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Modeling foreign direct investment returns and economic growth in Nigeria

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Abstract
Foreign Direct Investment (FDI) is a strategy used by the majority of developing nations, including Nigeria, to increase their foreign exchange reserves through investments, business ventures, and international aid. The main goal of this study is to model foreign investment returns and economic growth in Nigeria. To my knowledge, no prior research on the relationship between FDI and economic growth has used a simulation with differential equations, therefore our study adds to the body of knowledge by using both a robust regression model and the simulation approach. The robust regression model was used, and the outcome demonstrates that foreign direct investment positively affects Nigeria's economic expansion. According to the simulation results, an additional $1 billion increase in foreign direct investment will cause Nigeria's GDP to grow by around $3 billion. Robust regression and simulation models combined for improved precision show that FDI has a beneficial impact on economic growth. Consequently, the Nigerian government must step up by creating a more favourable environment and ensuring the safety of people and property to draw in foreign investors. It must also increase the country's foreign reserves by investing enough money from the removal of subsidies to give average Nigerians access to financial inclusion, infrastructure growth to boost productivity, and more employment opportunities to make it easier to payment of foreign debt.

Keywords: Foreign direct investment, Economic growth, Robust Regression model, Simulation

1. Introduction
Most developing countries employ foreign direct investment (FDI) as a strategy to obtain foreign reserves through investments, business ventures, and outside help from industrialized countries (World Bank, 2022) [22]. According to the World Bank, foreign direct investment (FDI) is a helpful tool for global trade as well as a key source of capital formation, technology transfer, and know-how exchange. Nigeria is one of the countries that gains from discoveries and scientific advancements as a result of the spillover effect. Even if it is uncertain whether faster growth is necessary to fulfil the Sustainable Development Goals, cross-border commerce is essential for economies to achieve sustainable and equitable development (UNCTAD, 2019) [8]. The largest host economy in Sub-Saharan Africa and the third largest host economy overall at the moment is Nigeria. Nigeria recently implemented a variety of trade policies intended to diversify the country's economy away from oil revenue. Austerity is imposed after the adoption of these programs, which are meant to support the manufacturing sector. Around 1.9 billion dollars in total foreign direct investment (FDI) entered the nation in 2018, down from roughly 3.5 billion dollars in 2017. The austerity measures put in place in 2018 were to blame for this decline. The amount of FDI in the total capital inflow for the third quarter of 2019 was only 3.37% (USD 200.08 million).
Foreign direct investment (FDI) is said to either stimulate economic growth or have little effect on it, depending on the study (Ali & Hussain, 2017; Louzi & Abadi, 2011) [1, 5]. However, it is impossible to overestimate the advantages of FDI in any global economy. "Foreign direct investment" (FDI) is a term used to describe an investment made by a company or an individual to launch a business or buy real estate in a country other than their own. Foreign direct investment, according to John (2016) [4], is the transfer of capital and technology from one wealthy or underdeveloped country to another. After Ethiopia and Egypt, Nigeria is the third host economy for FDI in Africa. Among the countries that invest in Nigeria are the United States, the United Kingdom, China, the Netherlands, and France (UNCTAD, 2018) [7].

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According to UNCTAD (2018) [7], Nigeria's FDI flows declined by 21% in 2017 to 3.5 billion USD. This decline is thought to have been triggered by political upheaval, a lack of openness in government operations, rife corruption, and inadequate infrastructure. In the belief that increasing their foreign investment portfolio will help their economies by enhancing the economies of the receiving nations and promoting economic growth and development, many developing countries actively seek out foreign investors. There is disagreement because the majority of empirical evidence on how FDI affects economic growth suggests the reverse. According to the available literature, some empirical results suggested a conflict between FDI and economic growth, while others thought that as FDI increased, output productivity increased, leading to a positive connection between the variables (Emmanuel, 2016) [3]. By simulating foreign direct investment returns and economic growth in Nigeria and using a robust regression model, this work adds to the body of current literature.

2. Literature review
This section presents a review of the previous literature works which includes both the theoretical and empirical review that were discussed accordingly in this study.

2.1 Theoretical review
The Keynesian economic growth theory, often known as the Harrod-Domar model, was created by Harrod in 1939 and was utilized in this study.

Marchand (2018) [6] asserts that the majority of developing countries allow for both domestic and foreign investment. Multinational firms with headquarters in developed countries commonly make foreign direct investments (FDIs). This research backs up the Harrod-Domar theory of growth, which states that a certain amount of GDP must be saved and invested for an economy to grow. In other words, a nation's pace of economic growth is determined by its capacity for saving money and capital-to-output ratio.

The national income growth rate is inversely or adversely correlated with the capital-output ratio; an increase in capital decreases the GDP growth rate. Investment and savings are favourably and directly correlated with growth, and a greater savings rate also encourages it. According to the Harrod-Dormar growth model, a low level of capital formation, which results in a low level of savings and investment, is the main barrier to growth. The main criticism of this approach is that it merely offers a necessary but insufficient prerequisite for development. Although it is essential for development, it is insufficient to sustain expansion.

2.2 Empirical review
Foreign direct investment (FDI) has done poorly in Nigeria due to its bad macroeconomic environment. The success of foreign investments in a given nation is influenced by several variables, such as the size of the market, the availability of human resources, the stability of the macroeconomic environment, and push and pull dynamics. FDI had minimal influence on Nigeria's economic growth because it only slightly increases output.

Akanegbu & Chizea (2017) [2] contend that despite the substantial gains, the nation only contributes a little amount to global FDI. The impact of FDI on economic growth in Nigeria was assessed using annual time series data spanning the years 1991 to 2014 and the neoclassical production function, where labour, capital, and FDI are all seen as production inputs. According to the findings, FDI has a negligibly positive impact on Nigeria's output productivity.

Egbo (2010) [12] analyzed annual secondary data spanning the years 1981–2007 using the OLS estimate method to analyze the impact of FDI inflows on economic growth in Nigeria. Since FDI fosters growth, the results indicate a favourable relationship.

Furthermore, utilizing time series data from 1981 to 2015 and a variety of regression estimate techniques, Emmanuel (2016) discovered a significant association between FDI and economic growth. A statistically significant correlation can be shown using the results.

Based on an analysis of annual secondary data between 1979 and 2013 using the Granger causality test and the Error Correction Model (ECM), Uwubanmwen and Ogiemudia (2016) [11] claim that FDI has a positive and significant short-term influence on growth in Nigeria.

In contrast to other capital inflows, FDI accounts for a significant amount of the volatility in Nigeria's economic development, according to a more recent analysis by Anetor (2019) [10]. Utilizing quarterly data from 1961Q1 to 2016Q4, the Structural Vector Autoregression model (SVAR) was built to evaluate the impact of shocks from private capital inflows on the expansion of the Nigerian economy. The results show a clear and statistically significant relationship between the expansion of the Nigerian economy and FDI and portfolio investment.

The VECM results show that FDI sharply quickens Nigeria's economic expansion. Akiri, Vehe, and Ijuo (2016) [13] used secondary data from the years 1981 to 2014 to correct the empirical evidence.

According to a study by Sokang (2018) [15], FDI and Cambodia's economic expansion are significantly related. The study evaluated the data using multiple regression analysis and a correlation matrix using annual time series data from 2006 to 2016. Gudaro, Chhapra, and Sheikh (2012) [16] produced a finding that is comparable for Pakistan from 1981 to 2010 using time series data and multiple regression techniques.

Melynk, Kubatko, and Pysarenko (2014) [17] assert that FDI also has a positive influence on economic growth in the communist transition countries using panel data on yearly transition report indicators from 1998 to 2010 using fixed-effects estimation to examine the data. Even while it momentarily slows economic growth in a small number of carefully chosen emerging countries, FDI promotes long-term growth.

Using multiple regression techniques, John (2016) [4] investigated the effects of FDI on economic growth in Nigeria from 1981 to 2015. According to the study, foreign direct investment has a significant and positive impact on Nigeria's GDP-based economic growth. Additionally, it was demonstrated that the gross domestic product is little impacted positively by exchange rates.

In their 2017 study, Ali and Hussain examined the impact of foreign direct investment (FDI) on Pakistan's economic expansion between 1991 and 2015. In the investigation, correlation and regression analytic techniques were also applied. They found that FDI has a favourable effect on Pakistan's economic expansion.

FDI and bilateral trade both contribute to economic growth, according to Leitao and Rasekhi's (2013) [19] panel data research. A study by Umeora (2011) [9] found that FDI did not support the theoretical and presumptive expectation of a positive relationship between FDI and economic growth. It

was discovered, in contrast to past findings that FDI slows down economic growth in Nigeria using secondary data from 1986 to 2011 and a model that was regressed using the OLS and multiple regression methodologies.

3. Data and Methodology
3.1. Data
Secondary data was used in this study which was collected from the World Bank development indicator as a reliable secondary source of data collection via the link data.worldbank.org from a period of 1970 to 2022 using a purposive sampling technique based on the data available within the selected period.

3.2 Methodology
The quantitative research design was adopted for this study to model the link between foreign direct investment returns and economic growth in Nigeria. The quantitative method applied for the analysis in this study includes descriptive statistics (Such as mean and standard deviation), the robust regression model and the simulation approach.

Table 1: Variable measurement

Variables	Measurement
GDP	Billion USD
FDI	Billion USD
Inflation	Percentage
Exchange rate	Naira per dollar

Source: World Bank

3.2.1 Model specification
The empirical model for this study can be adequately specified as:
Gross domestic product (GDP) = f (FDI, Inflation rate, Exchange rate).
The key economic growth indices are GDP, inflation and exchange rate where GDP is the dependent variable. This study contributes to the existing body of knowledge by applying both a robust regression model and the simulation approach because previous work about the connection between FDI and economic growth didn't adopt a simulation with differential equations to the best of my knowledge.

3.3 Robust regression model
To determine the link between FDI and economic growth in Nigeria, the robust regression model was used. It is suitable for predicting economic growth as well. If the parametric assumptions of the normality of the residual error, the constant variance of the error term, and the assumption that the error term should not be linked with explanatory variables-which frequently lead to autocorrelation-are met, the ordinary least regression is said to be robust. Last but not least, the fitted regression model shouldn't have the multicollinearity issue, which frequently results in deceptive p-values and R-squared.

The robust regression can be represented mathematically as follow

$GDP_t = \beta_0 + \beta_1 FDI_t + \beta_2 Inflation_t + \beta_3 Exchange\ rate_t + \mu_t$ (1)

Where GDP is the dependent variable while the independent variables are FDI, Inflation and exchange rate. The β_0 is the intercept or the constant term and β_1 to β_3 are the coefficient estimate of the independent variables. The μ is the error term

that takes care of the all-unaccounted factors in the model while t is the given period in years.

3.4 Simulation Approach
Simulation is simply the imitation of real-life process or operation over a period of time and the regression model can be simulated using the ordinary differential equation (ODE) as follow.

$$\frac{dGDP}{dt} = \alpha_0 + \alpha_1 FDI + \alpha_2 Inflation + \alpha_3\ Exchange\ rate \quad (2)$$

And therefore we need apply partial differential equation using python programming to estimate the parameters of the model since it is more than one independent variables according to Li (2021) [20].

4. Result and discussion

Table 2: Descriptive statistics

	N	Minimum	Maximum	Mean	Std. Deviation
GDP	53	9.182	574.184	172.825	174.158
FDI	53	-.739	8.841	2.050	2.421
Inflation	53	3.458	72.836	18.232	15.183
Exchange Rate	53	.547	472.400	93.799	121.817
Valid Observation	53				

Source: Author's computation using EViews software

Table 2 shows that average Nigeria's GDP during the period under review is about 173 Billion USD with a variability of about 174 Billion USD, the average FDI is about 2.1 Billion USD with a variability of about 2.4 Billion USD, the average inflation rate in Nigeria during the period under review is about 18 per cent with a variability of about 15 per cent while the average exchange rate in Nigeria is about 94 Naira per dollar during the period under review. The total number of valid observations for the period 1970 to 2022 under review is 53.

Table 3: Robust regression model

Dependent Variable: GDP					
Variable	Coefficient	Std. Error	T-Statistic	Prob.	VIF
C	50.39831	21.78732	2.313195	0.0250	NA
FDI	28.74704	5.096296	5.640771	0.0000	1.281
Inflation	-1.048306	0.737077	-1.422247	0.1613	1.054
Exchange rate	0.880642	0.101339	8.690059	0.0000	1.283
R-squared	0.808046	Mean dependent var		172.8246	
Adjusted R-squared	0.796293	S.D. dependent var		174.1581	
Prob(F-statistic)	0.000000				

Source: Author's computation using EViews software

Table 3 demonstrates that the coefficient estimate of FDI and the exchange rate is statistically significant at a 1% level and has a positive significant influence on Nigeria's GDP. This implies that Nigeria's gross domestic product will increase in proportion to its level of foreign direct investment. This supports the findings of John (2016) [4] and Ali & Hussain (2017) [1], whose research also demonstrates that FDI has a positive influence on the GDP. Since the P-value is less than 0.01, the overall regression model is statistically significant at a 1% level. This suggests that there is a significant linear relationship between foreign direct investment and Nigeria's economic expansion. 80.8% of the variance in GDP may be attributed to FDI, inflation, and exchange rate, according to the R-square value of 0.808, while the remaining 19.2% can be attributed to other factors not taken into account by the

model. The overall regression model is significant, and the R-squared is quite high, indicating that the fitted regression model is adequate for future GDP projection, which is a proxy for economic growth, and that the data are well matched by the model. All of the independent variables' variance inflation factors, or VIFs, are below 5, indicating that multicollinearity is not a concern for the model. Additionally, the OLS assumption test ($p>0.05$) in the appendix demonstrates that the residual error is normally distributed and that the fitted regression model is robust, valid, and reliable since the model is not affected by autocorrelation and heteroscedasticity.

4.1 Simulation results

Table 4: Model parameter values in Simulation

Variables	Model parameters symbol	Parameter estimate
GDP		
FDI	α1	2.73
Inflation	α2	-1.01
Exchange rate	α3	0.64

Source: Author's computation via Python programming language

The table 4 shows that for one billion USD rise in foreign direct investment, Nigeria GDP growth will also increase by about 3 Billion USD which suggest that FDI has a positive contribution to the gross domestic product of a nation.
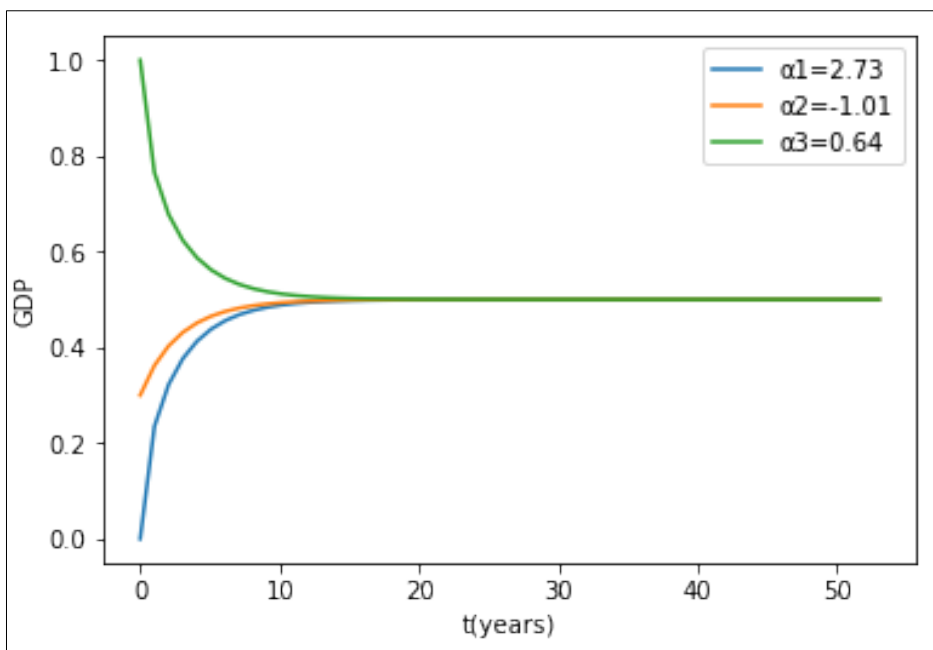

Fig 1: Graph of the simulation model

Figure 1 shows that the response of the GDP to the period (t) of the coefficient of foreign direct investment (α1) and the other independent variables such as inflation (α2) as well as the exchange rate (α3) converging around 0.5 which satisfies the steady state of the ordinary differential equation (ODE) and this also suggest that the simulation of the parameter estimate values is valid.
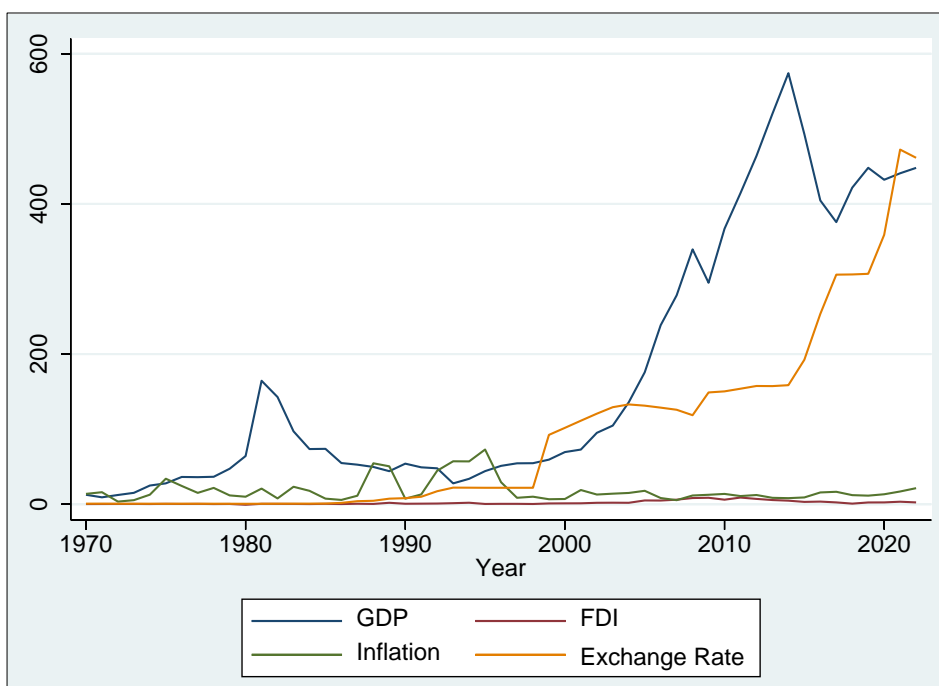
Fig 2: Graph of the foreign direct investment and the economic growth in Nigeria during the period under review

Figure 2 shows the pattern and the link between foreign direct investment and the economic growth indicators of Nigeria such as the GDP, inflation and exchange rate. The exchange rate can be seen to experience a sharp rise from 2020 the period of the COVID-19 and also rises more as a result of the supply shock caused by the Russia-Ukraine war. The GDP reveal a sharp decline in 2020 due to the lockdown measures that cripple the economic activities of the nation. The Inflation rate has also maintained a constant increase from the period of covid-19 till now and this has contributed to the rise in the general price of commodities in Nigeria and cause untold hardship in the country. The foreign direct investment has dropped in 2022 and also has not improved notably well in the previous year.

4.2 Discussion of findings
From Table 3, the coefficient estimate of foreign direct investment (FDI) and exchange rate have a positive significant impact on the gross domestic product (GDP) of Nigeria. This suggests that the higher the foreign direct investment in Nigeria, the higher will also be the country's gross domestic product. This support the works of John (2016) [4] and Ali & Hussain (2017) [1] whose work also shows that FDI has a positive significant effect on the gross domestic product. This is also consistent with the Keynesian economic growth theory that establishes the fact that FDI and Nigeria's economic growth are directly or positively correlated. The overall regression model is statistically significant which implies that there is a significant relationship between foreign direct investment and the economic growth of Nigeria. The OLS assumptions of the fitted regression model were also satisfied which implies that the model is robust.

The relationship between foreign direct investment and Nigeria's economic growth indices, such as the GDP, inflation, and exchange rate, is depicted in Figure 2. The exchange rate is anticipated to sharply increase starting in 2020, during the COVID-19 period, and increases even more as a result of the supply shock brought on by the Russia-Ukraine war. Due to the lockdown measures that cripple the country's economic activity, the GDP shows a significant fall in 2020. From the time of COVID-19 till the present, the inflation rate has also increased steadily, which has pushed up the overall price of goods in Nigeria and led to a great deal of misery there. Foreign direct investment declined in 2022 and did not much improve the year before. Therefore, the government need to pay serious attention to improving the country's foreign direct investment returns which will in turn contribute to Nigeria's economic growth.

However, comparing the simulation result with the real-life data analysis via the robust regression, we can also that the additional one billion USD increase in FDI increases the GDP by about 3 Billion USD which also implies a positive effect on the GDP and this also agrees with the real-life analysis. Therefore, adopting both the simulation and robust regression model provide better results and insight into the link between the FDI and economic growth in Nigeria and even to the other emerging economies of the world at large and thus contributes substantially to the existing body of Knowledge.

5. Conclusion and policy implication
This study contributes to the existing body of knowledge by applying both a robust regression model and the simulation approach because previous work about the connection between FDI and economic growth didn't adopt a simulation with differential equations to the best of my knowledge.

The robust regression model was applied and the result shows that foreign direct investment has a favourable impact on Nigeria's economic growth. The simulation result also indicates that Nigeria's GDP growth will increase by about 3 Billion USD for an additional one billion rise in the country's foreign direct investment. The combination of robust regression and simulation model provides better precision and both indicate that FDI positively influences economic growth. Therefore, the government of Nigeria must gear up by providing a more enabling environment as well as security of life and property that will attract foreign investors to the country and also improves on the foreign reserves by investing adequate money from the subsidy removal to provide access to financial inclusion to average Nigerians, infrastructural growth that will enhance productivity and create more employment opportunities which will make it easy for payment of tax and all this will contribute the increase in Nigeria foreign direct investment returns and brings about the significant economic growth of Nigeria.

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Appendix
Normality
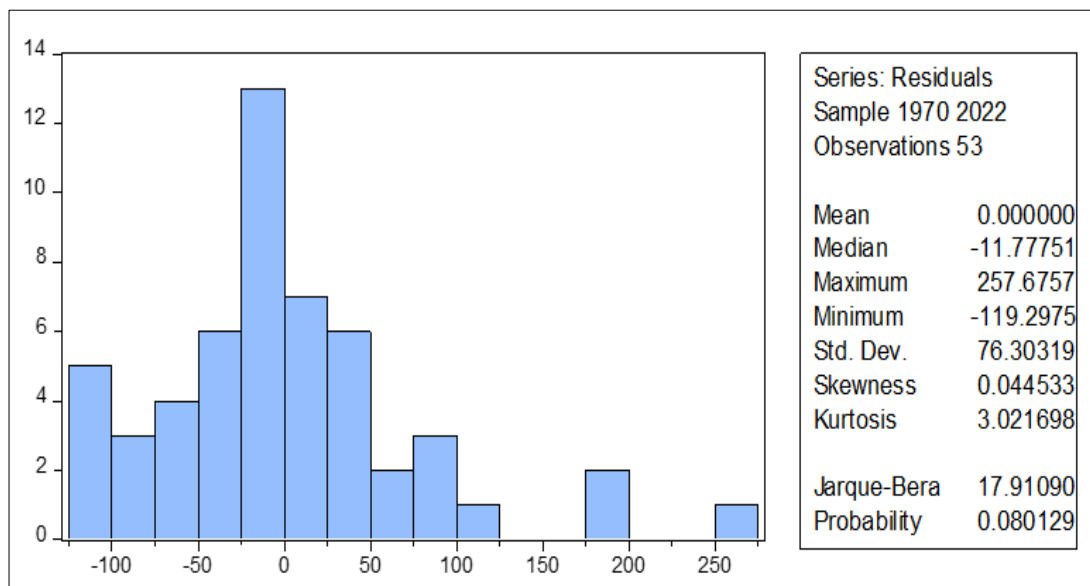
Heteroscedasticity test

Heteroskedasticity Test: Breusch-Pagan-Godfrey			
F-statistic	2.449907	Prob. F(3,49)	0.0746
OBS*R-squared	6.912814	Prob. Chi-Square(3)	0.0747
Scaled explained SS	11.29072	Prob. Chi-Square(3)	0.0103

Autocorrelation test

Breusch-Godfrey Serial Correlation LM Test			
F-Statistic	32.13855	Prob. F (2,47)	0.0842
OBS*R-squared	30.61444	Prob. Chi-Square (2)	0.0795