Economic analysis of system reliability model under operation in changing weather

Dr. Savita Deswal

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Abstract

The main concentration of the present work is to evaluate and analyze economic nature of a system reliability model which is allowed to operate under varying weather conditions – normal and abnormal. In this context a system reliability model consisting of two non-identical units is developed in which initially main unit is operative and the duplicate unit is at standby. Single repairmen/server comes immediately to rectify the fault in normal weather whereas in abnormal weather the repair is not feasible. The techniques used for the analysis are SMP & RPT.

Keywords: Economic analysis, varying weather conditions, repairable system, SMP, RPT

1. Introduction

The weather environment has a significant impact on the performance of the operating systems. The physical stresses created by adverse weather conditions deteriorate performance and efficiency of these systems. Besides, scientists and engineers have tried a lot to develop systems which can work in varying environmental conditions. And, the performance of such systems has been improved up to a considerable level by adopting better repair policies and redundancy techniques. Therefore, in the earlier study, stochastic Models of a system of non-identical units have been analyzed under two weather conditions - normal and abnormal. However, in these Models operation and repair activities are not allowed in abnormal weather. But, sometimes we may have emergency situations in which operation of the system becomes necessary irrespective of weather conditions. S Chander (2005) [1] have determined reliability measures of a single unit system with Priority for Operation and Repair with Arrival Time of Server. But, redundant systems and impact of weather on identical or non-identical units have not been studied so far by considering operation of the system in abnormal weather. Malik, S.C. and Barak, M.S (2009) [2] analyzed Reliability and Economic Analysis of a System Operating under Different Weather Conditions. Later Malik, S and Deswal, S (2012 – 2015) [3, 5] performed Stochastic Analysis of a Repairable System of Non-identical Units with Priority for Operation and Repair Subject to Weather Conditions. Deswal, S (2019) [8] measured the Cost Benefit Analysis of Reliability Models under Diverse Climatic Surroundings.

The purpose of the present study is not only to strengthen the existing literature on reliability but also to know the variations in reliability and economic measures of a system of non-identical units operating in different weather conditions. To meet out this objective a stochastic Model is developed under different set of assumptions on operation and repair policies. Initially, one original unit (called main unit) is operative and other substandard unit (called duplicate unit) is kept at spare in cold standby. Each unit has constant failure unit from normal mode. It is assumed that both units are capable of performing the same set of functions and activities but with different proficiencies. The system operates in two weather conditions - normal and abnormal. However, repair of the system is allowed only in normal weather by a server visits the system immediately as and when needed. The distribution of failure times of units and change of weather conditions are taken as negative exponential while that of repair times of the units follow arbitrary distributions. The units work as new after repair. All random variables are statistically independent. The semi-Markov and regenerative point technique are adopted to drive the expressions for the reliability measures such as availability, busy period of the server, expected number of visits by the server and profit function. The results are analyzed through graphs for particular values of various parameters and costs.
2. Notations

<table>
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<tr>
<th>Symbol</th>
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<td>The set of regenerative states</td>
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<td>MO/DO</td>
<td>Main/Duplicate unit is good and operative</td>
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<td>MO/DO</td>
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<td>MCs/DCs</td>
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<td>$M_i(t)$</td>
<td>Probability that the system is up initially in regenerative state $Si$ at time $t$ without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states</td>
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</tr>
<tr>
<td>$\mu_i$</td>
<td>The mean sojourn time in state $Si$ this is given by</td>
</tr>
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3. State Transition Diagram

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The diagram shows the state transition diagram of the regenerative system with various states and transitions. The states and transitions are labeled with their respective symbols and descriptions. The transitions are represented by arrows indicating the direction of state changes, with labels showing the conditions under which the transitions occur. The states are marked with specific symbols indicating their status (up-state, failed state, regenerative point).
4. Transition probabilities and mean sojourn times

The differential transition probabilities are:

\[ dQ_{01}(t) = \lambda e^{-(\lambda + \beta)t} G(t) \, dt, \]
\[ dQ_{02}(t) = \beta e^{-(\lambda + \beta)t} G(t) \, dt, \]
\[ dQ_{13}(t) = \beta e^{-(\lambda + \beta)t} \frac{1}{G(t)} \, dt, \]
\[ dQ_{14}(t) = \lambda e^{-(\lambda + \beta)t} \frac{1}{G(t)} \, dt, \]
\[ dQ_{15}(t) = g(t) e^{-(\lambda + \beta)t} \frac{1}{G(t)} \, dt, \]
\[ dQ_{20}(t) = \beta e^{-(\lambda + \beta)t} \frac{1}{G(t)} \, dt, \]
\[ dQ_{23}(t) = \lambda e^{-(\lambda + \beta)t} \frac{1}{G(t)} \, dt, \]
\[ dQ_{31}(t) = \beta e^{-(\lambda + \beta)t} \frac{1}{G(t)} \, dt, \]
\[ dQ_{36}(t) = \lambda e^{-(\lambda + \beta)t} \frac{1}{G(t)} \, dt, \]
\[ dQ_{47}(t) = g(t) e^{-(\beta + \lambda)t} \, dt, \]
\[ dQ_{48}(t) = \beta e^{-(\beta + \lambda)t} \, dt, \]
\[ dQ_{57}(t) = \lambda e^{-(\lambda + \beta)t} \frac{1}{G(t)} \, dt, \]
\[ dQ_{5,10}(t) = \beta e^{-(\lambda + \beta)t} \frac{1}{G(t)} \, dt, \]
\[ dQ_{69}(t) = \beta e^{-(\lambda + \beta)t} \frac{1}{G(t)} \, dt, \]
\[ dQ_{70}(t) = g(t) e^{-(\beta + \lambda)t} \, dt, \]
\[ dQ_{7,11}(t) = \lambda e^{-(\beta + \lambda)t} \frac{1}{G(t)} \, dt, \]
\[ dQ_{7,12}(t) = \beta e^{-(\beta + \lambda)t} \frac{1}{G(t)} \, dt, \]
\[ dQ_{89}(t) = \beta e^{-(\beta + \lambda)t} \frac{1}{G(t)} \, dt, \]
\[ dQ_{97}(t) = g(t) e^{-(\beta + \lambda)t} \, dt, \]
\[ dQ_{98}(t) = \beta e^{-(\beta + \lambda)t} \, dt, \]
\[ dQ_{10,5}(t) = \beta e^{-(\lambda + \beta)t} \frac{1}{G(t)} \, dt, \]
\[ dQ_{10,12}(t) = \lambda e^{-(\lambda + \beta)t} \frac{1}{G(t)} \, dt, \]
\[ dQ_{11,1}(t) = g(t) e^{-(\beta + \lambda)t} \, dt, \]
\[ dQ_{11,6}(t) = \beta e^{-(\beta + \lambda)t} \, dt, \]
\[ dQ_{12,7}(t) = \beta e^{-(\beta + \lambda)t} \frac{1}{G(t)} \, dt, \]
\[ dQ_{12,8}(t) = \lambda e^{-(\lambda + \beta)t} \frac{1}{G(t)} \, dt, \]

Simple probabilistic considerations yield the following expressions for the non-zero elements

\[ p_{ij} = Q_{ij}(\infty) = \int_0^\infty q_{ij}(t) \, dt. \]

We have

\[ p_{01} = \frac{\lambda}{\beta + \lambda}, \quad p_{02} = \frac{\beta}{\beta + \lambda}, \quad p_{13} = \frac{1}{\beta + \lambda}, \quad p_{14} = \frac{\lambda}{\beta + \lambda}(1 - g^*(\beta + \lambda)), \]
\[ p_{15} = g^*(\beta + \lambda), \quad p_{20} = \frac{\beta}{\beta + \lambda}, \quad p_{23} = \frac{\lambda}{\beta + \lambda}, \quad p_{31} = \frac{\lambda}{\beta + \lambda}, \quad p_{37.6} = \frac{\lambda}{\beta + \lambda}, \quad p_{47} = g^*(\beta), \]
\[ p_{48} = 1 - g^*(\beta), \quad p_{57} = \frac{\lambda}{\beta + \lambda}, \quad p_{5,10} = \frac{\lambda}{\beta + \lambda}, \quad p_{5,11} = 1, \quad p_{70} = g^*(\beta + \lambda), \quad p_{7,11} = \frac{\lambda}{\beta + \lambda}, \quad p_{7,12} = \frac{\lambda}{\beta + \lambda}, \]
\[ p_{71.11} = \frac{1}{\beta + \lambda}(1 - g^*(\beta + \lambda)), \quad g^*(\beta), \quad \]
\[ p_{71.11} = \frac{1}{\beta + \lambda}(1 - g^*(\beta + \lambda)), \quad p_{37.69} = \frac{\lambda}{\beta + \lambda}, \quad p_{37.69} = \frac{\lambda}{\beta + \lambda}, \quad p_{37.69} = \frac{\lambda}{\beta + \lambda}(1 - g^*(\beta + \lambda)), \]
\[ p_{71.11} = \frac{1}{\beta + \lambda}(1 - g^*(\beta + \lambda)), \quad g^*(\beta), \quad \]
\[ g^*(\beta), \quad \]
\[ \text{It can be easily verified that} \]

\[ p_{01} + p_{02} + p_{13} + p_{15} + p_{17.48} = 1, \quad p_{20} + p_{23} + p_{21} + p_{31} + p_{37.69} + p_{37.69} + p_{47} + p_{48} = 1, \]
\[ p_{70} + p_{71.11} + p_{71.11} + p_{71.11} = 1, \quad p_{12,7} + p_{12,7} + p_{12,7} + p_{12,7} = 1. \]
The mean sojourn times ($\mu_i$) in the state $S_i$ are

$$\mu_0 = m_{01} + m_{02} = \frac{1}{\beta + \lambda}, \quad \mu_1 = m_{11} + m_{13} + m_{14} + m_{15} = \frac{1}{\beta + \lambda} (1-g_{*}(\beta, \lambda_1)),$$

$$\mu_2 = m_{20} + m_{23} = \frac{1}{\beta + \lambda}, \quad \mu_3 = m_{31} + m_{35} = \frac{1}{\beta + \lambda}, \quad \mu_4 = m_{47} + m_{48} = \frac{1}{\beta + \lambda}, \quad \mu_5 = m_{57} + m_{5,10} = \frac{1}{\beta + \lambda}, \quad \mu_6 = m_{69} = \frac{1}{\beta}, \quad \mu_7 = m_{70} + m_{7,11} + m_{7,12} = \frac{1}{\beta + \lambda}, \quad \mu_8 = m_{89} = \frac{1}{\beta}, \quad \mu_9 = m_{97} + m_{98} = \frac{1}{\beta} (1-g_{*}(\beta)), \quad \mu_{10} = m_{10,5} + m_{10,12} = \frac{1}{\beta + \lambda}, \quad \mu_{11} = m_{11,1} + m_{11,6} = \frac{1}{\beta + \lambda}, \quad \mu_{12} = m_{12,7} + m_{12,8} = \frac{1}{\beta + \lambda}.$$

$$\mu'_1 = m_{13} + m_{15} + m_{17.4} + m_{17.4,(8,9)n} = \frac{1}{\beta + \lambda}, \quad \mu'_3 = m_{31} + m_{37.69} + m_{37.6,(9,8)n} = \frac{1}{\beta + \lambda}, \quad \mu'_5 = m_{57} + m_{5,10} = \frac{1}{\beta + \lambda}, \quad \mu'_7 = m_{70} + m_{7,11} + m_{7,12} + m_{7,11,6,9} + m_{7,11,6,(9,8)n} + m_{7,12} = \frac{1}{\beta + \lambda}, \quad \mu'_9 = m_{97} + m_{98} = \frac{1}{\beta} (1-g_{*}(\beta)), \quad \mu'_{12} = m_{12,7} + m_{12,8} = \frac{1}{\beta + \lambda}.$$

5. Steady state availability

Let $A_i(t)$ be the probability that the system is in up-state at instant ‘t’ given that the system entered regenerative state $S_i$ at $t = 0$. The recursive relations for $A_i(t)$ are given as:

$$A_0(t) = M_0(t) + q_{01}(t) \cdot A_1(t) + q_{02}(t) \cdot A_2(t)$$

$$A_1(t) = M_1(t) + q_{13}(t) \cdot A_3(t) + q_{15}(t) \cdot A_5(t) + (q_{17.4}(t) + q_{17.4,(8,9)n}(t)) \cdot A_7(t)$$

$$A_2(t) = M_2(t) + q_{20}(t) \cdot A_0(t) + q_{23}(t) \cdot A_3(t)$$

$$A_3(t) = M_3(t) + q_{31}(t) \cdot A_1(t) + (q_{37.69}(t) + q_{37.6,(9,8)n}(t)) \cdot A_7(t)$$

$$A_5(t) = M_5(t) + q_{57}(t) \cdot A_7(t) + q_{5,10}(t) \cdot A_{10}(t)$$

$$A_7(t) = M_7(t) + q_{70}(t) \cdot A_0(t) + q_{71.11}(t) \cdot A_1(t) + (q_{77.11,6,9}(t) + q_{77.11,6,(9,8)n}(t)) \cdot A_7(t) + q_{7,12}(t) \cdot A_{12}(t)$$

$$A_{10}(t) = M_{10}(t) + q_{10,5}(t) \cdot A_5(t) + q_{10,12}(t) \cdot A_{12}(t)$$

$$A_{12}(t) = M_{12}(t) + q_{12,7}(t) + q_{12,7,(8,9)n}(t)) \cdot A_7(t) \quad \ldots(5)$$

Where $M_i(t)$ is the probability that the system is up initially in state $S_i \in E$ is up at time $t$ without visiting to any other regenerative state, we have

$$M_0(t) = e^{-(\beta + \lambda)t} \cdot G(t), \quad M_1(t) = e^{-(\beta + \lambda)t}, \quad M_2(t) = e^{-(\beta + \lambda)t}, \quad M_3(t) = e^{-(\beta + \lambda)t}, \quad M_5(t) = e^{-(\beta + \lambda)t}, \quad M_7(t) = e^{-(\beta + \lambda)t}, \quad M_{10}(t) = e^{-(\beta + \lambda)t}, \quad M_{12}(t) = e^{-(\beta + \lambda)t} \quad \ldots(6)$$
Taking L.T. of above relations (5) and (6) and solving for \(A_0*(s)\), we have

\[
\begin{align*}
A'_0(s) &= \frac{\left(1-q_3^*(s)q_{31}^*(s)\right)
\left(1-q_{10,5}^*(s)q_{20}^*(s)\right)
\left(1-q_{10,5}^*(s)q_{5,10}^*(s)\right)
\left(1-q_{7,11,6,9,8}^*(s)\right)
\left(-q_{12,7}^*(s) + q_{12,7,6,8,9}^*(s)q_{7,12}^*(s) - q_{7,11}^*(s)q_{13}^*(s)q_{37,6,9,8}^*(s)\right)
\left(1-q_{12,7}^*(s)q_{20}^*(s)\right)\left(1-q_{10,5}^*(s)q_{5,10}^*(s)\right)
\left(1-q_{7,11,6,9,8}^*(s)\right)
\left(-q_{12,7}^*(s) + q_{12,7,6,8,9}^*(s)q_{7,12}^*(s) - q_{7,11}^*(s)q_{13}^*(s)q_{37,6,9,8}^*(s)\right)
\left(1-q_{12,7}^*(s)q_{20}^*(s)\right)\left(1-q_{10,5}^*(s)q_{5,10}^*(s)\right)
\left(1-q_{7,11,6,9,8}^*(s)\right)
\left(-q_{12,7}^*(s) + q_{12,7,6,8,9}^*(s)q_{7,12}^*(s) - q_{7,11}^*(s)q_{13}^*(s)q_{37,6,9,8}^*(s)\right)
\left(1-q_{12,7}^*(s)q_{20}^*(s)\right)\left(1-q_{10,5}^*(s)q_{5,10}^*(s)\right)
\left(1-q_{7,11,6,9,8}^*(s)\right)
\left(-q_{12,7}^*(s) + q_{12,7,6,8,9}^*(s)q_{7,12}^*(s) - q_{7,11}^*(s)q_{13}^*(s)q_{37,6,9,8}^*(s)\right)
\left(1-q_{12,7}^*(s)q_{20}^*(s)\right)\left(1-q_{10,5}^*(s)q_{5,10}^*(s)\right)
\left(1-q_{7,11,6,9,8}^*(s)\right)
\left(-q_{12,7}^*(s) + q_{12,7,6,8,9}^*(s)q_{7,12}^*(s) - q_{7,11}^*(s)q_{13}^*(s)q_{37,6,9,8}^*(s)\right)\right)
\end{align*}
\]

\[
A_0(\infty) = \lim_{s \to 0^+} A'_0(s) = \frac{N_2}{D_2}
\]

Where

\[
N_2 = (1-p_{510,5}) (1-p_{76,11,7,12}) ((\mu_0 + \mu_{1p_0} + p_{01} + p_{31} + \mu_1p_{01} + p_{23}p_{31} + \mu_0p_0p_0 + p_{02}p_{23}p_{31}) + \mu_1p_0p_1p_0p_2p_1p_2p_3p_3p_3)
\]

\[
D_2 = (1-p_{510,5}) (1-p_{76,11,7,12}) ((\mu_0 + \mu_{1p_0} + p_{01} + p_{31} + \mu_1p_{01} + p_{23}p_{31} + \mu_0p_0p_0 + p_{02}p_{23}p_{31}) + \mu_1p_0p_1p_0p_2p_1p_2p_3p_3p_3)
\]

The steady state availability is given by

\[
A_0(\infty) = \lim_{s \to 0} A_0'(s) = \frac{N_2}{D_2}
\]
6. Busy period analysis of the server

Let \( B_i(t) \) be the probability that the server is busy in repairing the unit at an instant ‘t’ given that the system entered regenerative state \( S_i \) at \( t=0 \). The recursive relations for \( B_i(t) \) are as follows:

\[
\begin{align*}
B_0(t) &= q_{01}(t) B_1(t) + q_{02}(t) B_2(t) \\
B_1(t) &= W_1(t) + q_{13}(t) B_3(t) + q_{15}(t) B_5(t) + (q_{17.4}(t) + q_{17.4, (8,9)}(t)) B_7(t) \\
B_2(t) &= q_{20}(t) B_0(t) + q_{23}(t) B_3(t) \\
B_3(t) &= q_{31}(t) B_1(t) + (q_{37.6}(t) + q_{37.6, (9,8)}(t)) B_7(t) \\
B_5(t) &= q_{57}(t) B_7(t) + q_{5,10}(t) B_{10}(t) \\
B_7(t) &= W_7(t) + q_{70}(t) B_0(t) + q_{71.11}(t) B_1(t) + (q_{77.11,6}(t) + q_{77.11,6, (9,8)}(t)) B_7(t) + q_{7,12}(t) B_{12}(t) \\
B_{10}(t) &= q_{10,5}(t) B_5(t) + q_{10,12}(t) B_{12}(t) \\
B_{12}(t) &= q_{12.7}(t) + q_{12.7, (8,9)}(t) B_7(t)
\end{align*}
\]

Where \( W_i(t) \) be the probability that the server is busy in state \( S_i \) due to failure up to time \( t \) without making any transition to any other regenerative state or returning to the same via one or more non regenerative states so,

\[
\begin{align*}
W_1(t) &= e^{-(\beta + \lambda) t} G(t) + (\lambda e^{-(\beta + \lambda) t} \circ G(t)),
W_7(t) &= (\beta + \lambda) e^{-(\beta + \lambda) t} G_1(t) + (\lambda e^{-(\beta + \lambda) t} \circ G_1(t))
\end{align*}
\]

Taking L.T. of above relations (8, 9) and solving for \( B_i^*(s) \), we obtain

\[
B_i^*(s) = \frac{\sum (1-q_{15}(s) q_{31}(s))(1-q_{12.7}(s) q_{22}(s))(1-q_{10.5}(s) q_{5.10}(s))(1-q_{7.11,6}(9,8) s) W_1(s)(1-q_{10.5}(s) q_{5.10}(s))}{(1-q_{15}(s) q_{31}(s))(1-q_{10.5}(s) q_{5.10}(s)) (1-q_{7.11,6}(9,8) s) W_1(s)(1-q_{10.5}(s) q_{5.10}(s))}
\]

\[
(1-q_{12.7}(s) q_{22}(s))(1-q_{10.5}(s) q_{5.10}(s)) - q_{7.11,6}(9,8) s R(s) \tau_{15}(s) q_{31}(s) q_{13}(s) q_{37.6}(9,8) s (1-q_{10.5}(s) q_{20}(s))
\]

\[
(1-q_{12.7}(s) q_{22}(s))(1-q_{10.5}(s) q_{5.10}(s)) - q_{7.11,6}(9,8) s R(s) \tau_{15}(s) q_{31}(s) q_{13}(s) q_{37.6}(9,8) s (1-q_{10.5}(s) q_{20}(s))
\]

\[
W_7(s) q_{5.10}(s) q_{5.10}(s) - q_{7.11,6}(9,8) s R(s) \tau_{15}(s) q_{31}(s) q_{13}(s) q_{37.6}(9,8) s (1-q_{10.5}(s) q_{20}(s))
\]

\[
W_7(s) q_{5.10}(s) q_{5.10}(s) - q_{7.11,6}(9,8) s R(s) \tau_{15}(s) q_{31}(s) q_{13}(s) q_{37.6}(9,8) s (1-q_{10.5}(s) q_{20}(s))
\]

\[
W_7(s) q_{5.10}(s) q_{5.10}(s) - q_{7.11,6}(9,8) s R(s) \tau_{15}(s) q_{31}(s) q_{13}(s) q_{37.6}(9,8) s (1-q_{10.5}(s) q_{20}(s))
\]

\[
W_7(s) q_{5.10}(s) q_{5.10}(s) - q_{7.11,6}(9,8) s R(s) \tau_{15}(s) q_{31}(s) q_{13}(s) q_{37.6}(9,8) s (1-q_{10.5}(s) q_{20}(s))
\]

\[
(1-q_{12.7}(s) q_{22}(s))(1-q_{10.5}(s) q_{5.10}(s)) - q_{7.11,6}(9,8) s R(s) \tau_{15}(s) q_{31}(s) q_{13}(s) q_{37.6}(9,8) s (1-q_{10.5}(s) q_{20}(s))
\]

\[
\cdots (10)
\]
The time for which server is busy due to repair is given by

$$B_0(\infty) = \lim_{s \to 0} sB_0(s) = \frac{N}{D_2} \quad \text{(11)}$$

$$N_0 = W_1^*(0)(1-p_{01}+p_{02}p_{15})(1-p_{01}^0+p_{02}p_{12})p_{01}p_{12}p_{21}p_{71.11} +$$

$$W_7^*(0)(1-p_{02}^0p_{12}p_{21})(1-p_{01}^0+p_{02}p_{12})p_{01}p_{12}p_{21}p_{71.11} \quad \text{and } D_2 \text{ is already mentioned.} \quad \text{(12)}$$

7. Expected number of visits by the server

Let $N_i(t)$ be the expected number of visits by the server in $(0,t]$ given that the system entered the regenerative state $S_i$ at $t=0$. The recursive relations for $N_i(t)$ are given as:

$$N_0(t) = Q_{01}(t) + Q_{02}(t)N_2(t) \quad \text{(13)}$$

Taking L.S.T. of relations (13) and solving for $N_0^{\infty}(s)$, we have

$$N_0^{\infty}(s) = \frac{(1-Q_{01}(s))Q_{02}(s)Q_{13}(s)Q_{15}(s)Q_{17.4}(s)Q_{17.4,6}(s)}{(1-Q_{01}(s))(1-Q_{13}(s))(1-Q_{15}(s))(1-Q_{17.4}(s))(1-Q_{17.4,6}(s))} \quad \text{(14)}$$
The expected numbers of visits per unit time by the server are given by

\[ N_0(\infty) = \lim_{s \to 0} s N_0^\ast (s) = \frac{N_4}{D_2} \]  

\[ \text{Where} \]

\[ N_4 = (1-p_{7,11}p_{7,12})(1-p_{5,10}p_{10,5})(p_{01}(1-p_{31}p_{13})+p_{17,4}(8,9)n(p_{01}+p_{02}p_{23}p_{31})+p_{02}p_{23}+p_{01}p_{13}) \]

\[ + p_{15}p_{57}(p_{01}+p_{02}p_{23}p_{31})+p_{36}p_{02}p_{23}p_{15}p_{71,11}(1-p_{5,10}p_{10,5})+ p_{7,11}p_{11,6}(p_{36}(p_{01}p_{13})+p_{14}(p_{01}+p_{02}p_{23})-p_{36}p_{13}p_{01}))(1-p_{5,10}p_{10,5}) \]

\[ - p_{7,11}(p_{5}(p_{10}+p_{02}p_{23}))(1-p_{5,10}p_{10,5}p_{7,12}(p_{36}(p_{01}p_{13})+p_{14}(p_{01}+p_{02}p_{23})+p_{14}(p_{01}+p_{02}p_{23}))) \]

\[ + p_{15}p_{57}(p_{01}+p_{02}p_{23}p_{31})(1-p_{7,11}p_{7,11}p_{7,11}p_{7,11}p_{10,12}) \]

D_2 is already specified.

\[ \text{8. Profit analysis} \]

The profit incurred to the system Model in steady state can be obtained as

\[ P = K_0A_0 - K_1B_0 - K_2N_0 \]

Where

\[ K_0 = \text{Revenue per unit up-time of the system} \]

\[ K_1 = \text{Cost per unit for which server is busy} \]

\[ K_2 = \text{Cost per unit visit by the server} \]

A_0, B_0, N_0 are already defined.

\[ \text{9. Particular cases} \]

Suppose \( g(t) = ae^{-\alpha t} \), \( g_1(t) = \frac{1}{(1+\beta t)^{\lambda}} \).

By using the non-zero elements \( p_{ij} \), we can obtain the following results:

\[ p_{01} = \frac{\lambda}{\beta + \lambda}, p_{02} = \frac{\beta}{\beta + \lambda}, p_{13} = \frac{\beta}{\alpha + \beta + \lambda}, p_{14} = \frac{\alpha}{\alpha + \beta + \lambda}, p_{15} = \frac{\lambda}{\alpha + \beta + \lambda}, p_{20} = \frac{\beta}{\beta + \lambda}. \]

\[ p_{23} = \frac{\beta}{\beta + \lambda}, p_{31} = \frac{\beta}{\beta + \lambda}, p_{36} = \frac{\lambda}{\beta + \lambda}, p_{47} = \frac{\lambda}{\beta + \lambda}, p_{48} = \frac{\beta}{\beta + \lambda}, p_{57} = \frac{\lambda}{\beta + \lambda}, p_{60} = 1, \]

\[ p_{70} = \frac{\alpha}{\alpha + \beta + \lambda}, p_{71,1} = \frac{\lambda}{\alpha + \beta + \lambda}, p_{7,12} = \frac{\beta}{\alpha + \beta + \lambda}, p_{89} = 1, p_{97} = \frac{\alpha}{\alpha + \beta + \lambda}, \]

\[ p_{98} = \frac{\beta}{\alpha + \beta}, p_{10,5} = \frac{\beta}{\alpha + \beta}, p_{10,12} = \frac{\beta}{\alpha + \beta}, p_{11,1} = \frac{\alpha}{\alpha + \beta}, p_{11,6} = \frac{\beta}{\alpha + \beta}, p_{12,5} = \frac{\beta}{\alpha + \beta}, p_{12,8} = \frac{\beta}{\alpha + \beta}, \]

\[ p_{17,4} = \frac{\lambda}{\alpha + \beta + \lambda}, p_{17,4}(8,9)n = \frac{\lambda}{\alpha + \beta + \lambda}, p_{37,6} = \frac{\lambda}{\alpha + \beta + \lambda}, p_{37,6}(9,8)n = \frac{\lambda}{\alpha + \beta + \lambda}, \]

\[ p_{71,11} = \frac{\alpha}{\alpha + \beta + \lambda}, p_{77,11}(6,9,8)n = \frac{\lambda}{\alpha + \beta + \lambda}, p_{12,7}(8,9)n = \frac{\lambda}{\alpha + \beta + \lambda}, \]

\[ \mu_0 = \frac{1}{\beta + \lambda}, \mu_1 = \frac{1}{\alpha + \beta + \lambda}, \mu_2 = \frac{1}{\beta + \lambda}, \mu_3 = \frac{1}{\alpha + \beta + \lambda}, \mu_4 = \frac{1}{\beta + \lambda}, \mu_5 = \frac{1}{\alpha + \beta + \lambda}, \mu_6 = \frac{1}{\beta + \lambda}, \]

\[ \mu_7 = \frac{1}{\alpha + \beta + \lambda}, \mu_8 = \frac{1}{\beta + \lambda}, \mu_9 = \frac{1}{\alpha + \beta + \lambda}, \mu_{10} = \frac{1}{\beta + \lambda}, \mu_{11} = \frac{1}{\alpha + \beta + \lambda}, \mu_{12} = \frac{1}{\beta + \lambda}. \]
\[ \mu_i = \frac{\alpha \beta_i + \lambda_i (\beta + \beta_i)}{\alpha \beta_i (\alpha + \beta + \beta_i)}, \mu_s = \frac{\alpha (\beta_i + \lambda_i) + \lambda_i (\beta + \beta_i)}{\alpha \beta_i (\beta_i + \lambda_i)}, \mu_i = \frac{\alpha \beta_i (\alpha_i + \beta_i) + \lambda (\alpha + \beta_i) (\beta + \beta_i)}{\alpha \beta_i (\alpha_i + \beta_i) (\alpha + \beta + \lambda_i)} \]

Steady state availability \( (A_0) = \frac{N_2}{D_2} \)

Busy period of the server \( (B_0) = \frac{N_3}{D_2} \)

Expected number of visits by the server \( (N_0) = \frac{N_4}{D_2} \)

Where

\[ N_2 = \alpha \beta_1 (\alpha_1, \lambda_1) (\beta + \beta_1, \lambda_1) (\beta_1 + \lambda_1) (\beta_1 + \beta + \lambda_1) (\alpha_1 + \beta + \lambda_1) + \alpha \beta_1 \beta C + \alpha \alpha_1 \beta_1 (B - \beta \beta_1) (\alpha_1 + \beta + \lambda_1) \]

\[ D_2 = \alpha_1 (\beta_1 + \lambda_1) (\alpha_1 + \beta + \lambda_1) (\alpha_1 + \beta + \lambda_1) (\alpha_1 + \beta + \lambda_1) E + \lambda (B + \beta C) + \alpha \alpha_1 \beta_1 (B - \beta \beta_1) \]

\[ N_3 = \alpha \beta_1 (\alpha_1 + \beta + \lambda_1) (\beta_1 + \lambda_1) (\beta + \beta_1, \lambda_1) \]

\[ N_4 = \alpha \beta_1 (\alpha_1 + \beta, B - \beta \beta_1) \]

\[ \lambda_1, \alpha_1^2 = \frac{(\alpha_1 + \beta_1 + \lambda_1) (B + \beta_1 + \lambda_1) (\alpha_1 + \beta + \lambda_1) (\alpha_1 + \lambda_1) (\beta_1 + \lambda_1) (\beta + \beta_1) + \lambda_1 (\beta + \beta_1) + \lambda_1 (\beta_1 + \lambda_1)}{\alpha_1} \]

\[ A = (\alpha_1 + \lambda_1) (\beta_1 + \lambda_1) + \beta \beta_1 \]

\[ B = (\beta_1 + \lambda_1) (\beta_1 + \lambda_1) + \beta \beta_1 \]

\[ C = \alpha_1 + \beta_1 + \beta_1 + \lambda_1 \]

\[ D = \alpha_1 + \beta_1 + \lambda_1 + \beta_1 \]

\[ E = (\alpha_1 + \lambda_1) (\beta_1 + \lambda_1) + \beta \lambda_1 \]
11. Conclusion
The graphs of the Availability and profit function of the system are drawn for fixed values of the various parameters and costs as shown respectively in figures 1, 2. These figures indicate that there is substantial positive change in these measures with increase of normal weather rate ($\beta_1$) and repair rates ($\alpha$ and $\alpha_1$) of the units. While their values decline with increase of abnormal weather rate ($\beta$) and failure rates ($\lambda$ and $\lambda_1$) of the units. However, the effect of repair rate of the main unit is more. On the basis of the above results, it can be concluded that the system can be made more profitable to use by increasing repair rate of the both units.
12. References