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# Study the conditions relating to the system's constricted motion under the combined influence of the solar radiation pressure and some turbulence

**Shyam Kishor Singh**

**Abstract**
His study examines the nonlinear effects of the solar radiation pressure combined with other natural perturbing forces on the motion and stability of two satellites connected by a thin, flexible, and inextensible string in the Earth's gravitational field.

**Keywords:** Solar radiation pressure, Keplerian, satellites

## Introduction

Musen (1960) [16] ignored the impact of the Earth's shadow while examining the impact of solar radiation pressure on the motion of an artificial Earth satellite. In their article from 1962, Radzievskii and Arremev investigated the impact of solar radiation pressure on changes in the perigee distance of an artificial Earth satellite while taking into account the impact of the Earth's shadow. Solar radiation pressure's influence on the mobility of artificial Earth satellites. By Divari and Klikh (1968) [5], among others, dust particles in the circumterrestrial dust cloud, moon satellites, spacecraft, and other objects have all been taken into consideration. In 1961, Minorsky *et al*. Beletsky (1969) [1] and Beletsky and Novikova (1969) [2] investigate the motion of a pair of satellites in a Keplerian elliptical orbit that are held together by a thin, flexible, and inextensible string in the centre of the gravitational field of force. He two satellites were believed to be travelling in the plane of the centre of mass for this investigation. Singh (1972, 1973) [13, 14] addressed this issue in its more generic form. Examining it in both two- and three-dimensional scenarios. The motion and stability of the linked satellite system under the influence of solar radiation pressure were explored by Sinha and Singh (1987, 1988) [15] in both circular and elliptical orbits. His research was only focused on linear impacts. The nonlinear oscillation of a network of linked satellites under the impact of solar radiation pressure was researched by Narayan and Singh in 1987, 1990, and 1992 [10-12]. Manaziruddin and Singh (1990, 1992) investigated how minor external pressures affected the planar oscillation of an orbiting system of cable-connected satellites.

### Relative Motion of the Centres of Mass

Let $r_1$ and $r_2$ be the radius vectors of the particles $m_1$ and $m_2$ with respect to the gravitating Earth's centre and 'I' be the length of the string connecting them.

$$\left|\vec{r}_1 - \vec{r}_2\right| \le l \tag{1.1}$$

Now the Lagrange's equation of motion of the first kind for the particles $m_1$ and $m_2$ are respectively.

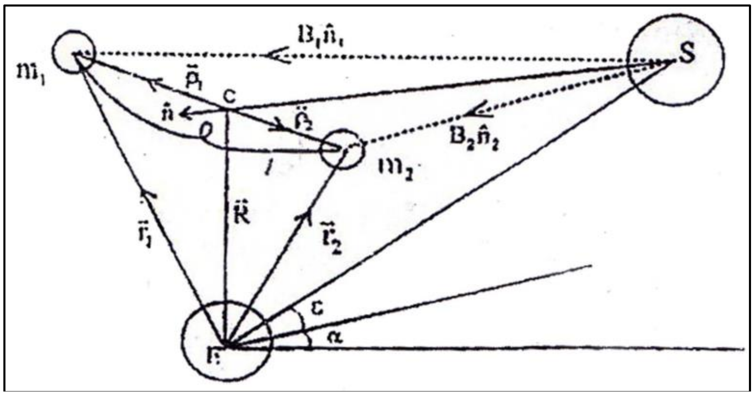


**Fig 1:** The System under the non-linear effects due to combined influence of the solar radiation pressure and some perturbing

$$m_1 \ddot{\vec{r}}_1 = \frac{\mu m_1 r_1}{r_1^3} + \lambda\left(\vec{r}_1 - \vec{r}_2\right) + Q_1(\dot{\vec{r}}_1 \text{ x H}) - \frac{3\mathrm{m}_1 \mu k_2 \dot{\vec{r}}_1}{r_1^5} + B_1\hat{n}_1 - Pac_1 m_1 \left|\dot{\vec{r}}_1\right| \dot{\vec{r}}_1$$

$$m_2 \ddot{\vec{r}}_2 = \frac{\mu m_2 r_2}{r_2^3} + \lambda\left(\vec{r}_2 - \vec{r}_1\right) + Q_2(\dot{\vec{r}}_2 \text{ x H}) - \frac{3\mathrm{m}_2 \mu k_2 \dot{\vec{r}}_2}{r_2^5} + B_2\hat{n}_2 - Pac_1 m_2 \left|\dot{\vec{r}}_2\right| \dot{\vec{r}}_2 \quad \text{………….. (1.2)}$$

Where $\lambda$ = Lagrange' undetermined multiplier

$$Q_1 = \frac{\text{Charge Qi on the ith Particle}}{\text{Velocity of the light (c)}} (i = 1,2)$$

$\mu$ = Product of the gravitation constant and the mass of the earth.

The Absolute values of the force due to the direct solar pressure exerted on $\mathrm{m}_1$ and $m_2$ respectively.

Unit vectors in the direction of the sun rays towards $m_1$ and $\mathrm{m}_2$.

$\rho_1$ and $\rho_2$ = Radius vectors of $m_1$ and $\mathrm{m}_2$ respectively
w.r.t. the centre of mass c.
Pa = Density of the atmosphere
$C_1$ = Ballistic co-efficient (i = 1, 2)
∈ = Inclination of the oscillating plane of the orbit of the centre of mass the system with the plane.
In order to linearise the relative equations of motion of the system with respect to their centre of mass, let us subtract the second equation of (1.2) from the first.

$$\ddot{\vec{r}}_1 - \ddot{\vec{r}}_2 + \mu\left[\frac{\vec{r}_1}{r_1^3} - \frac{\vec{r}_2}{r_2^3}\right] - \left(\vec{r}_1 - \vec{r}_2\right)\left(\frac{\lambda}{m_1} + \frac{\lambda}{m_2}\right)$$

$$= \quad -\left[\frac{Q_1}{m_1}\dot{\vec{r}}_1 \text{ x } \nabla\left(\frac{\vec{M}\cdot\vec{r}_1}{\vec{r}_1^{\,3}}\right) - \frac{Q_2}{m_2}\dot{\vec{r}}_2 \nabla\left(\frac{\vec{M}\cdot\vec{r}_2}{\vec{r}_2^{\,3}}\right)\right]$$

$$+\left(\frac{B_1}{m_1} - \frac{B_2}{m_2}\right)\hat{n} - 3\mu k_2\left[\frac{\vec{r}_1}{r_1^3} - \frac{\vec{r}_2}{r_3^3}\right] - Pa\left[c_1\left|\dot{\vec{r}}_1\right|\dot{\vec{r}}_1 - c_2\left|\dot{\vec{r}}_2\right|\dot{\vec{r}}_2\right] \quad \text{…… (1.3)}$$

Since $\ddot{\vec{r}}_1 - \ddot{\vec{r}}_2 = -\frac{m_1 + m_2}{m_1}\ddot{\vec{\rho}}_2$ ……. (1.4)

The equation of the relative motion of the particle $m_2$ can be obtained in the form :

$$\ddot{\vec{\rho}}_2 + \frac{\mu}{R_3}\vec{\rho}_2 - \frac{3\mu \ddot{\vec{R}}}{R^5}\left(\vec{R}.\vec{\rho}_2\right)\left(\frac{m_1+m_2}{m_1 m_2}\right)\lambda\vec{\rho}_2$$

$$= \quad -\left(\frac{m_1+m_2}{m_1 m_2}\right)\left(\frac{Q_1}{m_1} - \frac{Q_2}{m_2}\right)\ddot{R} \text{ x}\left[-\nabla\left(\frac{\overrightarrow{\text{M}}.\vec{R}}{R^3}\right)\right]$$

$$+\left(\frac{B_1}{m_1} - \frac{B_2}{m_2}\right)\hat{n} - \frac{3\mu K_2}{R^5}.\vec{\rho}_2 + \frac{15\mu k_2 \vec{R}}{R^7}\left(\vec{R}.\vec{\rho}_2\right) - a_1\vec{R} \quad \text{…. (1.5)}$$

Where $a_1 = Pa\dot{R}(C_2 - C_1)\frac{m_1}{m_1 + m_2}$ and since $q_i$, 's are all small so

$$Pa\dot{R}\left[\frac{\dot{\vec{R}}.\vec{\rho}_2}{R^2} + \dot{\vec{\rho}}_2\right]\left(\frac{c_1 m_2 + c_2 m_1}{m_1 + m_2}\right)$$ is neglected

If $\vec{K}_E$ = Unit vector along the axis of magnetic dipole of the earth = $\frac{\vec{M}}{[M]}$

$\vec{e}_r$ = The unit vector along the radius vector = $\frac{\vec{R}}{[\vec{R}]}$

$\mu_E$ = The value of the magnetic moment of the earth's dipole = $[\vec{M}]$

Thus the equation (1.5) can be linearised into the form

$$\ddot{\vec{\rho}}_2 + \frac{\mu}{R^3}\vec{\rho}_2 - \frac{3\mu\vec{R}}{R^5}(R.\rho_2) - \left(\frac{m_1+m_2}{m_1 m_2}\right)\lambda\vec{\rho}_2$$

$$= \quad -\frac{m_1}{m_1+m_3}\left(\frac{Q_1}{m_1} - \frac{Q_2}{m_2}\right)\left[-\frac{\mu_E}{R^3}\dot{\vec{R}}_2\, x\,\vec{k}_E - 3\left(\vec{k}_E.\vec{e}_r\right)\dot{\vec{R}}\vec{e}_r\right]$$

$$+\left(\frac{B_1}{m_1} - \frac{B_2}{m_2}\right)\hat{n} - \frac{3\mu K_2}{R^5}.\vec{\rho}_2 + \frac{15\mu k_2}{R^7}\left(\vec{R}\left(\vec{R}.\vec{\rho}_2\right)\right) - a_1\vec{R} \quad \text{….(1.6)}$$

This is the fundamental equation of motion for the system that explains the motion of the relative. With the aid of (1.6), it is simple to determine the motion of the other particles. In our ongoing research, we will employ a rotating system of coordinates with the origin at the system's centre of inertia, an axis along the radius vector, an axis in the direction of motion that is transverse to the orbit of the system's centre of mass, and an axis that is normal to the orbit plane of the system's centre of mass. The equation (1.6) system will be changed into in the rotating frame of reference.

$$\ddot{\xi} - 2\dot{v}\dot{\eta} + \ddot{v}2\eta - \dot{v}^2\xi - \frac{2\mu}{R^3}\xi - \frac{m_1+m_2}{m_1m_2}\lambda\xi = -\frac{m_1}{m_1+m_2}\left(\frac{Q_1}{m_1} - \frac{Q_2}{m_2}\right)\left[\frac{\mu_E}{R^3}\xi\cos iR\dot{v}\right]$$

$$+\left(\frac{B_2}{m_2} - \frac{B_1}{m_1}\right)\cos\in\cos(v-\alpha) - a_1\dot{R} + \frac{12\mu K_2}{R^5}\xi \qquad \text{...... (i)}$$

$$\ddot{n} + 2\dot{v}\dot{\xi} + \ddot{v}\xi - \dot{v}^2\eta + \frac{\mu}{R^3}\eta - \frac{m_1+m_2}{m_1m_2}\lambda\eta = -\frac{m_1}{m_1+m_2}\left(\frac{Q_1}{m_1} - \frac{Q_2}{m_2}\right)\frac{\mu_E}{R^3}R\dot{v}\cos i$$

$$+\left(\frac{B_2}{m_2} - \frac{B_1}{m_1}\right)\cos\in\sin(v-\alpha) - a_1R\dot{v} + \frac{3\mu k_2}{R^5}\eta \qquad \text{....... (ii)}$$

$$\ddot{\xi} + \frac{\mu}{R^3}\dot{\xi} - \frac{m_1}{m_1+m_2}\lambda\xi = -\frac{m_1}{m_1+m_2}\left(\frac{Q_1}{m_1} - \frac{Q_2}{m_2}\right)$$

$$\sin i\left[\frac{\mu_E}{R^3}\left\{\cos(v+\omega)\dot{R} \rightarrow -R\dot{v}\sin(v+\omega)\right\} + 3\sin(v+\omega)R\dot{v}\right]$$

$$+\left(\frac{B_1}{m_1} - \frac{B_2}{m_2}\right)\sin\in\frac{3\mu k_2}{R^5}\xi \qquad \text{...... (iii)} \qquad \text{.... (1.7)}$$

The condition of constraint (1.7 takes the form

$$\xi^2 + n^2 + \xi^2 \le 1 \qquad \text{... (1.8)}$$

The nechville's transformation given by the following equations

$$\xi = \rho x, \eta = \rho y \text{ and } \xi = \rho z \qquad \text{.... (1.9)}$$

Where $\rho = \frac{R}{p} = \frac{1}{1+e\cos v}$ and $p$ = focal parameter e = eccentricity of orbit,

Transforms the equations (1.7) into the following set of equation (1.10) where the true anomaly of the centre of mass is given by the relatives $v = \frac{dv}{dt} = \sqrt{\frac{\mu\rho}{\rho^2}.\frac{1}{\rho^2}}$ .... (1.10)

$$-2y' - 3\rho x - \lambda_\alpha\rho^4 = -\frac{A}{\rho}\cos i - f\rho\rho' - \frac{4B}{\rho}x + \left(\frac{B_2}{m_2} - \frac{B_1}{m_2}\right)\cos\in\cos(v-\alpha) \text{....(i)}$$

$$-2x' - \lambda_\alpha\rho^4 y = -f\rho^2 + \frac{B}{\rho}y - A\frac{\rho}{\rho^2} - +\cos i + \left(\frac{B_2}{m_2} - \frac{B_1}{m_2}\right)\cos\in\sin(v-\alpha) \text{...(ii)}$$

$$-2x''-\lambda_{\alpha}\rho^{4}z=-\frac{A}{\rho}\left[\frac{\rho}{\rho}\cos(v+\omega)+\left(\frac{3p^{2}\rho^{3}}{\mu_{E}}-1\right)\sin(v+\omega)\right]\sin i+\frac{B}{\rho}z$$

$$+\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right)\sin\in\ldots\ldots\quad \text{(iii)}\ \ldots\ldots \qquad (1.11)$$

Where $A=\frac{m_1}{m_1m_2}\left[\frac{Q_1}{m_1}-\frac{Q_2}{m_2}\right]\frac{\mu_E}{\sqrt{\mu p}}$ : magnetic force.

$B=-\frac{3k_2}{p^2}$ ; oblatesness of the earth.

$\lambda\alpha=\frac{P^3}{\mu}.\frac{m_1+m_2}{m_1m_2}\lambda$ ;

And $f=\frac{a_1P^3}{\sqrt{\mu P}}$ : Air Drag

The condition of constraint is $x^2+y^2+z^2\le 1$ …… (1.12)

**The condition regarding constrained motion**

Condition for motion of the particle $m_2$ under effective constraint is given by

$$-1\ge B(5\cos^2\Psi-1)+A\cos\Psi\cos i+f\sin\Psi \quad \ldots.\ (2.1)$$

In Our subsequent discussion, we shall use the term non-evolutional motion if the string remains always tight during the motion and evolution motion when the motion of the system is the combination of free motion and constrained motion

From (2.1), we shall have evolutional motion if

$$-1<\cos\Psi\cos i+f\sin\Psi+B(5\cos^2\Psi-1) \quad \ldots\ldots\ (2.2)$$

Again, the inequality (2.14) can be written in the form

$$\Psi'^2+2\Psi'+3\cos^2\Psi^2-B(5\cos^2\Psi-1)-f\sin\Psi\ge A\cos\Psi\cos i \quad \ldots..\ (2.3)$$

The evolution motion will take place if

$$\Psi'^2+2\Psi'+3\cos^2\Psi-B(5\cos^2\Psi-1)-f\sin\Psi<A\cos\Psi\cos i \quad \ldots..\ (2.4)$$

The inequality (3.4) represents a curve in the two dimensional phase space $(\Psi,\Psi')$ if the moving particle comes inside this curve in the phase space $(\Psi,\Psi')$, the string will become losse.

The equation (2.4) represents also a curve in the some phase space on which the particle is moving for some fixed value of h. Thus the set of real points of intersection of these two curves in the two dimensional phase space are the region of evolutional motion.

To obtain the points of intersection of these two curves, let us add (2.12) and (2.4)

$$2\psi'^2+2\psi'+f\sin\psi+B<h-A\cos\psi\cos i \qquad (2.5)$$

From this, it follows that the evolutional will take place if $\psi'$ lies between the positives and negatives real roots of the equation.

$$\psi'^2 + \psi' - \left(\frac{h}{2} - \frac{f}{2} - \frac{B}{2} - \frac{A}{2}\cos i\right) = 0 \quad (2.6)$$

The roots will be real if $h \le -\frac{1}{2} + A\cos i + f + B$ ……(2.7)

This, $h < -\frac{1}{2} + A\cos i + f + B$ ……(2.8)

The motion won't be evolutionary since the point of junction isn't actual. searching for negative values while investigating the true underpinnings of evolutionary motion. is higher than for a positive value of. Therefore, any movements for which evolution does not occur in the case of will continue to be non-evolutionary for.
By deducting (1.12 from 2.4), we obtain

$$2\psi' + 6\cos^2\psi + h < 3\cos\psi\cos i + B(10\cos^2\psi - 1) + 3f\sin\psi \quad (2.9)$$

On putting $\sin\psi = 1$ (max) we get.

$$(6 - 10B)\cos^2\psi - 3A\cos i\cos\psi + m < 0 \text{ where } m = 2\psi' + h - 3f + B$$

In order that evolution may take place, we must get real values of $\cos\psi$ and hence the inequality

$$(3f - B - h - 2\psi')\,(3 - 5B) + \frac{3}{8}A^2\cos^2 i \ge 0 \quad (2.10)$$

Consequently the motion will be non-evolutional if

$$(3f - B - h - 2\psi')\,(3 - 5B) + \frac{3}{8}A^2\cos^2 i \ge 0 \quad (2.11)$$

Substituting the roots of the equation (3.6) into (3.11) to eliminate $\psi'$, we obtain

$$\left[3f - B - h - \left\{-1 \pm \sqrt{1 + 2(h - f - B - A\cos i}\right\}\right](3 - 5B) + \frac{3}{8}A^2\cos^2 i < 0$$

(i) which provides the prerequisite for motion that is not evolutionary

$$-1 \ge A\cos i\cos\psi + f\sin\psi + B(5\cos^2\psi - 1) \quad (2.12)$$

(ii) $h < -\frac{1}{2} + A\cos i + f + B$ (2.13)

(iii) $\left[3f - B - h - \left\{-1 \pm \sqrt{1 + 2\left(h - f - B - A\cos i\right)}\right\}\right](3 - 5B)$

$$+ \frac{3}{8}A^2\cos^2 i < 0 \quad (2.14)$$

In we put A = 0, B = 0 and f = 0 in equations (2.12), (2.13) and (2.14), we get.

(i) $h < -\frac{1}{2}$

(ii) $(h - 2)^2 > 4$ for any $\psi'$

(iii) $(h - 2)^2 > 4$ for any $\psi' > 0$

Which are the outcomes that Singh, R. B. two cable-connected satellites moving non-evolutionarily in the presence of no perturbing forces in the main gravitational field

**Conclusion**

We get to the conclusion that the action of the earth's shadow causes a discontinuity in the area of the primary resonance where a system of two linked satellites oscillates. An oscillation's amplitude discontinuity happens at a frequency higher than the resonance frequency.

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