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Contribution of David Hilbert and Stefan Banach in mathematics education and literature

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Abstract
The two mathematicians David Hilbert (1862–1943) and Stefan Banach (1892–1945) are frequently cited regarding the importance of the mathematics in perspectives on contribution in mathematics literature. In the paper, we delve further, exploring the perspectives of these two mathematicians on contributions to mathematics education, how they articulated them, and potential overlaps. David Hilbert and Stefan Banach both made a more contribution in modern mathematics, but the founder of modern functional analysis was Stefan Banach while David Hilbert had direct consequences for important parts of modern functional analysis.

Keywords: Mathematics education, David Hilbert, Stefan Banach

Introduction

The great mathematicians who have come before us have worked tirelessly and consistently to bring mathematics to the heights it has reached now and throughout history. Many of them have achieved immortality by their enduring contributions to their field. They will live on in the gratitude of mankind. The students and intellectuals will continue to be inspired by their lives and deeds. These individuals were geniuses whose contributions would advance humanity forever [1].

The various ways of thinking in mathematics such as induction, algebraic thought, analytic geometry, infinitesimal calculus, topology, probability, etc. have developed in very interesting and peculiar historical contexts, frequently in the minds of very unique thinkers whose accomplishments are best known not for justice but for exemplarity. To situate the major issues, along with their causes and precedents, in time and space. Finally, I'll mention any outstanding issues [2].

David Hilbert and Stefan Banach were little contemporaries, and for a time worked in the same mathematical field. We have chosen these two mathematicians as a starting point for our investigations into past prominent mathematicians' views on contribution in mathematics related matters for two reasons. David Hilbert and Stefan Banach still seem to offer much inspiration to educators and researchers of today. Yet, they are often mentioned without explanation of their points of view and without going into further details. Second, despite the different periods in time, the pedagogical questions that these two mathematicians had to face were, in a certain sense, comparable, insofar as in both periods the deductive and formal structure of mathematics was a much-discussed educational issue. Math movement basing the entire field of mathematics, especially the concept of number, upon set theory (e.g., Beckers, 2019; Phillips, 2015) [3-4].

Recently, however, scholars have attempted to "describe ways in which a history of mathematics course can help prospective teachers of mathematics develop knowledge they will need for teaching" (Huntley & Flores, 2010) [5].

2. Research questions and methodology

A conjoint interpretative investigation of the ideas of several authors was necessary for our study of these two historical mathematicians.

(1) The reconstruction of their individual conceptions; (2) relating these views to present day research. Consequently, our study is guided by the following two research questions:

1. Role of David Hilbert (1862–1943) and Stefan Banach (1892-1945) and their contribution in mathematics education?

2. Concepts of these two mathematicians in mathematics?

The first and the second question is answered in the respective sections on the individual mathematicians.

In relation to methodology, our departure point was the known texts by these two mathematicians in which they touched upon aspects of the contribution of mathematics in relation to mathematics education.

The reader should be aware of the fact that these two mathematicians made a more contribution in modern mathematics and its teaching and learning.

Furthermore, hermeneutics tells us that interpreting texts requires a careful consideration of language and of the way an author expresses ideas (Jahnke, 1994) [6].

3. David Hilbert

David Hilbert [7] German 23 January 1862 – 14 February 1943) was a German mathematician, one of the most influential mathematicians of the 19th and early 20th centuries. Hilbert discovered and developed a broad range of fundamental ideas in many areas, including invariant theory, the calculus of variations, commutative algebra, algebraic number theory, the foundations of geometry, spectral theory of operators and its application to integral equations, mathematical physics, and the foundations of mathematics (particularly proof theory).

Hilbert and his students contributed significantly to establishing rigor and developed important tools used in modern mathematical physics. Hilbert is known as one of the founders of proof theory and mathematical logic [8].

Hilbert solves Gordan's Problem

Hilbert's initial research on invariant functions helped him prove his famous finiteness theorem in 1888. Twenty years prior, Paul Gordan had used a sophisticated computational strategy to prove the theorem of the finiteness of generators for binary forms. The extremely challenging nature of the calculations required prevented attempts to expand his method to functions with more than two variables from succeeding. Hilbert concluded that it was necessary to adopt an entirely new approach in order to solve what had come to be known in some circles as Gordan's Problem. He then proved Hilbert's basis theorem in an abstract form, demonstrating the existence of a finite set of generators for the invariants of quantics in any number of variables. In other words, while proving the existence of such a set, it was an existence proof [9] and depended on the usage of the law of excluded middle in an infinite extension; it did not present "an object."

Axiomatization of geometry

Hilbert suggests a formal set, known as Hilbert's axioms, to replace the conventional Euclid axioms in his 1899 book Foundations of Geometry. They do not have the flaws seen in Euclid's works, which at the time were still taught in textbook fashion. Since Hilbert updated and adjusted his axioms multiple times, it is challenging to define them without consulting the Grundlagen's publishing history. The French version of the original monograph, to which Hilbert added V.2, the Completeness Axiom, came immediately after [10-11].

The current axiomatic technique began to take shape with Hilbert's methodology. This is where Moritz Pasch's 1882 work predates Hilbert. Axioms are not considered to be self-evident facts. Geometry can deal with things for which we have strong intuitions, but it is not required to provide the vague concepts any clear definition. As Hilbert reportedly told Schoenflies and Kötter, the elements, such as point, line, plane, and others, might be replaced by tables, chairs, beer glasses, and other similar objects [12]. We talk about their clearly established relationships.

Point, line, plane, resting on (a relation between points and lines, points and planes, and lines and planes), between-ness, congruence of pairs of points (line segments), and congruence of angles are the first undefined notions listed by Hilbert. The axioms combine Euclid's solid geometry and plane geometry into a single framework.

The 23 problems

At the International Congress of Mathematicians in Paris in 1900, Hilbert presented the most significant list, which included 23 open issues. This is regarded as the most accomplished and carefully thought-out collection of open questions ever created by a single mathematician.

Hilbert could have expanded to other areas of mathematics after reworking the fundamentals of classical geometry. The subsequent "foundationalist" Russell-Whitehead or "encyclopedist" Nicolas Bourbaki, as well as his contemporaries Giuseppe Peano, took a different approach. He highlighted several of the issues as vital components of significant areas of mathematics, and the mathematical community as a whole may engage in them.

The talk "The Problems of Mathematics" that was delivered as part of the Second International Congress of Mathematicians taking place in Paris served as the starting point for the problem set. In his speech's introduction, Hilbert stated:

Who among us would not be glad if we could pull back the curtain that hides the future and look forward to seeing how our research will advance over the coming centuries? What goals will the spirit of the next generation of mathematicians be driven by? What new techniques and discoveries in the wide and diverse field of mathematical thought will the new century bring? [13].

Hilbert's program

Hilbert's agenda is a research initiative in mathematics that he suggested in 1920. He desired a full and sound logical foundation for mathematics. In theory, he thought, this might be accomplished by demonstrating that:

1. All of mathematics follows from a correctly chosen finite system of axioms.
2. That some such axiom system is provably consistent through some means such as the epsilon calculus.

He appears to have developed this approach for both philosophical and technological reasons. It confirmed his distaste for the ignorabimus, a concept that had come to be associated with German philosophy throughout his lifetime and whose conception could be traced back to Emil du Bois-Reymond.

The most common school of mathematics philosophy, where this programme is still understood, calls it formalism. As an illustration, the Bourbaki group embraced a diluted and selected version of it as meeting the needs of their dual aims of (a) creating encyclopaedic foundational books and (b) promoting the axiomatic approach as a research methodology. While Hilbert's work in algebra and functional analysis has benefited from and been influenced by this method, his interests in physics and logic have not been similarly piqued.

Hilbert wrote in 1919:

In no way are we discussing arbitrariness here. Mathematics is not like a game where the objectives are decided by arbitrary rules. Instead, it is a conceptual framework with an underlying imperative that can only be this way and in no other way [14].

Hilbert published his views on the foundations of mathematics in the 2-volume work, Grundlagen der Mathematik.

Functional analysis

Hilbert committed himself to the study of differential and integral equations about 1909, and his work directly influenced a significant portion of contemporary functional analysis. Hilbert developed the idea of an infinite dimensional Euclidean space, subsequently known as Hilbert space, in order to conduct this research. His work in this area of analysis served as the foundation for significant advancements in physics mathematics during the following two decades, though in an unanticipated way. Later, Stefan Banach expanded the idea by introducing the idea of Banach spaces. In the field of functional analysis, particularly in the spectrum theory of self-adjoint linear operators, which developed around it in the 20th century, Hilbert spaces are a significant class of objects.

4. Stefan Banach

Stefan Banach, a Polish mathematician who lived from 30 March 1892 to 31 August 1945 [15], is regarded as one of the most significant and influential mathematicians of the 20th century [16]. He was a founding member of the Lwów School of Mathematics and the creator of contemporary functional analysis. His most important publication was the first monograph on the general theory of functional analysis, Théorie des opérations linéaires (Theory of Linear Operations), published in 1932 [15].

The idea of a complete normed vector space was formally axiomatized by Banach in his dissertation, which was finished in 1920 and published in 1922. It also lay the groundwork for the field of functional analysis. Such spaces were referred to as "class E-spaces" by Banach in this work, but he referred to them as "spaces of type B" in his 1932 book Théorie des opérations linéaires, which is likely why these spaces were later given his name [17]. The work of the Hungarian mathematician Frigyes Riesz, which was published in 1916, and contemporaneous contributions by Hans Hahn and Norbert Wiener were the forerunners of the theory of what later came to be known as Banach spaces [18]. In reality, based on terminology Wiener himself proposed, complete normed linear spaces were briefly referred to as "Banach-Wiener" spaces in mathematical literature. However, because to Wiener's scant research on the subject, the established name was changed to Banach spaces [17].

Also included in his dissertation was Banach's fixed point theorem, which was later expanded upon by his students (for example, in the Banach-Schauder theorem) and other mathematicians (particularly Brouwer, Poincaré, and Birkhoff). This theorem was based on earlier techniques created by Charles Émile Picard. The theorem applied to any full Cauchy space and did not require that the space be linear (in particular to any complete metric space) [18].

The Hahn–Banach theorem is one of the fundamental theorems of functional analysis [18]. Further theorems related to Banach are:

- Banach–Tarski paradox
- Banach–Steinhaus theorem
- Banach–Alaoglu theorem
- Banach–Stone theorem

Some of the notable mathematical concepts that bear Banach's name include Banach spaces, Banach algebras, Banach measures, the Banach–Tarski paradox, the Hahn–Banach theorem, the Banach–Steinhaus theorem, the Banach–Mazur game, the Banach–Alaoglu theorem, and the Banach fixed-point theorem.

Stefan Banach contributed to the theory of orthogonal series and made innovations in the theory of measure and integration. Together with his coworkers Banach summarized the previously developed concepts and theorems of functional Analysis and integrated them into a comprehensive system. His work started, of course, from what was achieved during the decades following Vito Volterra's work of the 1890's on integral equation [19]. Before Banach there were either rather specific individual results that only much later were obtained as applications of general theorems, or relatively vague general concepts. Ivar Fredholm's and David Hilbert papers on integral equations marked the most substantial progress [20]. The concepts and theorems they had discovered later became an integral part of the Functional Analysis, but most of them concern only a single linear space (later called Hilbert Space) [21]. Banach himself introduced the concept of normed linear space, which are also known as Banach space. He also proved serval fundamental theorems in the field, and his applications of theory inspired much of the work in Functional Analysis for the next few decades [22].

5. Conclusion

The results of our analysis are presented in according with research questions. First, we solicit opinions on the relative contributions of the two concepts of mathematics (research question 2). We have considered, David Hilbert and Stefan Banach both made a more contribution in modern mathematics, but if we talk about functional Analysis, Stefan Banach who is generally considered one of the 20th century's most important and influential mathematicians. He was the founder of modern functional analysis. While David Hilbert had direct consequences for important parts of modern functional analysis and also David Hilbert is known as one of the founders of proof theory and mathematical logic.

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