/©/©^{n}	Symbol for Laplace Stieltjes convolution/Laplace convolution/Laplace convolution n times
** / *	Symbol for Laplace Steiltjes Transform (L.S.T.)/Laplace transform (L.T.)
´(Desh)	Used to represent alternative result

3. State Transition Diagram



4. Transition probablities and mean sojourn times

The differential transition probabilities are:

$dQ_{01}(t)=\lambda e^{-(\lambda+\beta)t}dt$, $dQ_{02}(t)=\beta e^{-(\lambda+\beta)t}dt$, $dQ_{13}(t)=\beta e^{-(\beta+\lambda_1)t}\overline{G(t)}\,dt$,

$dQ_{14}(t)=\lambda_1 e^{-(\beta+\lambda_1)t}\overline{G(t)}\,dt$, $dQ_{15}(t)=g(t)\,e^{-(\beta+\lambda_1)t}\,dt$, $dQ_{20}(t)=\beta_1 e^{-(\beta_1+\lambda)t}\,dt$,

$dQ_{23}(t)=\lambda e^{-(\beta_1+\lambda)t}\,dt$, $dQ_{31}(t)=\beta_1 e^{-(\beta_1+\lambda_1)t}\,dt$, $dQ_{36}(t)=\lambda_1 e^{-(\beta_1+\lambda_1)t}\,dt$, $dQ_{47}(t)=g(t)e^{-\beta t}dt$, $dQ_{48}(t)=\beta e^{-\beta t}\overline{G(t)}\,dt$,

$dQ_{57}(t)=\lambda_1 e^{-(\beta+\lambda_1)t}\,dt$, $dQ_{5,10}(t)=\beta e^{-(\beta+\lambda_1)t}\,dt$, $dQ_{69}(t)=\beta_1 e^{-\beta_1 t}\,dt$, $dQ_{70}(t)=g_1(t)e^{-(\beta+\lambda)t}dt$, $dQ_{7,11}(t)=\lambda e^{-(\beta+\lambda)t}\overline{G_1(t)}\,dt$,
$dQ_{7,12}(t)=\beta e^{-(\beta+\lambda)t}\overline{G_1(t)}\,dt$,

$dQ_{89}(t)=\beta_1 e^{-\beta_1 t}\,dt$, $dQ_{97}(t)=g(t)\,e^{-\beta t}\,dt$, $dQ_{98}(t)=\beta e^{-\beta t}\,\overline{G(t)}\,dt$, $dQ_{10,5}(t)=\beta_1 e^{-(\beta_1+\lambda_1)t}\,dt$, $dQ_{10,12}(t)=\lambda_1 e^{-(\beta_1+\lambda_1)t}\,dt$,
$dQ_{11,1}(t)=g_1(t)e^{-\beta t}dt$, $d\,Q_{11,6}(t)=\beta e^{-\beta t}\,\overline{G_1(t)}\,dt$, $dQ_{12,7}(t)=\beta_1 e^{-(\beta_1+\lambda)t}\,dt$, $dQ_{12,8}(t)=\lambda e^{-(\beta_1+\lambda)t}\,dt$

…(1)

Simple probabilistic considerations yield the following expressions for the non-zero elements

$p_{ij}=Q_{ij}(\infty)=\int_0^\infty q_{ij}(t)dt,$

We have

$p_{01}=\dfrac{\lambda}{\beta+\lambda}$, $p_{02}=\dfrac{\beta}{\beta+\lambda}$, $p_{13}=\dfrac{\beta}{\beta+\lambda_1}(1-g^*(\beta+\lambda_1))$, $p_{14}=\dfrac{\lambda_1}{\beta+\lambda_1}(1-g^*(\beta+\lambda_1))$,

$p_{15}=g^*(\beta+\lambda_1)$, $p_{20}=\dfrac{\beta_1}{\beta_1+\lambda}$, $p_{23}=\dfrac{\lambda}{\beta_1+\lambda}$, $p_{31}=\dfrac{\beta_1}{\beta_1+\lambda_1}$, $p_{36}=\dfrac{\lambda_1}{\beta_1+\lambda_1}$, $p_{47}=g^*(\beta)$,

$p_{48}=1-g^*(\beta)$, $p_{57}=\dfrac{\lambda_1}{\beta+\lambda_1}$, $p_{5,10}=\dfrac{\beta}{\beta+\lambda_1}$, $p_{69}=1$, $p_{70}=g_1^*(\beta+\lambda)$, $p_{7,11}=\dfrac{\lambda}{\beta+\lambda}(1-g_1^*(\beta+\lambda))$, $p_{7,12}=\dfrac{\beta}{\beta+\lambda}(1-g_1^*(\beta+\lambda))$,
$p_{89}=1$, $p_{97}=g^*(\beta)$, $p_{98}=1-g^*(\beta)$, $p_{10,5}=\dfrac{\beta_1}{\beta_1+\lambda_1}$, $p_{10,12}=\dfrac{\lambda_1}{\beta_1+\lambda_1}$, $p_{11,1}=g_1^*(\beta)$, $p_{11,6}=1-g^*(\beta)$, $p_{12,7}=\dfrac{\beta_1}{\beta_1+\lambda}$, $p_{12,8}=\dfrac{\lambda}{\beta_1+\lambda}$,
$p_{17.4}=\dfrac{\lambda_1}{\beta+\lambda_1}(1-g^*(\beta+\lambda_1))\,g^*(\beta)$,

$p_{17.4(8,9)}{}^{n}=\dfrac{\lambda_1}{\beta+\lambda_1}(1-g^*(\beta+\lambda_1))(1-g^*(\beta))$, $p_{37.69}=\dfrac{\lambda_1}{\beta_1+\lambda_1}g^*(\beta)$, $p_{37.6(9,8)}{}^{n}=\dfrac{\lambda_1}{\beta_1+\lambda_1}(1-g^*(\beta))$,
$p_{71.11}=\dfrac{\lambda}{\beta+\lambda}(1-g_1^*(\beta+\lambda))$

$g_1^*(\beta)$,

$p_{77.11,6(9,8)}{}^{n}=\dfrac{\lambda}{\beta+\lambda}(1-g_1^*(\beta+\lambda))(1-g_1^*(\beta))$, $p_{12,7.(8,9)}{}^{n}=\dfrac{\lambda}{\beta_1+\lambda}$

…(2)

It can be easily verified that

$p_{01}+p_{02}=p_{13}+p_{15}+p_{17.4}+p_{17.4(8,9)}{}^{n}=p_{20}+p_{23}=p_{31}+p_{37.69}+p_{37.6(9,8)}{}^{n}=p_{47}+p_{48}=p_{57}+p_{5,10}=p_{69}=1$

$p_{70}+p_{71.11}+p_{77.11,6(9,8)}{}^{n}+p_{7,12}=p_{89}=p_{97}+p_{98}=p_{10,5}+p_{10,12}=p_{11,1}+p_{11,6}=p_{12,7}+p_{12,7.(8,9)}{}^{n}=1$

…(3)

The mean sojourn times (μ_i) in the state S_i are

$\mu_0=m_{01}+m_{02}=\frac{1}{\beta+\lambda}$, $\mu_1= m_{13}+m_{14}+m_{15}=\frac{1}{\beta+\lambda_1}(1-g^*(\beta+\lambda_1))$, $\mu_2 = m_{20}+m_{23}=\frac{1}{\beta_1+\lambda}$,

$\mu_3= m_{31}+m_{36}=\frac{1}{\beta_1+\lambda_1}$, $\mu_4= m_{47}+m_{48}=\frac{1}{\beta}(1-g^*(\beta)$, $\mu_5= m_{57}+ m_{5,10}=\frac{1}{\beta+\lambda_1}$, $\mu_6= m_{69}=\frac{1}{\beta_1}$,

$\mu_7=m_{70}+m_{7,11}+m_{7,12}=\frac{1}{\beta+\lambda}(1-g_1^*(\beta+\lambda))$, $\mu_8=m_{89}=\frac{1}{\beta_1}$, $\mu_9=m_{97}+m_{98}=\frac{1}{\beta}(1-g^*(\beta))$,

$\mu_{10}=m_{10,5}+m_{10,12}=\frac{1}{\beta_1+\lambda_1}$, $\mu_{11}=m_{11,1}+m_{11,6}=\frac{1}{\beta}(1-g_1^*(\beta))$, $\mu_{12}=m_{12,7}+m_{12,8}=\frac{1}{\beta_1+\lambda}$

$$\mu'_1=m_{13}+m_{15}+m_{17.4}+m_{17.4,(8,9)^n}=\frac{(1-g^*(\beta+\lambda_1))(\beta\beta_1 g^*(\beta)+\lambda_1(\beta+\beta_1)(1-g^*(\beta)))}{\beta\beta_1(\beta+\lambda_1)g^*(\beta)},$$

$$\mu_3'=m_{31}+m_{37.69}+m_{37.6(9,8)^n}=\frac{\beta(\beta_1+\lambda_1)g^*(\beta)+\lambda_1(\beta+\beta_1)(1-g^*(\beta))}{\beta\beta_1(\beta_1+\lambda_1)g^*(\beta)},$$

$$\mu_7'=m_{70}+m_{71.11}+m_{77.11,6,9}+m_{77.11,6,(9,8)^n}+m_{7,12}=\frac{\beta\beta_1 g^*(\beta)(1-g_1^*(\beta+\lambda))((\beta\beta_1+\lambda(\beta_1(1-g_1^*(\beta))+\beta(1-g^*(\beta))))+\lambda(1-g^*(\beta))^2(\beta+\beta_1))}{\beta\beta_1(\beta+\lambda)g^*(\beta)},$$

$$\mu'_{12}=m_{12,7}+m_{12,7.(8,9)^n}=\frac{\beta(\beta_1+\lambda)g^*(\beta)+\lambda(\beta+\beta_1)(1-g^*(\beta))}{\beta\beta_1(\beta_1+\lambda)g^*(\beta)} \quad \ldots(4)$$

5. Steady state availability

Let $A_i(t)$ be the probability that the system is in up-state at instant 't' given that the system entered regenerative state S_i at t = 0.

The recursive relations for $A_i(t)$ are given as:

$A_0(t) =M_0(t) +q_{01}(t) © A_1(t) + q_{02}(t) © A_2(t)$

$A_1(t) =M_1(t) +q_{13}(t)©A_3(t)+ q_{15}(t) © A_5(t) +(q_{17.4}(t)+q_{17.4,(8,9)^n}(t)) ©A_7(t)$

$A_2(t) =M_2(t)+q_{20}(t)©A_0(t)+q_{23}(t)©A_3(t)$

$A_3(t)=M_3(t)+q_{31}(t)©A_1(t)+(q_{37.69}(t)+q_{37.6,(9,8)^n}(t))©A_7(t)$

$A_5(t)=M_5(t)+q_{57}(t)©A_7(t)+q_{5,10}(t)©A_{10}(t)$

$A_7(t)=M_7(t)+q_{70}(t)©A_0(t)+q_{71.11}(t)©A_1(t)+(q_{77.11,6,9}(t)+q_{77.11,6,(9,8)^n}(t))©A_7(t)+q_{7,12}(t)©A_{12}(t)$

$A_{10}(t)=M_{10}(t)+q_{10,5}(t)©A_5(t)+q_{10,12}(t)©A_{12}(t)$

$A_{12}(t)=M_{12}(t)+(q_{12,7}(t)+q_{12,7.(8,9)^n}(t))©A_7(t)$ …(5)

Where $M_i(t)$ is the probability that the system is up initially in state $S_i \in E$ is up at time t without visiting to any other regenerative state, we have

$M_0(t)=e^{-(\beta+\lambda)t}$, $M_1(t)=e^{-(\beta+\lambda_1)t}\overline{G(t)}$, $M_2(t)=e^{-(\beta_1+\lambda)t}$, $M_3(t) = e^{-(\beta_1+\lambda_1)t}$, $M_5(t)=e^{-(\beta+\lambda_1)t}$,

$M_7(t)=e^{-(\beta+\lambda)t}\overline{G_1(t)}$, $M_{10}(t)=e^{-(\beta_1+\lambda_1)t}$, $M_{12}(t)=e^{-(\beta_1+\lambda)t}$ …(6)

Taking L.T. of above relations (5) and (6) and solving for $A_0^*(s)$, we have

$$A_0^*(s)=\frac{\begin{array}{l}((1-q_{13}^*(s)q_{31}^*(s))(M_0^*(s)+q_{02}^*(s)M_2^*(s))+M_1^*(s)(q_{01}^*(s)+q_{02}^*(s)q_{23}^*(s)q_{31}^*(s))\\ +M_3^*(s)(q_{02}^*(s)q_{23}^*(s)+q_{01}^*(s)q_{13}^*(s)))(1-q_{10,5}^*(s)q_{5,10}^*(s))(1-q_{77.11,6(9,8)^n}^*(s)-\\ (q_{12,7}^*(s)+q_{12,7.(8,9)^n}^*(s))q_{7,12}^*(s))-q_{71.11}^*(s)q_{13}^*(s)q_{37.6(9,8)^n}^*(s)(M_0^*(s)+q_{02}^*(s)M_2^*(s))\\ (1-q_{10,5}^*(s)q_{5,10}^*(s))-q_{71.11}^*(s)(M_0^*(s)+q_{02}^*(s)M_2^*(s)+M_3^*(s)q_{02}^*(s)q_{23}^*(s))\\ (q_{15}^*(s)(q_{57}^*(s)+q_{5,10}^*(s)q_{10,12}^*(s)(q_{12,7}^*(s)+q_{12,7.(8,9)^n}^*(s)))+(q_{17.4}^*(s)\\ +q_{17.4(8,9)^n}^*(s))(1-q_{10,5}^*(s)q_{5,10}^*(s)))+(q_{37.6(9,8)^n}^*(s)(q_{02}^*(s)q_{23}^*(s)+q_{01}^*(s)q_{13}^*(s))\\ +(q_{17.4}^*(s)+q_{17.4(8,9)^n}^*(s))(q_{01}^*(s)+q_{02}^*(s)q_{23}^*(s)q_{31}^*(s)))(M_7^*(s)+M_{12}^*(s)q_{7,12}^*(s))\\ (1-q_{10,5}^*(s)q_{5,10}^*(s))+q_{15}^*(s)(q_{01}^*(s)+q_{02}^*(s)q_{23}^*(s)q_{31}^*(s))(M_5^*(s)(1-q_{77.11,6(9,8)^n}^*(s)\\ -(q_{12,7}^*(s)+q_{12,7.(8,9)^n}^*(s))q_{7,12}^*(s))+q_{5,7}^*(s)(M_7^*(s)+M_{12}^*(s)q_{7,12}^*(s))+q_{5,10}^*(s)(\\ M_7^*(s)(q_{12,7}^*(s)+q_{12,7.(8,9)^n}^*(s))q_{10,12}^*(s)+(1-q_{77.11,6(9,8)^n}^*(s))(M_{10}^*(s)+M_{12}^*(s)q_{10,12}^*(s))\\ -q_{7,12}^*(s)M_{10}^*(s)(q_{12,7}^*(s)+q_{12,7.(8,9)^n}^*(s))))+q_{71.11}^*(s)q_{02}^*(s)q_{23}^*(s)q_{37.6(9,8)^n}^*(s)\\ (M_1^*(s)(1-q_{5,10}^*(s)q_{10,5}^*(s))+q_{15}^*(s)(M_5^*(s)+q_{5,10}^*(s)(M_{10}^*(s)+M_{12}^*(s)q_{10,12}^*(s))))\end{array}}{\begin{array}{l}((1-q_{13}^*(s)q_{31}^*(s))(1-q_{02}^*(s)q_{20}^*(s))(1-q_{10,5}^*(s)q_{5,10}^*(s))(1-q_{77.11,6(9,8)^n}^*(s)\\ -(q_{12,7}^*(s)+q_{12,7.(8,9)^n}^*(s))q_{7,12}^*(s))-q_{71.11}^*(s)q_{13}^*(s)q_{37.6(9,8)^n}^*(s)(1-q_{02}^*(s)q_{20}^*(s))\\ (1-q_{10,5}^*(s)q_{5,10}^*(s))-q_{71.11}^*(s)(1-q_{02}^*(s)q_{20}^*(s))((q_{15}^*(s)(q_{57}^*(s)+q_{5,10}^*(s)q_{10,12}^*(s)\\ (q_{12,7}^*(s)+q_{12,7.(8,9)^n}^*(s)))+(q_{17.4}^*(s)+q_{17.4(8,9)^n}^*(s))(1-q_{10,5}^*(s)q_{5,10}^*(s)))-q_{70}^*(s)\\ (q_{37.6(9,8)^n}^*(s)(q_{02}^*(s)q_{23}^*(s)+q_{01}^*(s)q_{13}^*(s))+(q_{17.4}^*(s)+q_{17.4(8,9)^n}^*(s))\\ (q_{01}^*(s)+q_{02}^*(s)q_{23}^*(s)q_{31}^*(s)))(1-q_{5,10}^*(s)q_{10,5}^*(s))-q_{70}^*(s)q_{15}^*(s)(q_{01}^*(s)+\\ q_{02}^*(s)q_{23}^*(s)q_{31}^*(s))(q_{57}^*(s)+q_{5,10}^*(s)q_{10,12}^*(s)(q_{12,7}^*(s)+q_{12,7.(8,9)^n}^*(s)))\end{array}}$$

The steady state availability is given by

$$A_0(\infty)=\lim_{s\to 0} sA_0^*(s)=\frac{N_2}{D_2}$$

Where

$N_2=(1-p_{5,10}p_{10,5})(1-p_{76.11}-p_{7,12})((\mu_0+\mu_2p_{02})(1-p_{13}p_{31})+\mu_1(p_{01}+p_{02}p_{23}p_{31})+\mu_3(p_{02}p_{23}+p_{01}p_{13}))+\mu_5p_{15}(p_{01}+p_{02}p_{23}p_{31})$

$(1-p_{71.11}-p_{7,12})+p_{36}p_{02}p_{23}p_{15}p_{71.11}(p_{5,10}(\mu_{10}+\mu_{12}p_{10,12})+\mu_5)+(\mu_7(p_{36}(p_{01}p_{13}+p_{02}p_{23})+p_{14}(p_{01}+p_{02}p_{23}p_{31}))+$

$p_{71.11}(\mu_1p_{36}p_{02}p_{23}-p_{14}(\mu_0+\mu_2p_{02}+\mu_3p_{02}p_{23})-p_{36}p_{13}(\mu_0+\mu_2p_{02})))(1-p_{5,10}p_{10,5})+p_{71.11}(p_{5,10}-p_{57})(p_{15}(\mu_0+\mu_2p_{02}+\mu_3p_{02}p_{23})$

$+\mu_{12}(1-p_{5,10}p_{10,5})p_{7,12}(p_{36}(p_{01}p_{13}+p_{02}p_{23})+p_{14}(p_{01}+p_{02}p_{23}p_{31}))+p_{15}p_{57}(p_{01}+p_{02}p_{23}p_{31})(\mu_7+p_{7,12}\mu_{12})+p_{15}p_{5,10}(p_{01}+p_{02}p_{23}p_{31})$

$(\mu_7p_{10,12}+(1-p_{76.11})(\mu_{10}+p_{10,12}\mu_{12})-p_{7,12}\mu_{10})$ and

$D_2=(1-p_{5,10}p_{10,5})(1-p_{76.11}-p_{7,12})((\mu_0+\mu_2p_{02})(1-p_{13}p_{31})+\mu_1'(p_{01}+p_{02}p_{23}p_{31})+\mu_3'(p_{02}p_{23}+p_{01}p_{13})+\mu_5p_{15}(p_{01}+p_{02}p_{23}p_{31})(1-p_{71.11}-p_{7,12})+p_{36}p_{02}p_{23}p_{15}p_{71.11}(p_{5,10}(\mu_{10}+\mu_{12}'p_{10,12})+\mu_5)+(\mu_7'(p_{36}(p_{01}p_{13}+p_{02}p_{23})+p_{14}(p_{01}+p_{02}p_{23}p_{31}))+p_{71.11}(\mu_1'p_{36}p_{02}p_{23}-p_{14}(\mu_0+\mu_2p_{02}+\mu_3'p_{02}p_{23})-{}_{36}p_{13}(\mu_0+\mu_2p_{02})(1-p_{5,10}p_{10,5})+p_{71.11}(p_{5,10}-p_{57})(p_{15}(\mu_0+\mu_2p_{02}+\mu_3'p_{02}p_{23})+\mu_{12}'(1-p_{5,10}p_{10,5})p_{7,12}(p_{36}(p_{01}p_{13}+p_{02}p_{23})+p_{14}(p_{01}+p_{02}p_{23}p_{31}))+p_{15}p_{57}(p_{01}+p_{02}p_{23}p_{31})(\mu_7'+p_{7,12}\mu_{12}')+p_{15}p_{5,10}(p_{01}+p_{02}p_{23}p_{31})(\mu_7'p_{10,12}+(1-p_{76.11})$

$(\mu_{10}+p_{10,12}\mu_{12}')-p_{7,12}\mu_{10})$

…(7)

6. Busy period analysis of the server

Let $B_i(t)$ be the probability that the server is busy in repairing the unit at an instant 't' given that the system entered regenerative state S_i at t=0.The recursive relations for $B_i(t)$ are as follows:

$B_0(t) = q_{01}(t) © B_1(t) + q_{02}(t) © B_2(t)$

$B_1(t) = W_1(t) + q_{13}(t)©B_3(t)+ q_{15}(t) © B_5(t) +(q_{17.4}(t)+q_{17.4,(8,9)^n}(t)) ©B_7(t)$

$B_2(t) = q_{20}(t)©B_0(t)+q_{23}(t)©B_3(t)$

$B_3(t)=q_{31}(t)©B_1(t)+(q_{37.69}(t)+q_{37.6,(9,8)^n}(t))©B_7(t)$

$B_5(t)=q_{57}(t)©B_7(t)+q_{5,10}(t)©B_{10}(t)$

$B_7(t)=W_7(t)+q_{70}(t)©B_0(t)+q_{71.11}(t)©B_1(t)+(q_{77.11,6,9}(t)+q_{77.11,6,(9,8)^n}(t))©B_7(t)+q_{7,12}(t)©B_{12}(t)$

$B_{10}(t)=q_{10,5}(t)©B_5(t)+q_{10,12}(t)©B_{12}(t)$

$B_{12}(t)=(q_{12,7}(t)+q_{12,7.(8,9)^n}(t))©B_7(t)$

…(8)

Where

$W_i(t)$ be the probability that the server is busy in state S_i due to failure up to time t without making any transition to any other regenerative state or returning to the same via one or more non regenerative states so,

$$W_1(t)= e^{-(\beta+\lambda_1)t}\,\overline{G(t)} +(\lambda_1 e^{-(\beta+\lambda_1)t} ©1)\,\overline{G(t)}\ ,W_7(t)=e^{-(\beta+\lambda)t}\,\overline{G_1(t)} +(\lambda e^{-(\beta+\lambda)t}©1)\,\overline{G_1(t)} \quad …(9)$$

Taking L.T. of above relations (8, 9) and solving for $B_0^*(s)$,we obtain

$$B_0^*(s)=\frac{\begin{gathered}W_1^*(s)(q_{01}^*(s)+q_{02}^*(s)q_{23}^*(s)q_{31}^*(s))(1-q_{10,5}^*(s)q_{5,10}^*(s))(1-q_{77.11,6(9,8)^n}^*(s)\\-(q_{12,7}^*(s)+q_{12,7.(8,9)^n}^*(s))q_{7,12}^*(s))+(q_{37.6(9,8)^n}^*(s)(q_{02}^*(s)q_{23}^*(s)+\\q_{01}^*(s)q_{13}^*(s))+(q_{17.4}^*(s)+q_{17.4(8,9)^n}^*(s))(q_{01}^*(s)+q_{02}^*(s)q_{23}^*(s)q_{31}^*(s)))\\W_7^*(s)(1-q_{10,5}^*(s)q_{5,10}^*(s))+q_{15}^*(s)(q_{01}^*(s)+q_{02}^*(s)q_{23}^*(s)q_{31}^*(s))\\W_7^*(s)(q_{5,7}^*(s)+q_{5,10}^*(s)((q_{12,7}^*(s)+q_{12,7.(8,9)^n}^*(s))q_{10,12}^*(s))+q_{71.11}^*(s)\\q_{02}^*(s)q_{23}^*(s)q_{37.6(9,8)^n}^*(s)W_1^*(s)(1-q_{5,10}^*(s)q_{10,5}^*(s))\end{gathered}}{\begin{gathered}((1-q_{13}^*(s)q_{31}^*(s))(1-q_{02}^*(s)q_{20}^*(s))(1-q_{10,5}^*(s)q_{5,10}^*(s))(1-q_{77.11,6(9,8)^n}^*(s)\\-(q_{12,7}^*(s)+q_{12,7.(8,9)^n}^*(s))q_{7,12}^*(s))-q_{71.11}^*(s)q_{13}^*(s)q_{37.6(9,8)^n}^*(s)(1-q_{02}^*(s)q_{20}^*(s))\\(1-q_{10,5}^*(s)q_{5,10}^*(s))-q_{71.11}^*(s)(1-q_{02}^*(s)q_{20}^*(s))((q_{15}^*(s)(q_{57}^*(s)+q_{5,10}^*(s)q_{10,12}^*(s)\\(q_{12,7}^*(s)+q_{12,7.(8,9)^n}^*(s)))+(q_{17.4}^*(s)+q_{17.4(8,9)^n}^*(s))(1-q_{10,5}^*(s)q_{5,10}^*(s)))-q_{70}^*(s)\\(q_{37.6(9,8)^n}^*(s)(q_{02}^*(s)q_{23}^*(s)+q_{01}^*(s)q_{13}^*(s))+(q_{17.4}^*(s)+q_{17.4(8,9)^n}^*(s))\\(q_{01}^*(s)+q_{02}^*(s)q_{23}^*(s)q_{31}^*(s)))(1-q_{5,10}^*(s)q_{10,5}^*(s))-q_{70}^*(s)q_{15}^*(s)(q_{01}^*(s)+\\q_{02}^*(s)q_{23}^*(s)q_{31}^*(s))(q_{57}^*(s)+q_{5,10}^*(s)q_{10,12}^*(s)(q_{12,7}^*(s)+q_{12,7.(8,9)^n}^*(s)))\end{gathered}} \quad ...(10)$$

The time for which server is busy due to repair is given by

$$B_0^*(\infty)=\lim_{s\to 0} sB_0^*(s)=\frac{N_3}{D_2} \qquad \ldots(11)$$

$N_3=W_1^*(0)(1-p_{5,10}p_{10,5})((1-p_{76.11}-p_{7,12})(p_{01}+p_{02}p_{23}p_{31})+p_{02}p_{36}p_{23}p_{71.11})+$ $W_7^*(0)(p_{36}(p_{01}p_{13}+p_{02}p_{23})+(1-p_{13})(p_{01}+p_{02}p_{23}p_{31}))$ and D_2 is already mentioned.

…(12)

7. Expected number of visits by the server

Let $N_i(t)$ be the expected number of visits by the server in (0,t] given that the system entered the regenerative state S_i at t=0. The recursive relations for $N_i(t)$ are given as:

$N_0(t) = Q_{01}(t)\circledS(1+N_1(t)) + Q_{02}(t)\circledS N_2(t)$

$N_1(t) = Q_{13}(t)\circledS N_3(t) + Q_{15}(t)\circledS N_5(t) + Q_{17.4}(t)\circledS N_7(t) + Q_{17.4,(8,9)^n}(t)\circledS(1+N_7(t))$

$N_2(t) = Q_{20}(t)\circledS N_0(t) + Q_{23}(t)\circledS N_3(t)$

$N_3(t)=Q_{31}(t)\circledS(1+N_1(t))+(Q_{37.69}(t)+Q_{37.6,(9,8)^n}(t))\circledS(1+N_7(t))$

$N_5(t)=Q_{57}(t)\circledS(1+N_7(t))+Q_{5,10}(t)\circledS N_{10}(t)$

$N_7(t) = Q_{70}(t)\circledS N_0(t)+Q_{71.11}(t)\circledS N_1(t)+(Q_{77.11,6,9}(t)+Q_{77.11,6,(9,8)^n}(t))\circledS(1+N_7(t))+Q_{7,12}(t)\circledS N_{12}(t)$

$N_{10}(t)=Q_{10,5}(t)\circledS N_5(t)+Q_{10,12}(t)\circledS N_{12}(t)$

$N_{12}(t)=(Q_{12,7}(t)+Q_{12,7.(8,9)^n}(t))\circledS(1+N_7(t))$ … (13)

Taking L.S.T. of relations (13) and solving for $N_0^{**}(s)$,we have

$$N_0^{**}(s)=\frac{\begin{gathered}((1-Q_{13}^{**}(s)Q_{31}^{**}(s))Q_{01}^{**}(s)+Q_{17.4(8,9)^n}^{**}(s)(Q_{01}^{**}(s)+Q_{02}^{**}(s)Q_{23}^{**}(s)Q_{31}^{**}(s)) \\ (Q_{31}^{**}(s)+Q_{37.6(9,8)^n}^{**}(s))(Q_{02}^{**}(s)Q_{23}^{**}(s)+Q_{01}^{**}(s)Q_{13}^{**}(s)))(1-Q_{10,5}^{**}(s) \\ Q_{5,10}^{**}(s))(1-Q_{77.11,6(9,8)^n}^{**}(s)-(Q_{12,7}^{**}(s)+Q_{12,7.(8,9)^n}^{**}(s))Q_{7,12}^{**}(s))-Q_{01}^{**}(s) \\ Q_{37.6(9,8)^n}^{**}(s)Q_{71.11}^{**}(s)Q_{13}^{**}(s)(1-Q_{10,5}^{**}(s)Q_{5,10}^{**}(s))-Q_{71.11}^{**}(s)(Q_{01}^{**}(s)+(Q_{31}^{**}(s) \\ +Q_{37.6(9,8)^n}^{**}(s))Q_{02}^{**}(s)Q_{23}^{**}(s))(Q_{15}^{**}(s)(Q_{57}^{**}(s)+Q_{5,10}^{**}(s)Q_{10,12}^{**}(s)(Q_{12,7}^{**}(s)+ \\ Q_{12,7.(8,9)^n}^{**}(s)))+(Q_{17.4}^{**}(s)+Q_{17.4(8,9)^n}^{**}(s))(1-Q_{5,10}^{**}(s)Q_{10,5}^{**}(s)))+(Q_{37.6(9,8)^n}^{**}(s) \\ (Q_{02}^{**}(s)Q_{23}^{**}(s)+Q_{01}^{**}(s)Q_{13}^{**}(s))+(Q_{17.4}^{**}(s)+Q_{17.4(8,9)^n}^{**}(s))(Q_{01}^{**}(s)+Q_{02}^{**}(s)Q_{23}^{**}(s) \\ Q_{31}^{**}(s)))(Q_{77.11,6(9,8)^n}^{**}(s)+(Q_{12,7}^{**}(s)+Q_{12,7.(8,9)^n}^{**}(s))Q_{7,12}^{**}(s))(1-Q_{10,5}^{**}(s)Q_{5,10}^{**}(s)) \\ +Q_{15}^{**}(s)(Q_{01}^{**}(s)+Q_{02}^{**}(s)Q_{23}^{**}(s)Q_{31}^{**}(s))(Q_{57}^{**}(s)(1-Q_{77.11,6(9,8)^n}^{**}(s)-(Q_{12,7}^{**}(s)+ \\ Q_{12,7.(8,9)^n}^{**}(s))Q_{7,12}^{**}(s))+Q_{5,7}^{*}(s)(Q_{77.11,6(9,8)^n}^{**}(s)+(Q_{12,7}^{**}(s)+Q_{12,7.(8,9)^n}^{**}(s))Q_{7,12}^{**}(s)) \\ +Q_{5,10}^{**}(s)(Q_{77.11,6(9,8)^n}^{**}(s)(Q_{12,7}^{**}(s)+Q_{12,7.(8,9)^n}^{**}(s))Q_{10,12}^{**}(s))+(1-Q_{77.11,6(9,8)^n}^{**}(s)) \\ (Q_{12,7}^{**}(s)+Q_{12,7.(8,9)^n}^{**}(s))Q_{10,12}^{**}(s)))+Q_{71.11}^{**}(s)Q_{02}^{**}(s)Q_{23}^{**}(s)Q_{37.6(9,8)^n}^{**}(s)(Q_{17.4(8,9)^n}^{**}(s) \\ (1-Q_{5,10}^{**}(s)Q_{10,5}^{**}(s))+Q_{15}^{**}(s)(Q_{57}^{**}(s)+Q_{5,10}^{**}(s)Q_{10,12}^{**}(s)(Q_{12,7}^{**}(s)+Q_{12,7.(8,9)^n}^{**}(s)))\end{gathered}}{\begin{gathered}((1-Q_{13}^{**}(s)Q_{31}^{**}(s))(1-Q_{02}^{**}(s)Q_{20}^{**}(s))(1-Q_{10,5}^{**}(s)Q_{5,10}^{**}(s))(1-Q_{77.11,6(9,8)^n}^{**}(s) \\ -(Q_{12,7}^{**}(s)+Q_{12,7.(8,9)^n}^{**}(s))Q_{7,12}^{**}(s))-Q_{71.11}^{**}(s)Q_{13}^{**}(s)Q_{37.6(9,8)^n}^{**}(s)(1-Q_{02}^{**}(s)Q_{20}^{**}(s)) \\ (1-Q_{10,5}^{**}(s)Q_{5,10}^{**}(s))-Q_{71.11}^{**}(s)(1-Q_{02}^{**}(s)Q_{20}^{**}(s))((Q_{15}^{**}(s)(Q_{57}^{**}(s)+Q_{5,10}^{**}(s)Q_{10,12}^{**}(s) \\ (Q_{12,7}^{**}(s)+Q_{12,7.(8,9)^n}^{**}(s)))+(Q_{17.4}^{**}(s)+Q_{17.4(8,9)^n}^{**}(s))(1-Q_{10,5}^{**}(s)Q_{5,10}^{**}(s)))-Q_{70}^{**}(s) \\ (Q_{37.6(9,8)^n}^{**}(s)(Q_{02}^{**}(s)Q_{23}^{**}(s)+Q_{01}^{**}(s)Q_{13}^{**}(s))+(Q_{17.4}^{**}(s)+Q_{17.4(8,9)^n}^{**}(s)) \\ (Q_{01}^{**}(s)+Q_{02}^{**}(s)Q_{23}^{**}(s)Q_{31}^{**}(s)))(1-Q_{5,10}^{**}(s)Q_{10,5}^{**}(s))-Q_{70}^{**}(s)Q_{15}^{**}(s)(Q_{01}^{**}(s)+ \\ Q_{02}^{**}(s)Q_{23}^{**}(s)Q_{31}^{**}(s))(Q_{57}^{**}(s)+Q_{5,10}^{**}(s)Q_{10,12}^{**}(s)(Q_{12,7}^{**}(s)+Q_{12,7.(8,9)^n}^{**}(s)))\end{gathered}} \qquad \ldots(14)$$

The expected numbers of visits per unit time by the server are given by

$$N_0(\infty)=\lim_{s\to 0} sN_0^{**}(s)=\frac{N_4}{D_2} \quad \ldots(15)$$

Where

$N_4=(1-p_{76.11}-p_{7,12})((1-p_{5,10}p_{10,5})(p_{01}(1-p_{13}p_{31})+p_{17.4(8,9)}{}^{n}(p_{01}+p_{02}p_{23}p_{31})+p_{02}p_{23}+p_{01}p_{13})$

$+p_{15}p_{57}(p_{01}+p_{02}p_{23}p_{31}))+p_{36}p_{02}p_{23}p_{15}p_{71.11}(p_{5,10}p_{10,12}+p_{57})+p_{7,11}p_{11,6}(p_{36}(p_{01}p_{13}+p_{02}p_{23})$

$+p_{14}(p_{01}+p_{02}p_{23}p_{31}))+p_{71.11}(p_{17.4(8,9)}{}^{n}p_{36}p_{02}p_{23}-p_{14}(p_{01}+p_{02}p_{23})-p_{36}p_{13}p_{01}))(1-p_{5,10}p_{10,5})$

$-p_{71.11}(p_{15}(p_{01}+p_{02}p_{23}))-(1-p_{5,10}p_{10,5})p_{7,12}(p_{36}(p_{01}p_{13}+p_{02}p_{23})+p_{14}(p_{01}+p_{02}p_{23}p_{31}))$

$+p_{15}p_{57}(p_{01}+p_{02}p_{23}p_{31})(1-p_{7,11}-p_{71.11}-p_{76.11})p_{10,12}$ and D_2 is already specified.

…(16)

8. Profit analysis

The profit incurred to the system Model in steady state can be obtained as

$P=K_0A_0-K_1B_0-K_2N_0$

Where

K_0=Revenue per unit up-time of the system

K_1=Cost per unit for which server is busy

K_2= Cost per unit visit by the server and A_0, B_0, N_0 are already defined.

9. Particular cases

Suppose $g(t)=\alpha e^{-\alpha t}$, $g_1(t)=\alpha_1 e^{-\alpha_1 t}$

By using the non-zero elements p_{ij},we can obtain the following results:

$p_{01}=\frac{\lambda}{\beta+\lambda}$, $p_{02}=\frac{\beta}{\beta+\lambda}$, $p_{13}=\frac{\beta}{\alpha+\beta+\lambda_1}$, $p_{14}=\frac{\lambda_1}{\alpha+\beta+\lambda_1}$, $p_{15}=\frac{\alpha}{\alpha+\beta+\lambda_1}$, $p_{20}=\frac{\beta_1}{\beta_1+\lambda}$,

$p_{23}=\frac{\lambda}{\beta_1+\lambda}$, $p_{31}=\frac{\beta_1}{\beta_1+\lambda_1}$, $p_{36}=\frac{\lambda_1}{\beta_1+\lambda_1}$, $p_{47}=\frac{\alpha}{\alpha+\beta}$, $p_{48}=\frac{\beta}{\alpha+\beta}$, $p_{57}=\frac{\lambda_1}{\beta+\lambda_1}$, $p_{5,10}=\frac{\beta}{\beta+\lambda_1}$, $p_{69}=1$,

$p_{70}=\frac{\alpha_1}{\alpha_1+\beta+\lambda}$, $p_{7,11}=\frac{\lambda}{\alpha_1+\beta+\lambda}$, $p_{7,12}=\frac{\beta}{\alpha_1+\beta+\lambda}$, $p_{89}=1$, $p_{97}=\frac{\alpha}{\alpha+\beta}$,

$p_{98}=\frac{\beta}{\alpha+\beta}$, $p_{10,5}=\frac{\beta_1}{\beta_1+\lambda_1}$, $p_{10,12}=\frac{\lambda_1}{\beta_1+\lambda_1}$, $p_{11,1}=\frac{\alpha_1}{\alpha_1+\beta}$, $p_{11,6}=\frac{\beta}{\alpha_1+\beta}$, $p_{12,7}=\frac{\beta_1}{\beta_1+\lambda}$, $p_{12,8}=\frac{\lambda}{\beta_1+\lambda}$,

$p_{17.4}=\frac{\lambda_1}{\alpha+\beta+\lambda_1}\ \frac{\alpha}{\alpha+\beta}$, $p_{17.4(8,9)}{}^{n}=\frac{\lambda_1}{\alpha+\beta+\lambda_1}\ \frac{\beta}{\alpha+\beta}$, $p_{37.69}=\frac{\lambda_1}{\beta_1+\lambda_1}\ \frac{\alpha}{\alpha+\beta}$, $p_{37.6(9,8)}{}^{n}=\frac{\lambda_1}{\beta_1+\lambda_1}\ \frac{\beta}{\alpha+\beta}$,

$p_{71.11}=\frac{\lambda}{\alpha_1+\beta+\lambda}\ \frac{\alpha_1}{\alpha_1+\beta}$, $p_{77.11,6(9,8)}{}^{n}=\frac{\lambda}{\alpha_1+\beta+\lambda}\ \frac{\beta}{\alpha_1+\beta}$, $p_{12,7.(8,9)}{}^{n}=\frac{\lambda}{\beta_1+\lambda}$

$\mu_0=\frac{1}{\beta+\lambda}$, $\mu_1=\frac{1}{\alpha+\beta+\lambda_1}$, $\mu_2=\frac{1}{\beta_1+\lambda}$, $\mu_3=\frac{1}{\beta_1+\lambda_1}$, $\mu_4=\frac{1}{\alpha+\beta}$, $\mu_5=\frac{1}{\beta+\lambda_1}$, $\mu_6=\frac{1}{\beta_1}$,

$\mu_7=\frac{1}{\alpha_1+\beta+\lambda}$, $\mu_8=\frac{1}{\beta_1}$, $\mu_9=\frac{1}{\alpha+\beta}$, $\mu_{10}=\frac{1}{\beta_1+\lambda_1}$, $\mu_{11}=\frac{1}{\alpha_1+\beta}$, $\mu_{12}=\frac{1}{\beta_1+\lambda}$,

$$\mu_1' = \frac{\alpha\beta_1 + \lambda_1(\beta+\beta_1)}{\alpha\beta_1(\alpha+\beta+\lambda_1)}, \mu_3' = \frac{\alpha(\beta_1+\lambda_1)+\lambda_1(\beta+\beta_1)}{\alpha\beta_1(\beta_1+\lambda_1)}, \mu_7' = \frac{\alpha\beta_1(\alpha_1+\beta)+\lambda(\alpha+\beta)(\beta+\beta_1)}{\alpha\beta_1(\alpha_1+\beta)(\alpha_1+\beta+\lambda)},$$

$$\mu_{12}' = \frac{\alpha\beta_1+\lambda(\alpha+\beta+\beta_1)}{\alpha\beta_1(\beta_1+\lambda)}, W^*_1(0) = \frac{(\alpha+\lambda_1)}{\alpha(\alpha+\beta+\lambda_1)}, W^*_7(0) = \frac{(\alpha_1+\lambda)}{\alpha_1(\alpha_1+\beta+\lambda)}$$

Steady state availability (A_0) = $\dfrac{N_2}{D_2}$,

Busy period of the server (B_0) = $\dfrac{N_3}{D_2}$,

Expected number of visits by the server (N_0) = $\dfrac{N_4}{D_2}$

Where

$N_2=\alpha\beta_1(\alpha_1\lambda_1(\beta+\beta_1+\lambda_1)(\beta_1+\lambda)(\alpha_1+\beta+\lambda)((\beta+\beta_1+\lambda)E+\lambda(B+\beta C)+\alpha\alpha_1\lambda\beta(B-\beta\beta_1)(\alpha_1+\beta+\lambda)+\alpha\alpha_1\beta\lambda^2\lambda_1(\beta(\beta_1+\lambda+\lambda_1)+(B-\beta\beta_1))+\lambda\lambda_1^2(\beta+\beta_1+\lambda_1)((\alpha_1+\beta)(\beta C+B)-\alpha_1(\beta+\beta_1+\lambda)(\beta+\beta_1+\lambda_1))(\beta_1+\lambda)+\alpha\alpha_1\lambda(\beta_1-\lambda_1)((\beta+\beta_1+\lambda)(\beta_1+\lambda_1)+\beta\lambda)(B-\beta\beta_1)+\beta\lambda\lambda_1^2(\beta_1+\lambda_1+\beta)(\alpha_1+\beta)(\beta C+B)+\alpha\lambda\lambda_1B(\beta+\beta_1+\lambda)(\alpha_1+\beta)(\beta_1+\lambda_1)+\alpha\beta\lambda B((\beta_1+\lambda)((\alpha_1+\beta)+\alpha_1(\alpha_1+\beta+\lambda))+(\alpha_1+\beta)^2+\alpha_1\lambda)\lambda_1))$

$D_2=\alpha_1\lambda_1(\beta_1+\lambda)(\alpha_1+\beta+\lambda)(\beta+\beta_1+\lambda_1)(\alpha\beta_1(\beta+\beta_1+\lambda)E)+\lambda(\alpha\beta_1+\lambda_1(\beta+\beta_1))B+\beta\lambda(\alpha(\beta_1+\lambda_1)+\lambda_1(\beta+\beta 1))C+\alpha^2\alpha_1\lambda\beta_1(B-\beta\beta_1)(\alpha_1+\beta+\lambda)B+\alpha\alpha_1\beta\lambda^2\lambda_1(\alpha\beta_1(\beta_1+\lambda)(\beta+\beta_1+\lambda_1)+\beta\lambda_1(\alpha\beta_1+\lambda(\alpha+\beta+\beta_1)))+\lambda\lambda_1^2(\beta+\beta_1+\lambda_1)(\beta_1+\lambda)((\alpha\beta_1(\alpha_1+\beta+\lambda)+\beta\lambda(\alpha+\beta+\beta_1))(\beta C+B)-\alpha_1(\beta\beta_1(\beta+\beta_1)+\beta_1(\beta_1+\lambda_1)(\beta_1+\beta+\lambda)+\beta\lambda(\beta_1+\lambda_1)))+\alpha\alpha_1\lambda(\beta-\lambda_1)(\alpha\beta_1(\beta_1+\lambda_1)(\beta+\beta_1+\lambda)+\beta\lambda(\alpha(\beta_1+\lambda_1)+\lambda_1(\beta+\beta_1)))(B-\beta\beta_1)+\beta\lambda\lambda_1^2(\alpha_1+\beta)(\beta+\beta_1+\lambda_1)(\alpha\beta_1+\lambda(\alpha+\beta+\beta_1)))(\beta C+B)+\alpha\lambda\lambda_1B((\alpha\beta_1(\alpha_1+\beta+\lambda)+\beta\lambda(\alpha+\beta+\beta_1))(\beta_1+\lambda)+\beta(\alpha\beta_1+\lambda(\alpha+\beta+\beta_1))(\alpha_1+\beta))(\beta_1+\lambda_1)+\alpha\beta\lambda B((\beta_1+\lambda)(\lambda_1(\alpha\beta_1(\alpha_1+\beta+\lambda)+\beta\lambda(\alpha+\beta+\beta_1))+\alpha\alpha_1\beta_1(\alpha_1+\beta+\lambda))+\lambda_1((\alpha_1+\beta)^2+\alpha_1\lambda)(\alpha\beta_1+\lambda(\alpha+\beta+\beta_1)))$

$$N_3 = \frac{\begin{gathered}\lambda\lambda_1\alpha_1^2(\alpha+\lambda_1)(\beta+\beta_1+\lambda_1)(B(\alpha_1+\beta+\lambda)+\lambda\lambda_1\beta)+\lambda(\alpha_1+\lambda)(\beta\lambda_1C+(\alpha+\lambda_1)B)\\ \alpha\beta_1(\alpha_1+\beta)(\beta+\lambda_1)(B-\beta\beta_1)\end{gathered}}{\alpha_1}$$

$$N_4 = \frac{\begin{gathered}\alpha\beta_1(\beta_1+\lambda)\lambda\lambda_1\alpha_1(\beta+\beta_1+\lambda_1)(E(\beta_1+\lambda)(\alpha+\beta)+\beta\lambda B+\beta(B-\beta\beta_1)C)\\ +\alpha\alpha_1\lambda\lambda_1B+\lambda_1^2\lambda^2\beta\alpha\alpha_1(\beta+\beta_1+\lambda_1)+\lambda_1(\beta+\beta_1+\lambda_1)(\beta\lambda\lambda_1(\alpha+\beta)(\beta(\beta_1+\lambda)C+\\ B\lambda)+\alpha_1\lambda^2\lambda_1(\beta^2\lambda_1-(\alpha+\beta)(\beta_1+\lambda_1)(\beta+\beta_1+\lambda)-\beta(\alpha+\beta)(\beta_1+\lambda)))+\alpha\alpha_1\lambda^2(\beta-\lambda_1)\\ (\beta+\beta_1+\lambda)+\alpha\beta\lambda\lambda_1(B(\lambda(\beta_1+\lambda_1)+(\alpha_1+\beta)D)+\beta\lambda\lambda_1^2(\beta+\beta_1+\lambda_1)(\beta C+B)))\end{gathered}}{(\alpha+\beta)}$$

$A=(\alpha_1+\lambda)(\beta_1+\lambda)+\beta\lambda$

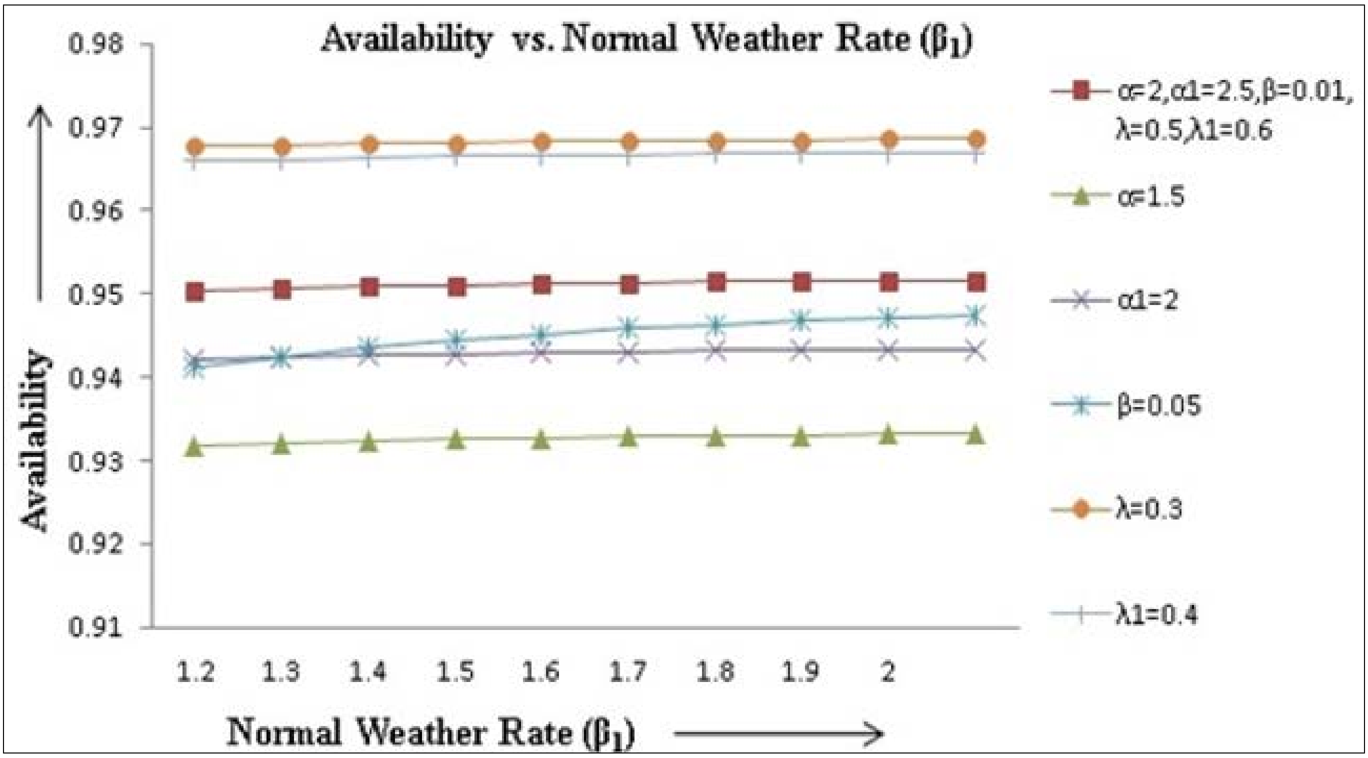
$B=(\beta_1+\lambda)(\beta_1+\lambda_1)+\beta\beta_1$

$C=\alpha+\beta+\lambda+\beta_1+\lambda_1$

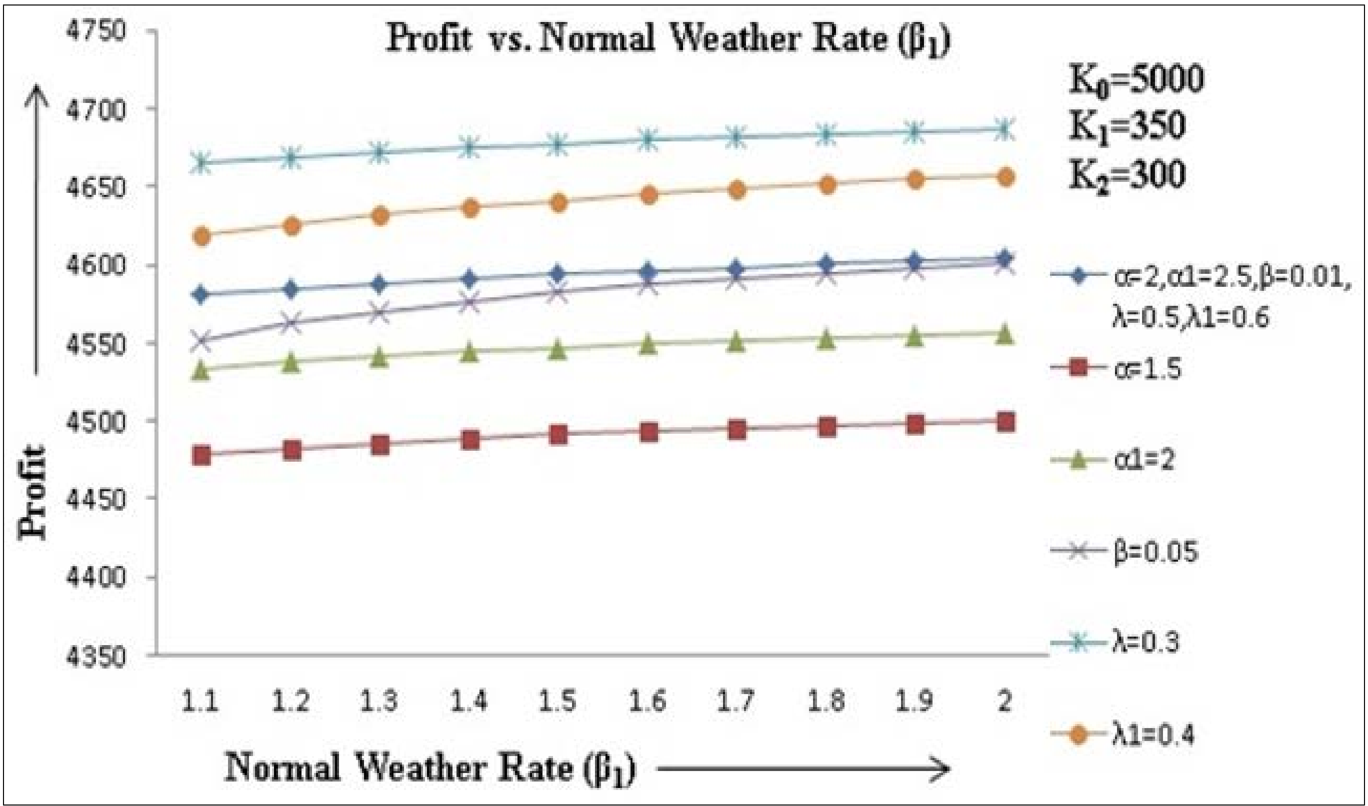
$D=\alpha_1+\beta_1+\lambda_1+\beta+\lambda$

$E=(\alpha+\lambda_1)(\beta_1+\lambda_1)+\beta\lambda_1$

10. Graphs



Graph 1: Availability vs. Normal Weather Rate (β_1)



Graph 2: Profit vs. Normal Weather Rate (β_1)

11. Conclusion

The graphs of the Availability and profit function of the system are drawn for fixed values of the various parameters and costs as shown respectively in figures 1, 2. These figures indicate that there is substantial positive change in these measures with increase of normal weather rate (β1) and repair rates (α and α1) of the units. While their values decline with increase of abnormal weather rate (β) and failure rates (λ and λ1) of the units. However, the effect of repair rate of the main unit is more. On the basis of the above results, it can be concluded that the system can be made more profitable to use by increasing repair rate of the both units.

12. References

1. Chander S. Reliability Models with Priority for Operation and Repair with Arrival Time of Server. Pure and Applied Mathematika Sciences. 2005;LXI(1-2):9-22.
2. Malik SC, Barak MS. Reliability and Economic Analysis of a System Operating under Different Weather Conditions. Journal of Proc. Mat. Acad. Sci, India, Sect. A. 2009;79(II):205-213.
3. Malik S, Deswal S. Stochastic Analysis of a Repairable System of Non-identical Units with Priority for Operation and Repair Subject to Weather Conditions. International Journal of Computer Applications. 2012;49(14):0975-8887.
4. Deswal S, Malik S, Sureria JK. Stochastic Analysis of a System of Non-Identical Units with no Repair Activity in Abnormal Weather, International Journal of Agricultural and Statistical Sciences. 2013;(Supplement 1):193-201
5. Deswal S, Malik S. Reliability Measures of A System of two Nonidentical Units With Priority Subject To Weather Conditions. Journal of Reliability and Statistical Studies. 2015;8(1):181-190. ISSN (Print): 0974-8024, (Online):2229-5666.
6. Malik S. Stochastic Modeling of a Repairable System Under Different Weather Conditions. Recent Advances in Mathematics, Statistics and Computer Science; c2016. p. 155-163.
7. Ashok K, Dheeraj P, Malik CS. Weathing server system with non-identical units and priority to repair of main unit. Journal of Advanced research in Dynamical and Control system; c2019.
8. Deswal S. Cost Benefit Analysis of Reliability Models under Diverse Climatic Surroundings. IJRAR, 2019, 6(2).
9. Kumar A, Pawar D, Malik SC. Reliability analysis of a redundant System with ‘FCFS’ repair policy subject to weather conditions International Journal of Advanced Science and Technology. 2020;29(3):7568-7578.