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Temperature distribution of viscous liquid under osciliation rate of heat addition superposed on the steady temperature incompressible fluid two elltptic cylinders

Sarvesh Nigam

Abstract
In this paper expression for temperature distribution in a channel bounded by two elliptical cylinders similarly situated for viscous incompressible fluid flowing through it neglecting for viscous incompressible fluid flowing through it neglecting the dissipation due to friction when as oscillatory rate of head addition is superposed on the steady temperature.

Keywords: Viscous liquid, temperature incompressible, fluid two elltptic cylinders

Introduction
Solutions for temperature distribution in a circular pipe have been given by various authors notably Gretz, Nusselts, Goldstein. All these are cited in (1). Krishna Lal (2) considered the temperature distribution in Co-axial cylinders. S. U. Dube (3) considered temperature distribution in channel bounded by co-axial circular pipe for viscous incompressible fluid flowing through it by neglecting the dissipation due to friction when an oscillatory rate of heat addition is superposed on the steady temperature. Nigam S. (.I, II, III, IV). As the above had been a step forward in continuity of Nigam S. (V, VI, VII)

Energy equation and its solution
The equation of energy, in the present case is

$$\frac{\partial T}{\partial t} = \frac{1}{\rho C_v}\frac{\partial Q}{\partial t} + K'\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \tag{1.1}$$

Where $K' = \frac{K}{\rho C_v}$ a constant and dissipation due to friction is neglected.
Now assume that

$$\frac{1}{\rho C_v}\frac{\partial Q}{\partial t} = \sum_{n=1}^{\infty} a_n e^{int} \tag{1.2}$$

and

$$T = T_\circ + \sum_{n=1}^{\infty} T_n e^{int} \tag{1.3}$$

a_n and T_n are real and $T_n$is function of x and y only.

Substituting equation (1.2) and (1.3) and comparing the terms of the same family the differential equations for the coefficients are.

and

$$\frac{\partial^2 T_\circ}{\partial x^2} + \frac{\partial^2 T_\circ}{\partial y^2} = 0 \tag{1.4}$$

and

$$\frac{\partial^2 T_n}{\partial x^2}+\frac{\partial^2 T_n}{\partial y^2}-\frac{in}{K'}T_n+\frac{a_n}{K\prime}=0 \tag{1.5}$$

How since the boundary of the tube are the ellipses let us change the equation to elliptic coordinates and the equation transform to

$$\frac{\partial^2 T_\circ}{\partial \xi^2}+\frac{\partial^2 T_\circ}{\partial \eta^2}=0 \tag{1.6}$$

Before superposing the oscillatory flow, we must have fully developed steady motions with the following boundary conditions

$T_\circ = T_1$, When $\xi = \xi_1$
$T_\circ = T_2$, When $\xi = \xi_2$

So (1.6) gives

$$T_\circ = \frac{T_1 \sinh 2(\xi_2-\xi)+T_2 \sinh 2(\xi_1-\xi)}{\sinh 2(\xi_2-\xi_1)} \tag{1.7}$$

Now the boundary conditions are

$$T_\circ = T_1 e^{int}+T_1,\ \text{When } \xi = \xi_1 \tag{1.8a}$$

$$T_\circ = T_2 e^{int}+T_2,\ \text{When } \xi = \xi_2 \tag{1.8b}$$

Equation (1.5) on changing to elliptic coordination becomes

$$\frac{2}{C^2(\cosh 2\xi-\cos 2\eta)}\left(\frac{\partial^2 T_n}{\partial \xi^2}+\frac{\partial^2 T_n}{\partial \eta^2}\right)-\frac{in}{K'}T_n+\frac{a_n}{K\prime}=0,$$
$$\left(\frac{\partial^2 T_n}{\partial \xi^2}+\frac{\partial^2 T_n}{\partial \eta^2}\right)-\frac{in}{K'}\frac{C^2}{2}(\cosh 2\xi-\cos 2\eta)T_n+\frac{C_n}{K'}\frac{C^2}{2}(\cosh 2\xi-\cos 2\eta)=0 \tag{1.9}$$

$$\int_{\xi_1}^{\xi_2}\int_{\circ}^{2\Pi}\left(\frac{\partial^2}{\partial \xi^2}+\frac{\partial^2}{\partial \eta^2}\right)T_n \beta_{2p}(\xi,q)C_{e2p}(\eta,q)\,d\xi\, d\eta$$

$$=\int_{\xi_1}^{\xi_2}\beta_{2p}(\xi,q)\left[C_{e2p}(\eta,q)\frac{\partial T_n}{\partial \eta}-T_n\frac{\partial}{\partial \eta}C_{e2p}(\eta,q)\right]_{\circ}^{2\Pi}d\xi$$

$$+\int_{\xi_1}^{\xi_2}\int_{\circ}^{2\Pi}T_n\frac{\partial^2}{\partial \eta^2}\beta_{2p}(\xi,q_{2p,n})C_{e2p}(\eta,q)\,d\eta\, d\xi$$

$$+\left[\beta_{2p}(\xi,q)\frac{\partial T_n}{\partial \xi}-T_n\frac{\partial}{\partial \xi}\beta_{2p}(\xi,q)\right]_{\xi_1}^{\xi_2}2\Pi A_\circ^{2p}$$

$$+\int_{\xi_1}^{\xi_2}\int_{\circ}^{2\Pi}T_n\beta_{2p}\frac{\partial^2}{\partial \xi^2}\left[\beta_{2p}(\xi,q)C_{e2p}(\eta,q)\right]d\eta\, d\xi$$

$$=-4\Pi A_\circ^{(2p)}(T_2e^{int}+T_2-T_1e^{int}-T_1)\beta'_{2p}(\xi,q_{2p,m})$$

$$+\int_{\xi_1}^{\xi_2}T_n\left(\frac{\partial^2}{\partial \xi^2}+\frac{\partial^2}{\partial \eta^2}\right)\xi_{2p,m} \tag{1.10}$$

When

$$\beta'_{2p}(\xi_2,q_{2p,m})=\beta'_{2p}(\xi,q_{2p,m})$$

and

$\beta_{2p}(\xi,q)$ and $C_{e2p}(\eta,q)$ have

Usual meaning where

$$\xi_{2p,m} = \beta_{2p}(\xi, q)C_{e2p}(\eta, q)$$

is the solution of differential equation

$$\frac{\partial^2 \xi}{\partial \xi^2} + \frac{\partial^2 \xi}{\partial \eta^2} + 2q_{2p,m}(\cosh 2\xi - \cos 2\eta)\xi = 0.$$

Hence

$$\int_{\xi_1}^{\xi_2}\int_{\circ}^{2\Pi}\left(\frac{\partial^2 T_n}{\partial \xi^2} + \frac{\partial^2 T_n}{\partial \eta^2}\right)\beta_{2p}(\xi, q)C_{e2p}(\eta, q)d\eta d\xi$$

$$= -2q_{2p,m}\int_{\xi_\circ}^{\xi_1}\int_{\circ}^{2\Pi} T_n(\cosh 2\xi - \cos 2\eta)\beta_{e2p}(\xi, q)C_{e2p}(\eta, q)d\eta d\xi$$

$$+4\Pi A_\circ^{2p}\beta'_{2p}(\xi, q)(T_n e^{int} + T_2 - T_1 e^{int} - T_1)$$

$$= -2q_{2p,m}\bar{T}_n + 4\Pi A_\circ^{2p}(T_2 e^{int} + T_2 - T_1 e^{int} - T_1) \tag{1.11}$$

Where,

$$\bar{T}_n = \int_{\xi_1}^{\xi_2}\int_{\circ}^{2\Pi} T_n(\cosh 2\xi - \cos 2\eta)\beta_{2p}(\xi, q)C_{e2p}(\eta, q)d\eta d\xi.$$

Now to solve equation 1.9 multiply both sides of 1.9 by $\beta_{2p}(\xi, q_{2n,m})C_{e2p}(\eta, q_{2p,m})$and integrating ηwithin the limits $\circ$ to 2Π and ξ with in the limits ξ_1 and ξ_2 we find equations becomes

$$4\Pi A_\circ^{2p}\beta'_{2p}(\xi, q_{2p,m})(T_n e^{int} + T_2 - T_1 e^{int} - T_1) - 2q_{2p,m}\bar{T}_n$$

$$-\frac{in}{K'}\frac{C^2}{2}\bar{T}_n + \frac{a_n}{K'}\frac{C^2}{2}\int_{\xi_1}^{\xi_2}\int_{\circ}^{2\Pi}(\cosh 2\xi - \cos 2\eta)\beta_{e2p}(\xi, q)C_{e2p}(\eta, q)d\eta d\xi = 0$$

Or

$$\left(2q + \frac{in}{K'}\right)\bar{T}_n = 4\Pi A_\circ^{2p}\beta'_{2p}(\xi, q)(T_2 e^{int} + T_2 - T_1 e^{int} - T_1)$$

$$+\frac{a_n}{K'}\frac{C^2}{2}\int_{\xi_1}^{\xi_2}\beta_{2p}(\xi, q)\{2A_\circ^{2p}\cosh 2\xi - A_2^{2p}\}d\xi$$

Or

$$\bar{T}_n = \frac{4\Pi A_\circ^{2p}\beta'_{2p}(\xi, q)(T_2 e^{int} + T_2 - T_1 e^{int} - T_1) + \frac{a_n}{K'}\frac{C^2}{2}\int_{\xi_1}^{\xi_2}\beta_{2p}(\xi, q)\{2A_\circ^{2p}\cosh 2\xi - A_2^{2p}\}d\xi}{\left(2q + \frac{in}{K'}\frac{C^2}{2}\right)}$$

Or

$$\bar{T}_n = \frac{\left(2q - \frac{inC^2}{K'\,2}\right)\left[4\Pi A_\circ^{2p}\beta'_{2p}(\xi, q)(T_2 e^{int} + T_2 - T_1 e^{int} - T_1) + \frac{a_n C^2}{K'\,2}\int_{\xi_1}^{\xi_2}\beta_{2p}(\xi, q)\{2A_\circ^{2p}\cosh 2\xi - A_2^{2p}\}d\xi\right]}{\left(4q^2 - \frac{n^2C^4}{4K'^2}\right)} \tag{1.12}$$

So,

$$T_n = \sum_{p=0}^{\infty}\frac{\sum_{n=1}^{\infty}\beta_{2p}(\xi, q_{2p,m})C_{e2p}(\eta, q_{2p,m})\bar{T}_n}{\Pi\int_{\xi_1}^{\xi_2}\beta_{2p}^2(\xi, q_{2p,m})[\cosh 2\xi - \Theta_{2p,m}]d\xi} \tag{1.13}$$

Now substituting the values of $\bar{T}_n$ in (1.13), we get

$$T_n = \sum_{p=0}^{\infty} \frac{\sum_{n=1}^{\infty} \beta_{2p}(\xi, \dot{q}_{2p,m}) C_{e2p}(\eta, q_{2p,m})}{\Pi \int_{\xi_1}^{\xi_2} \beta_{2p}^2(\xi, q_{2p,m})[\cosh 2\xi - \Theta_{2p,m}] d\xi}$$

$$\times \frac{\left(2q + \frac{in}{K'} \frac{C^2}{2}\right)}{\left(4q^2 - \frac{n^2 C^4}{4K'^2}\right)} \Bigg[4\,\Pi\, A_\circ^{2p} \beta'_{2p}(\xi, q)(T_2 e^{int} + T_2 - T_1 e^{int} - T_1)$$

$$+ \frac{a_n}{K'} \frac{C^2}{2} \int_{\xi_1}^{\xi_2} \beta_{2p}(\xi, q)\{2A_\circ^{2p} \cosh 2\xi - A_2^{2p}\} d\xi \Bigg] \quad (1.14)$$

Thus

$$T = T_\circ + \sum_{n=1}^{\infty} T_n e^{int}$$

$$T = \frac{T_1 \sinh 2(\xi_2 - \xi) + T_2 \sinh 2(\xi_1 - \xi)}{\sinh 2(\xi_2 - \xi)}$$

$$+ \left[e^{in} \frac{\left(2q + \frac{inC^2}{K'\,2}\right)}{\left(4q^2 - \frac{n^2C^4}{4K'^2}\right)} \frac{\sum_{\beta=1}^{\infty} \sum_{m=1}^{\infty} (\xi, q_{2p,m}) C_{e2p}(\eta, q_{2p,m}) \left[4\Pi A_\circ^{2p} \beta'_{2p}(\xi, q)(T_2 e^{int} + T_2 - T_1 e^{int} - T_1) + \frac{a_n C^2}{K'\,2} \int_{\xi_1}^{\xi_2} \beta_{2p}(\xi, q)\{2A_\circ^{2p} \cosh 2\xi - A_2^{2p}\} d\xi \right]}{\Pi \int_{\xi_1}^{\xi_2} \beta_{2p}^2(\xi, q_{2p,m})[\cosh 2\xi - \Theta_{2p,m}] d\xi} \right] \quad (1.15)$$

Conclusion

As in present case it's in the form of double series. The rapidity of convergence is observed in above cases. It can be easily seen that first few terms are sufficient to give the shape of curve. Further it is observed the result hold good for positive Raileigh number. The result obtained shall be beneficial for the industrialist for selecting beneficial type of tube for giving higher output at lower cost.

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