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A simple proof on Fermat's last theorem in case of n=3

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Abstract
Fermat's last theorem was proposed more than 350 years ago, it attracted the interests of a lot of researchers [1-11]. The simplest case of Fermat's last theorem is n=3, but the previous proofs on it are generally complex or not easy to understand. The present work through the transformation x=t+1, firstly proves that when the values of x and $t \in \{t_{min}, t_{max}\} \cap \{x_{min}, x_{max}\}$, the Fermat's last theorem in the case of n=3 is true. Furthermore, the paper also proves that when x take other positive integers besides $x \in \{t_{min}, t_{max}\} \cap \{x_{min}, x_{max}\}$, the Fermat's last theorem in the case of n=3 is true as well. Therefore, there are no a group of positive integers of x, y and z to satisfy the Diophantine equation in the case of n=3; Fermat's last theorem in the case of n=3 is true.

Keywords: Fermat's last theorem, n=3, $\{t_{min}, t_{max}\} \cap \{x_{min}, x_{max}\}$, algebraic equation, induction, disprove method

1. Introduction
Fermat's last theorem was proposed more than 350 years ago, but Pierre de Fermat has never given a proof on this theorem by himself. A lot of people who are familiar with mathematics have put forward various studies on this theorem [1-11]. At first, they like to prove the simplest case of n=3, but in general, their proofs are complex and their theories are not easy to understand. Although Andrew Wiles completely proved the Fermat's last theorem in 1994, his proof published in annual of mathematics with long long 104 pages [6, 7]. Therefore, it is significant to find simpler proofs on the theorem. In consideration of this, the present work will present a simple proof on Fermat's last theorem in case of n=3 by use of the mathematics which are familiarly known by pupils of middle school.

2. The proof
The Diophantine equation in the case of n=3 is written

$$x^3 + y^3 = z^3 \quad (1),$$

Using "the disprove method" if y and z are integers, x can also be an integer, through the transformation x=t+1, eq.(1) becomes

$$(t+1)^3 + y^3 = z^3 \quad (2),$$

namely,

$$3t^2 + 3t + 1 + t^3 + y^3 = z^3 \quad (3).$$

Comparing eq.(3) with eq.(1), $t \in \{t_{min}, t_{max}\}$, $x \in \{x_{min}, x_{max}\}$; but for x and $t \in \{t_{min}, t_{max}\} \cap \{x_{min}, x_{max}\}$, $t^3 + y^3 = z^3$ is completely same as eq.(1),
hence, from eq.(3) it obtains

$$3t^2 + 3t + 1 = 0 \quad (4),$$

According to the theory of algebraic equation, $b^2 - 4ac = 3^2 - 4\times3\times1 = -3 < 0$, there is no significant solution of eq.(4), t and therefore x can't be positive integers.

It demonstrates that when $x \in \{t_{min}, t_{max}\} \cap \{x_{min}, x_{max}\}$, there are no a group of integers for x, y, and z to satisfy eq.(1).

For Fermat's last theorem, t and x can take any positive integers, so $t_{min}=1$, $x_{min}=t_{min}+1=2$; $x_{max}=t_{max}+1$. It is still necessary to discuss the cases of x=1, and $x=t_{max}+1$ for eq.(1). At first, according to the above discussions, when y and z are positive integers, t can't be an integer, therefore, t_{max} is not an integer, and $x_{max}=t_{max}+1$ can't be an integer as well.

Thus, when x=1, eq.(1) is written as

$$1+y^3=z^3 \qquad (5),$$

it is easy to use "the mathematical induction" to discuss whether both y and z can be positive integers,

When y=1, eq. (5) becomes 1+1=2, z can't be an integer,

Supposing that when y=k (k is an arbitrary positive integer), z can't be an integer in eq. (5), thus,

When y=k+1, eq.(5) becomes

$$1+(k+1)^3=z^3 \qquad (6).$$

Using "the disprove method" again, if z can be an integer, from eq. (6) it obtains

$$3k^2+3k+1+k^3=1=z^3 \qquad (7),$$

according to the supposition for y=k, $1+k^3=z^3$, z can't be an integer, so for an arbitrary value of k, it can always obtain a value of z which is not required to be an integer, therefore, $1+k^3=z^3$ is an identity. From eq.(7) it arrives

$$3k^2+3k+1=0 \qquad (8),$$

but $3^2 - 4\times 3\times 1=-3<0$, there is no significant solution of eq.(8), this contradicts the proposition of y=k (k is an arbitrary positive integer), so z can't be an integer if y is an integer in eq.(5).

In conclusion, when y and z are integers in eq.(1), thus, x can't be an integer, therefore, Fermat's last theorem in the case of n=3 is true.

3. Conclusion

The paper proved Fermat's last theorem in case of n=3 by use of the simple algebraic theory, the proof was carried out in two steps, at first, the paper discussed when x and $t \in \{t_{min}, t_{max}\} \cap \{x_{min}, x_{max}\}$, it proved that there are no a group of positive integers of x, y, and z to satisfy eq.(1). Furthermore, it still needs to discuss the cases of x=1, and $x_{max}=t_{max}+1$, the paper proved that there are also no a group of positive integers of x, y, and z to satisfy eq.(1). In general, the paper proved that with respect to all of the positive integers of x, y, and z, there are no a group of positive integers of x, y, and z to satisfy Diophantine equation in case of n=3; Fermat's last theorem in the case of n=3 is true.

4. References

1. Harold M Edwards. The background of Kummer's proof of Fermat's last theorem for regular primes [J]. Archive for History of Exact Science 1975;14(3):219-236.
2. Quang Nguyen Van. Euler's proof of Fermat's last theorem for n=3 is incorrect [J]. viXra.org > Number Theory > viXra:1605.0123, 2016-05.
3. Roy Barbara. Fermat's Last Theorem for n=3 Revised [J]. The Mathematical Gazette 2017;101(550):90-93.
4. Anatoly A. Grinberg. Elementary proof of Fermat's last theorem for even exponents [J]. arXiv.org > math > arXiv: 1707.01772, 2017-07.
5. Kenneth A. Ribet and Brian Hayes. Fermat's last theorem and modern arithmetic [J]. American Scientist, 1994;82 (2):144-156.
6. Davide Castelvecchi. Fermat's last theorem earns Andrew Wiles the Abel prize [J]. Nature, 2016;531:287.
7. Andrew Wiles. Modular Elliptic Curves and Fermat's Last Theorem [J]. Annals of Mathematics (second series), 1995;141(3):443-551.
8. James E Joseph. A proof of Fermat's last theorem using elementary algebra [J]. International Journal of Algebra and statistics 2015;4:39-41.
9. Angel Isaac Cruz Escalante. A Simple Proof of Fermat's Last Theorem [J]. viXra.org > Number Theory > viXra: 1808.0074, 2018-08-07.
10. Perera B B, Upeksha P, Piyadasa RAD. Proof of Fermat's Last Theorem for n=3 Using Tschirnhaus Transformation [C]. The 3th International Conference on Computational Mathematics and Computational Geometry, Springer proceedings in mathematics & statistics, Springer 2016;124:133-136.
11. Levesque C. On a few Diophantine equations, in particular, Fermat's last theorem [J]. Int. J. Math. Math. Sci., 2003;71:4473-4500.