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Model specification test against non-nested univariate and multivariate nonlinear regression models

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Abstract
In an econometric model, Non-nested hypothesis tests give the best path to test the specification of univariate and multivariate Regression models. The model introduced by Cox for evaluate different set of hypotheses was used to the alternative between two non-nested linear regression models. This paper examines the current literature on non-nested univariate and multivariate hypothesis testing in the context of nonlinear regression and related models. The paper also covered testing the hypothesis for non-nested univariate and multivariate nonlinear regression models. The principal part of the article derives the results and explains that they are identifiable as generalizations of the univariate-equation case. It is also revealed that the computation of the test statistic involves very little calculation beyond that necessary to estimate the models.

Keywords: Non-nested, univariate, multivariate, nonlinear regression

1. Introduction

Applied econometrician had the experience of assessing a relapse model which appeared, from the outset, to be exceptionally acceptable, however which accordingly ended up, on nearer examination, to be bogus and deluding. The idea of economic data makes this inescapable. In recent era researchers are facing lot problems to find the best model to fit the linear and nonlinear regression models. In addition, especially when the research is at the beginning stages, the researcher may not know whether the existing models could possibly be true or not. The first step is to set the test the specification of each of the available models. Tests for multicollinearity, Consistency of parameter, heteroskedasticity, etc., obviously may be plays vita role in this perspective.

In economic models, non-nested hypothesis provides a path to test the one or more non nested alternative. In the year (1962, 1963) Cox discussed briefly about non-nested models. In the regression analysis parameter, stability, multicolinearilty, serial correlation, heteroscedasticity etc are playing major role. But there are many tests cannot utilize the data that the model being tested is only one of the few model to make sense of similar sort of information. In much case, on the off chance that H_0 is true, and then it follows any non-nested test, say H_1 must be false. Such test commonly named as "Non-nested hypothesis" test.

In an econometric model, if $H_{0,}$ tested with H_1, then H_1 can be condensed to H_0 based on the few multiple constraints on its parameters. Similarly I production functions, Cobb-Douglas production function is nested with in C.E.S production function, because, the elasticity of substitution is unity. Based on this we will consider H_0 and H_1 may be non-nested, if H_0 is not nested with H_0.

Test for model specification (or)te sting of no nested hypothesis are the models for correlation (or) omitted variables. We have sufficient literature is also available on model specification tests. Anselin L (1984) [25], Bera, A. and M. McAleer (1989) [3], Sawa (1978) [23], Sawyer (1980) [24] are contributed their effort on non-nested hypothesis test.

2. Cox test

The hypothetical writing on non-nested testing was basically initiated by Cox (1961, 1962). The principal benefit of the Cox test is, it made sense of overall and simple in nature. The fundamental course of action of this test is that one might test the legitimacy of an null hypothesis (H_0) about how a group of data was created by looking at the noticed proportion of the values of the likelihood functions for H0 and for some non-nested alternative hypothesis, H1, with an estimate of the expected value of this likelihood ratio if H0 were true.

If H1 fits either better or worse than it should if H0 were true, then H0 must be false.
Consider the Cox test statistic

$$T_0 = L(\hat{\theta}_0) - L(\hat{\theta}_1) - T\left(P\lim_{n\to\infty} o\frac{1}{T}\left(L(\hat{\theta}_0) - L(\hat{\theta}_1)\right)\right)_{\theta=\theta_0} \quad (2.1)$$

Here θ_0 and θ_1 vector parameters under the null and alternative hypothesis respectively. T is the no. of observations. $L(\hat{\theta}_i)\ \forall i = 0,1$ is the likelihood function and T_0 is asymptotically normally distributed, with mean zero.
The major problem in Cox test is, to evaluate the third term in (2.1). In (1973) Amemiya tested unconventional specifications of the distribution error term in regression models. Consequently Persaran and Deaton (1978) [15] expanded the Persaran derivation to the case of nonlinear univariate and nonlinear multivariate models.

3. Non-nested univariate nonlinear regression model

Consider univariate nonlinear regression model

$$H_0 : Y = f(\beta) + \varepsilon_0$$

Where

$$\varepsilon_0 \sim N(0, \sigma^2 I) \quad (3.1)$$

Here Y is dependent variable vector, $f(\theta)$ is function of vector

$$H_1 : Y = g(\gamma) + \varepsilon_1$$

Where

$$\varepsilon_1 \sim N(0, \sigma^2 I) \quad (3.2)$$

The model H_0 and H_1 are assumed to be non-nested, then

$$TP\lim_{n\to\infty} o\frac{1}{T}L(\hat{\theta}_0) \quad (3.3)$$

Evaluated at $\hat{\theta}_0$ is simply $L(\hat{\theta}_0)$ then (3.1) becomes

$$T_0 = -L(\hat{\theta}_1) + T\left[P\lim_{n\to\infty} o\frac{1}{T}L(\hat{\theta}_1)\right]\theta_0 = \hat{\theta}_0 \quad (3.4)$$

The concentrated log likelihood function for H_1 is

$$L(\hat{\theta}_1) = -\frac{T}{2}\log\hat{\sigma}_1^2 \quad (3.5)$$

$$\hat{\sigma}_1^2 = \frac{1}{T}(Y - \hat{g})^T (y - \hat{g}) \quad (3.6)$$

Here

Here $\hat{g}$ denotes $g(\hat{\gamma})$, i.e., the estimated values of H_1 computed at maximum likelihood estimates $\hat{r}$.
As we know, $L(\hat{\theta}_1)$ depends only on $\hat{\sigma}_1^2$, then we have to find the Plim of $\hat{\sigma}_1^2$ under H_0, in order to calculate the second term in (3.4).

$$\tilde{\sigma}_1^2 \equiv \frac{1}{T}(Y - \tilde{g})^T (Y - \tilde{g}) \quad (3.7)$$

$$= \frac{1}{T}(Y - f + f - \tilde{g})^T (Y - f + f - \tilde{g}) \quad (3.8)$$

$$= \frac{1}{T}(Y-f)^T(Y-f) + (f-\tilde{g})^T(f-\tilde{g}) + 2(Y-f)^T(f-\tilde{g}) \quad (3.9)$$

Here f is the function of $f(\theta)$. Here $(Y - f)$ a vector follow normal distribution with mean zero and variance σ_0^2 then the estimate $P\lim\hat{\sigma}_1^2$ is

$$\tilde{\sigma}_{10} = \hat{\sigma}_0^2 + \frac{1}{T}(\hat{f} - \tilde{g})^T(\hat{f} - \tilde{g}) \quad (3.10)$$

Based on the (3.4) (3.5) and (3.10), the numerator for Cox test

$$T_0 = \frac{T}{2}\log\frac{\hat{\sigma}_1^2}{\tilde{\sigma}_{10}^2} \quad (3.11)$$

Since nonlinear regression

$$\hat{f} = g(\gamma) + e \quad (3.12)$$

An estimative variance of 'To

$$\hat{V}(To) = \frac{\hat{\sigma}_0^2}{\tilde{\sigma}_{10}^4}(\hat{f} - \tilde{g})^T\left[I - \hat{F}(\hat{F}^T\hat{F})^{-1}\hat{F}^T\right](\hat{f} - \tilde{g}) \quad (3.13)$$

Where 'F' is the Marix derivatives of $f(\theta)$ w.r. to θ
$\therefore$ According to the Pesaran, M. H. and Deaton, A. S. (1978), Cox test statistic for nonlinear regression model is

$$N_o = \frac{T_o}{\sqrt{\hat{V}_o(T_o)}} \quad (3.14)$$

The above N_o provides a test of H_o and it explains about the validity of 'H_1'. If N_o is less than zero, it is explaining that, H is rejected under the directions away from 'H_1' and N_o is greater than zero then H_o is rejected in the favour of H_1

4. Non-nested multivariate nonlinear regression model

Cox test and a number of methods of the t-test may be applied in many cases of multivariate nonlinear regression models. So

according to that,
The null hypothesis is

$$H_0 : Y_{ti} = f_{t1}(\theta) + e_{ti}^0$$

Where

$$e_{ti}^0 \sim N(0, \Omega_0) \quad (4.1)$$

The Alternative Hypothesis is

$$H_1 : Y_{ti} = g_{ti}(-r) + e_{ti}^1$$

Where

$$e_{ti}^1 \sim N(0, \Omega_1) \quad (4.2)$$

Here 'i' tends to m equations and t tends to 'T' observation and Ω_j is the mxn covariance matrix for the error terms corresponding to the hypothesis. H_j.
The numerator for the test statistic is

$$T_o = T/2 \log\left|\frac{\hat{\Omega}_1}{\tilde{\Omega}_{10}}\right| \quad (4.3)$$

Where $\hat{\Omega}_1$ is maximum likelihood estimate of Ω_1 and $\tilde{\Omega}_{10}$ is analogously to $\tilde{\sigma}_{10}$.
There are several multivariate cases of t-test are available in the literature. The easiest artificial compound model analogous to the uni-variable model is

Y = $(1-\alpha) f(\theta) + \alpha \hat{g} + e$

i.e., H_0: $Yti = (1-\alpha) fti(\theta) + \alpha \hat{g}_{ti} + eti$ (4.4)

Under H_0, Yt is the Covariance Matrix Ω_0. Linearizing around the point $\alpha = 0, \theta = \hat{\theta}$ yields the Multivariable linear regression

$$Y_{ti} - \hat{f}_{ti} = \hat{F}_{ti}^T b + \alpha\left(\hat{g}_{ti} - \hat{f}_{ti}\right) + eti \quad (4.5)$$

$$Y_{ti} - \hat{f}_{ti} = \hat{F}_{ti}^T b + \alpha \hat{h}_{ti} + eti \quad (4.6)$$

When h_{ti} is an element matrix of the Txm.

$$\therefore \hat{h} \equiv \left(\hat{g} - \hat{f}\right) \hat{\Omega}_1^{-1} \hat{\Omega}_0 \quad (4.7)$$

Here $\hat{g}$ and $\hat{f}$ are the Txm matrices with the elements of $\hat{g}_{ti}$ and $\hat{f}_{ti}$ respectively.

5. Conclusion

This paper intended at contributing to the literature existing form of testing nonnested multivariate nonlinear regression models. Non-nested univariate and multivariate models take place regularly in practice and researchers are using a wide variety of methods to test such models against one or more alternatives. This paper has explained the significance of testing of non-nested univariate and multivariate models, especially in the context of linear, and nonlinear regression models.

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