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## Implementation of fuzzy multi: Criteria decision making for recommendation student selection

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### Abstract

Multi Criteria Decision Making (MCDM) uses different techniques to find a best alternative from multi-alternative and multi-criteria conditions. TOPSIS is an important practical technique for ranking and selection of different alternatives by distance measures. Fuzzy set theories are also employed due to the presence of vagueness and imprecision of information. This paper brings out the application of Multi Criteria Decision Making (MCDM) method known as technique for order preference by similarity to ideal solution (TOPSIS) using pentagonal intuitionistic fuzzy numbers. Therefore the objective of this paper is to select a best student on the basis of the student's performance using ranking techniques for pentagonal intuitionistic fuzzy numbers can be converted into crisp value with the help of this value using TOPSIS method we get the best student of college. An example has been worked out to illustrate the application of TOPSIS for a multi-criteria decision making scenario.

**Keywords:** MCDM, TOPSIS, vagueness, fuzzy numbers, intuitionistic fuzzy numbers, pentagonal intuitionistic fuzzy numbers etc.

### 1. Introduction

Multi Criteria Decision Making is based upon formation and designing decision and outlining problems composed of complex multi pattern. The whole purpose is to give decision makers a feasible solution to such problems. Predictably, there does not exist an exclusive optimal answer for such matter and it is mandatory to utilize the choice maker's performance to evaluate and characterize between solutions. MCDM is a dynamic region of research since the 1960's. Different approach has been proposed by distinct scholars to solve the MCDM problems.

In general, Multi Criteria Group Decision Making (MCGDM) problems are frequently evaluated. To solve problems related to decisions making several optimization methods are use in practices. But, in case where decision activity is based on similar options it becomes critical to analyze various factors, alternatives with similar category.

One simple example, a group of three person (say A, B, and C) intends to determine which mobiles phone is to buy based on certain criteria, Let they have various criteria like price, model quality, screen size, battery life and memory etc. But each person among A, B and C may give different importance to different criteria. So, now it becomes challenge to decision makers (i.e. A, B and C) to find which alternative best meets the group's criteria.

Here in this paper, we shall be working on pentagonal intuitionistic fuzzy numbers and its Accuracy Function to solve the TOPSIS. Initially the rating of choice is represented as pentagonal intuitionistic fuzzy numbers. The Accuracy Function is developed for the decision making applied to TOPSIS method with pentagonal intuitionistic fuzzy numbers.

### 2. Literature review

The TOPSIS (Technique for order of preference by Similarity to Ideal Solution) is a Multi-Criteria Decision analysis method proposed by <sup>[1]</sup> which was further extended by <sup>[2]</sup>. TOPSIS is set upon the concept that the selected alternative should have the minimum distance from Positive Ideal Solution (PIS) and maximum distance from Negative Ideal Solution (NIS).

Fuzzy set theory was proposed by <sup>[3]</sup> to represent non exact information into a better form. Later, <sup>[4-5]</sup> gave the idea of Intuitionistic fuzzy set (IFS) as more compact and precise form of fuzzy set. Different types of fuzzy numbers and various actions on them were researched by

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many researchers. They investigated on various properties and fluctuations of intuitionistic fuzzy numbers and the first property of correlation between these numbers. Intuitionistic fuzzy sets are already proven to be commodious deal with vagueness and perplexity. Both the degree of membership and non-membership functions in IFS combined by the sum is less than one.

Many researchers used fuzzy numbers in decision making by considering new parameters and present their précised application in MCGDM, consisting of medical and smart phone selections [6-7]. Prediction of games is very curious topic and fuzzy numbers can be used to predict a sport is proposed by [8-9]. Soft sets are considered more precise in vague and hesitate environment. Many researchers discussed the applications, considering MCDM problems but in recent, using accuracy function in uncertain and vague environment a generalized TOPSIS is proposed [10-11]. But still there are some problems which are solved by fuzzy numbers due to their graphical representations. Ranking of optimal solution using octagonal numbers is also proposed by [12]. Fuzzy numbers are used in the problems having fluctuations. Triangular, Trapezoidal, pentagonal numbers are used in uncertain environment to deal with the fluctuations [13-14]. Development of fuzzy to intuitionistic and into neutrosophic and then further divisions of numbers are done by [15]. Ye. Worked in intuitionistic environment and developed a new theory to tackle the problems having uncertain environment [16].

**3. Preliminaries**

In this section, we review the fundamental definitions of fuzzy set theory, initiated by and Zadeh, [3].

**3.1 Fuzzy Set (FS)**

Let X be a set. A fuzzy set A on X is defined to be a function  $A : X \rightarrow [0; 1]$  or  $A\mu : X \rightarrow [0; 1]$ , Equivalently, a fuzzy set A is defined to be the class of objects having the following representation  $A = \{(x, A\mu x) : x \in X\}$  where  $A\mu : X \rightarrow [0; 1]$ , is a function called the membership function of A [16].

**3.2 Intuitionistic Fuzzy Set (IFS)**

Let X is a non-empty set. An intuitionistic fuzzy set A in X is an object having the form A is equal to  $\{(x, A\mu(X), \nu_A(X) ; X \in X\}$  where the function  $A\mu(X), \nu_A(X) : X \rightarrow [0,1]$  define respectively, [4] the degree of membership and the degree of non-membership of the element  $x \in X$  to the set A, which is a subset of X, and for every element  $x \in X, 0 \leq A\mu(X) + \nu_A(X) \leq 1$ .

A decision maker may not get deterministic alternatives in many real life situations. To overcome this problem, the best suitable tool is fuzzy numbers.

**3.3 Fuzzy Number (FN) [3]**

A fuzzy number A is a fuzzy set of the real line with a normal, (fuzzy) convex and continuous membership function of bounded support.

**3.4 Pentagonal Fuzzy Number (PFN)**

A pentagonal fuzzy number (PFN) of a fuzzy set A is defined as  $A_p = \{a, b, c, d, e\}$ , and its membership function is

given by: [16]

$$\mu_{A_p}(X) = \begin{cases} 0 & , \text{ for } x < a ; \\ \frac{(x-a)}{(b-a)} & , \text{ for } a \leq x \leq b ; \\ \frac{(x-b)}{(c-b)} & , \text{ for } b \leq x \leq c ; \\ 1 & , \text{ for } x = c ; \\ \frac{(d-x)}{(d-c)} & , \text{ for } c \leq x \leq d ; \\ \frac{(e-x)}{(e-d)} & , \text{ for } d \leq x \leq e ; \\ 0 & , \text{ for } x > e . \end{cases}$$

**3.5 Pentagonal Intuitionistic Fuzzy Number (PIFN)**

A pentagonal intuitionistic fuzzy number  $A^1$  of an intuitionistic fuzzy set is defined as  $A^1 = (a_1, b_1, c_1, d_1, e_1) (a_2, b_2, c_2, d_2, e_2)$  Where all  $a_1, b_1, c_1, d_1, e_1$  and  $a_2, b_2, c_2, d_2, e_2$  are real numbers and its membership function  $\mu_{A^1}(X)$ , non-membership function  $\nu_{A^1}(X)$  are given by: [12]

$$\mu_{A_p}(X) = \begin{cases} 0 & , \text{ for } x < a_1 ; \\ \frac{(x-a_1)}{(b_1-a_1)} & , \text{ for } a_1 \leq x \leq b_1 ; \\ \frac{(x-b_1)}{(c_1-b_1)} & , \text{ for } b_1 \leq x \leq c_1 ; \\ 1 & , \text{ for } x = c_1 ; \\ \frac{(d_1-x)}{(d_1-c_1)} & , \text{ for } c_1 \leq x \leq d_1 ; \\ \frac{(e_1-x)}{(e_1-d_1)} & , \text{ for } d_1 \leq x \leq e_1 ; \\ 0 & , \text{ for } x > e_1 . \end{cases}$$

$$\nu_{A^1}(X) = \begin{cases} 1 & , \text{ for } x < a_1 ; \\ \frac{(b_2-x)}{(b_2-a_2)} & , \text{ for } a_2 \leq x \leq b_2 ; \\ \frac{(c_2-x)}{(c_2-b_2)} & , \text{ for } b_2 \leq x \leq c_2 ; \\ 0 & , \text{ for } x = c_1 ; \\ \frac{(x-c_2)}{(d_2-c_2)} & , \text{ for } c_2 \leq x \leq d_2 ; \\ \frac{(x-d_2)}{(e_2-d_2)} & , \text{ for } d_2 \leq x \leq e_2 \end{cases}$$

**4. Accuracy function of an pentagonal intuitionistic fuzzy number**

Let  $A^1 = \{(a_1, b_1, c_1, d_1, e_1), (a_2, b_2, c_2, d_2, e_2)\}$  be a pentagonal intuitionistic fuzzy number. Then its accuracy

function  $H(A^1)$  is given by:

$$H(A^1) = \frac{(a_1 + a_2 + b_1 + b_2 + c_1 + c_2 + d_1 + d_2 + e_1 + e_2)}{5}$$

**5. Proposed TOPSIS Algorithm**

Technique for Order Performance by Similarity to Ideal Solution (TOPSIS) is one of the most classical methods for solving MCDM problem. According to this technique, the best alternative would be the one that is nearest to the ‘Positive Ideal Solution (PIS)’ and the farthest from the ‘Negative Ideal Solution (NIS)’ for solving a Multiple Criteria Decision Making Problem (MCDMP). In short, the positive ideal solution is composed of all best values attainable of criteria; whereas the negative ideal solution is made up of all worst values attainable of criteria. In this method two artificial alternatives are hypothesized. It is based on a principle where the chosen alternative should have the longest distance from the negative-ideal solution, i.e. *the solution that maximizes the cost criteria and minimizes the benefits criteria*; and the shortest distance from the positive-ideal solution, i.e., *the solution that maximizes the benefit criteria and minimizes the cost criteria* [17-18].

The procedures of calculation for this proposed fuzzy TOPSIS model can be described as following steps:

**Step 1.** Using accuracy function of pentagonal intuitionistic fuzzy number, fuzzy value can be converted into crisp value.

**Step 2. Construction of Decision Matrix**

First of all, a decision matrix  $DM = [X_{ij}]_{m \times n}$ , where  $i = 1, 2, 3, \dots, m$  and  $j = 1, 2, 3, \dots, n$  comprising of m alternatives and n criteria is designed as:

$$\begin{matrix} C_1 & C_2 & \dots & C_n \\ \begin{bmatrix} X_{11} & X_{12} & \dots & X_{1n} \\ X_{21} & X_{22} & \dots & X_{2n} \\ \vdots & \vdots & & \vdots \\ X_{m1} & X_{m2} & \dots & X_{mn} \end{bmatrix} \end{matrix} \quad (1)$$

$[X_{ij}]_{m \times n}$  represents score of the  $i^{th}$  alternative regarding the  $j^{th}$  criteria. Our environment is intuitionistic fuzzy, so the initial decision matrix would be in IFN.

Whatever the intuitionistic fuzzy score, it can be reduced to crisp value using the accuracy formula for intuitionistic fuzzy number. The final decision matrix obtained in this step would now be in crisp environment after the application of accuracy function formula.

**Step 3. Normalization**

Decision Matrix is then normalized to form a normalized

decision matrix  $R = [r_{ij}]_{m \times n}$  by:

$$r_{ij} = \frac{X_{ij}}{\sqrt{\sum_{i=1}^m X_{ij}^2}} ; i = 1, 2, 3, \dots ; n \text{ and } j = 1, 2, 3, \dots ; m \quad (2)$$

Where  $X_{ij}$  and  $r_{ij}$  are original and the normalized score of decision matrix respectively.

**Step 4. Computation of Weighted Normalized Decision Matrix**

Weighted normalized decision matrix by multiplying the weights  $w_i$  of evaluation criteria with the normalized decision matrix  $r_{ij}$ .

$$V_{ij} = W_j X r_{ij} \quad j = 1, 2, 3 \dots, m \text{ and } i = 1, 2, 3 \dots, n \quad (3)$$

**Step 5. Calculation of PIS and NIS**

Positive Ideal Solution:

$$A^+ = \{ V_1^+, V_2^+, \dots, V_n^+ \} \text{ maximum values} \quad (4)$$

Where

$$V_i^+ \{ \max (V_{ij}) \text{ if } j \in J ; \min (V_{ij}) \text{ if } j \in J^- \}$$

Negative Ideal Solution:

$$A^- = \{ V_1^-, V_2^-, \dots, V_n^- \} \text{ minimum values} \quad (5)$$

Where

$$V_i^- \{ \min (V_{ij}) \text{ if } j \in J ; \max (V_{ij}) \text{ if } j \in J^- \}$$

**Step 6. Determination of separation measure for each alternative**

Separation Measure of each alternative is to be measured from PIS and NIS respectively.

$$S_i^+ = \sqrt{\sum_{j=1}^n (V_j^+ - V_{ij})^2} \quad i = 1, 2, 3, \dots, m \quad (6)$$

$$S_i^- = \sqrt{\sum_{j=1}^n (V_j^- - V_{ij})^2} \quad i = 1, 2, 3, \dots, m \quad (7)$$

**Step 7. Computation of Relative Closeness to Ideal Solution  $C_i$**

For each alternative, Closeness coefficient is calculated by:

$$C_i = \frac{S_i^-}{(S_i^+ + S_i^-)} , \quad 0 \leq c_i < 1, \quad i = 1, 2, \dots, m \quad (8)$$

**Step 8 Result**

Alternatives get ranked depending upon the closeness coefficient from most beneficial to least value. The alternative possessing highest value of closeness coefficient is then taken into account.

**6. Numerical analysis**

In this section, in order to demonstrate the calculation process of the proposed approach, an example is provided. The hierarchical structure of decision making problem is formed as shown in figure 1.

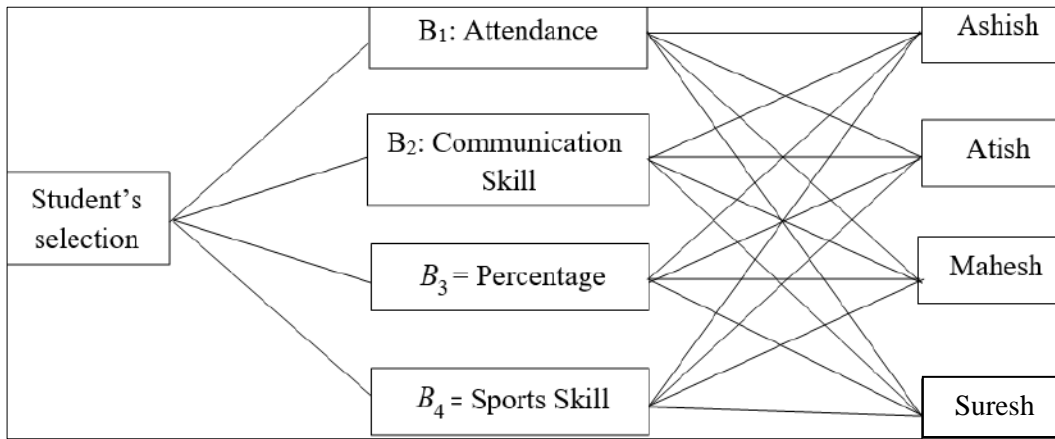


Fig 1: Hierarchical structure student’s selection decision making

**Statement 6.1:** A teacher is desirable to select the best student on the basis of choice parameter. After pre-evaluation four students  $A_i$  ( $i = 1, 2, 3, 4$ ) have remind as alternatives for further evaluation. Four criteria are considered as:

$B_1 =$  Attendance,  $B_2 =$  Communication Skill;  $B_3 =$  Percentage  $B_4 =$  Sports Skill. (Whose weighting vector is completely unknown) the following data form 6.1 is constructed.

**Initial decision matrix in pentagonal intuitionistic fuzzy environment**

Performance Students	Attendance ( $B_1$ )	Communication Skill ( $B_2$ )	Percentage ( $B_3$ )	Sport Skills ( $B_4$ )
Ashish	{(3,4,5,6,7); (4,5,6,7,8)}	{(4,6,7,8,9); (7,9,12,13,15)}	{(2,10,12,13,14); (7,10,11,12,14)}	{(2,4,6,8,9); (7,8,9,10,12)}
Atish	{(2,5,6,7); (3,4,5,6,9)}	{(8,9,10,11,12); (9,11,13,15,17)}	{(6,8,14,15,16); (8,9,13,15,16)}	{(4,5,6,9,8); (5,6,7,8,9)}
Mahesh	{(5,7,10,11,12); (6,7,8,9,10)}	{(1,4,5,6,8); (8,9,10,11,13)}	{(7,9,12,13,14); (4,6,10,12,13)}	{(2,4,7,8,9); (7,8,9,10,11)}
Suresh	{(5,6,7,8,9); (10,13,15,17)}	{(2,3,4,5,6); (5,6,7,8,9)}	{(7,11,13,14,16); (10,12,13,14,15)}	{(4,8,10,11,12); (7,78,9,10,11)}

Proposed fuzzy TOPSIS decision making model is applied to solve this problem, and the computational procedure is described step by step as given below:

**Step 1:** By the use of accuracy function, we defuzzified the above values into crisp notation given by:

Performance Students	Attendance	Communication Skills	Percentage	Sport Skills
Ashish	0.0005	0.0009	0.0007	0.0016
Atish	0.0009	0	00.0003	0.0040
Mahesh	0.00001	0.0021	0.0010	0.0016
Suresh	0	0.0049	0	0

**Defuzzified Decision Matrix**

Performance Students	Attendance ( $B_1$ )	Communication Skill ( $B_2$ )	Percentage ( $B_3$ )	Sport Skills ( $B_4$ )
Ashish	11	18	21	15
Atish	10	23	24	13
Mahesh	17	15	20	15
Suresh	18	11	25	18
Weight	0.1	0.2	0.3	0.4

**Step 2:** Calculate  $(\sum x_{ij}^2)^{1/2}$  for each column and divide each column by that to get  $r_{ij}$

Performance Students	Attendance ( $B_1$ )	Communication Skill ( $B_2$ )	Percentage ( $B_3$ )	Sport Skills ( $B_4$ )
Ashish	0.38	0.51	0.46	0.48
Atish	0.31	0.66	0.53	0.42
Mahesh	0.58	0.4	0.44	0.48
Suresh	0.62	0.31	0.55	0.58
Weight	0.1	0.2	0.3	0.4

Then it is multiplied weight criteria. Therefore it is  $V_{11} = 0.1 \times 0.38 = 0.038$ ,  $V_{12} = 0.2 \times 0.51 = 0.102$ ,  $V_{13} = 0.3 \times 0.46 = 0.138$ ,  $V_{14} = 0.4 \times 0.48 = 0.192$

**Step 5:** Find the Positive Ideal Solution (PIS)  $A^+$  and Negative Ideal Solution (NIS)  $A^-$   
 $A^+ = \{0.062, 0.132, 0.165, 0.232\}$

$A^- = \{0.031, 0.062, 0.132, 0.168\}$

Performance Students	Attendance	Communication Skills	Percentage	Sport Skills
Ashish	0.007	0.0016	0.00003	0.0005
Atish	0	0.0049	00.0007	0
Mahesh	0.007	0.005	0.007	0.005
Suresh	0.0009	0	0.0010	0.004

**Step 6:** Determine the separation from ideal solution  $S_i^+$

**Separation of Positive Ideal Solution (PIS)  $S_i^+$**

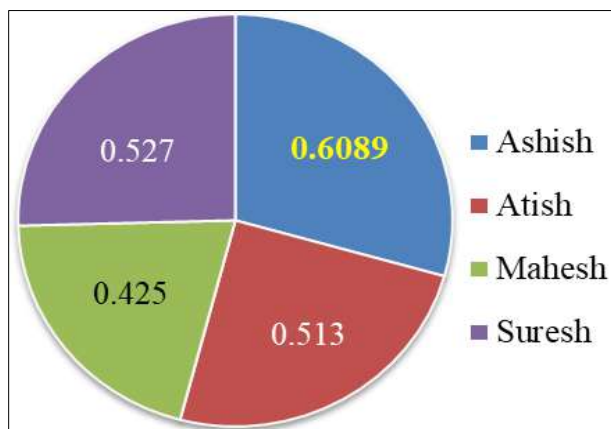
	$\sum_{j=1}^n (V_j^+ - V_{ij})^2$	$S_i^+ = \left[ \sum_{j=1}^n (V_j^+ - V_{ij})^2 \right]^{1/2}$
Ashish	0.0038	0.061
Atish	0.0050	0.071
Mahesh	0.0048	0.069
Suresh	0.0049	0.07

**Separation of Negative Ideal Solution (NIS)  $S_i^-$**

	$\sum_{j=1}^n (V_j^- - V_{ij})^2$	$S_i^- = \left[ \sum_{j=1}^n (V_j^- - V_{ij})^2 \right]^{1/2}$
Ashish	0.0092	0.095
Atish	0.0056	0.075
Mahesh	0.0026	0.051
Suresh	0.0061	0.078

**Step 7:** Calculate the relative closeness to the ideal solution  
 $C_i^+ = S_i^- / (S_i^+ + S_i^-)$

	$S_i^- / (S_i^+ + S_i^-)$	$C_i^+$
Ashish	0.095/0.156	0.6089 *
Atish	0.075/0.146	0.513
Mahesh	0.051/0.12	0.425
Suresh	0.078/0.148	0.527



**Fig 2:** Best students selection

**Result:** Ashish > Suresh > Atish > Mahesh

Hence it is concluded that the Ashish is the best student for this TOPSIS method.

**7. Conclusion**

This research paper focused on MCDM issues in intuitionistic fuzzy environment problems. In which the assessment of alternatives are represented as pentagonal intuitionistic fuzzy numbers. The accuracy of ranking method is developed for the MCDM and applied to TOPSIS technique with pentagonal intuitionistic fuzzy numbers which reduces the complexity of the environment from complex intuitionistic fuzzy to crisp. With the help of derived results we conclude the teacher can select the best student by using their choice factors.

Furthermore, the proposed method may be suitable for different MCDM problems, such as management problems (e.g., location selection and project management) and supply chain problems (e.g., supplier selection problems) when available data are inaccurate, vague, imprecise and ambiguous by nature.

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