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An introduction to signal compression by using 'Haar Wavelets'

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Abstract
In present era Robotics, Computer Vision, Machine learning and Data Science are some key areas, who have major contribution in Advance Technology Optimization Theory and Classical Linear Algebra are main tools for understanding and application in above areas, Haar Wavelets. Haar basis, Hadamard matrices are fundamental and important tools/ topics which are required for application in Computer science, Graphics, Signal processing, Image processing and Error Correcting Codes.

Keywords: 1) Haar Matrix 2) Haar Wavelet 3) Hadamard matrix

1. **Haar Matrix:** A matrix which is 2 x 2 DCT matrix or N x N DCT (II) matrix is treated as haar matrix for block size.
2. **Haar Wavelet:** Proposed by 'Alfred Haar' in 1909 "Haar Wavelet" is a sequence of Square shaped function together forming Wavelet family or basis like Fourier analysis. It allows a target function over an interval which can be represented in terms of orthonormal basis. It sec also known as Db 1, being special case of Daubechies Wavelet.
3. **Hadamard Matrix:** Any Square matrix having entries +1 or -1 and Mutually orthogonal rows is known as 'Hadamard matrix' named after 'Jacques Hadamard' a French Mathematician Order of Hadamard matrix should be of order 1,2 or multiple of 4.

Introduction

Wavelets play an important role in audio and video signal processing, especially for compressing long signals into Smaller but even after compressing they retain enough information so that while even playing we can't find any difference in playing audio-visual effect.

Description with example

We shall start by Considering Haar Wavelets in R^4
Let four vectors $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ from R^4 given by

$$\alpha_1, = \begin{pmatrix}1\\1\\1\\1\end{pmatrix} \alpha_2 = \begin{pmatrix}1\\1\\-1\\-1\end{pmatrix} \alpha_3 = \begin{pmatrix}1\\-1\\0\\0\end{pmatrix} \alpha_4 = \begin{pmatrix}0\\0\\1\\-1\end{pmatrix}$$

Easily we can see these vectors are pairwise orthogonal as their inner product is zero, so by property of vectors they form linearly independence. Also if we take B = $\{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$ as Haar basis and U = $\{e_1, e_2, e_3, e_4\}$ as Canonical are standard basis for R^4 where

$$e_1 = \begin{pmatrix}1\\0\\0\\0\end{pmatrix} e_2, = \begin{pmatrix}0\\1\\0\\0\end{pmatrix} e_3 = \begin{pmatrix}0\\0\\1\\0\end{pmatrix} e_4 = \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

If W be the change of basis matrix

Then w = P_{BU} from u to v will be written as

$$W = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & -1 & 0 \\ 1 & -1 & 0 & 1 \\ 1 & -1 & 0 & -1 \end{bmatrix}$$

Inverse of W can be using

$W^{-1} = (W'W)^{-1}W'$

$$= \begin{bmatrix} 1/4 & 0 & 0 & 0 \\ 0 & 1/4 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1/2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

So any vector v = (6, 4, 5, 1) over the basis U becomes c = (c_1,c_2, c_3 , c_4) over the haar basis B, must be

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} 1/4 & 0 & 0 & 0 \\ 0 & 1/4 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1/2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 6 \\ 4 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 1 \\ 2 \end{bmatrix}$$

Let we have a signal vector v = (v_1,v_2, v_3 , v_4) it can be transformed into coefficient vector c as c = (c_1,c_2, c_3 , c_4) over haar basis by relation

c= $W^{-1}v$

where

$$c_1 = \frac{V1+V2+V3+V4}{4}$$

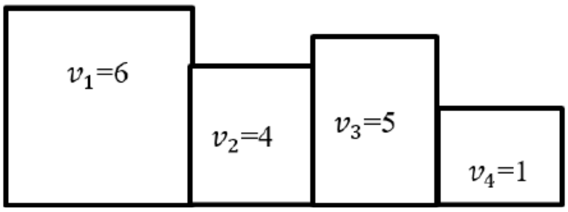
Which is average value of signal V, coefficient c_1 corresponds to background of the image (or of the Sound) whereas coarse detail of v is given by c_2,c_3 gives detail in first part of v and c_4 elaborates about second half of v.

Reconstruction of Signal

AS c = $W^{-1}v$ so by Computing v = Wc we can reconstruct the Signal for good compression, the trick is to take some Coefficients of c as zero, so by obtaining a Compressed signal $\hat{c}$ which retain enough crucial information such that reconstructed signal $\hat{v} = W\hat{c}$ behaves almost as good as original signal v, so compressing steps are as given below

Input v → Coeff's c =$W^{-1}v$ → Compressed $\hat{c}$→ compressed $\hat{v} = W\hat{c}$

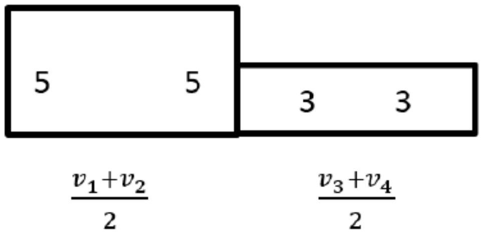
Modern video conferencing is possible by without this kind of compression scheme; so, it becomes a faster way to find c = $W^{-1}v$ without actually using W^{-1},multiscale nature of Haar wavelet can be done by this, it can also be shown by figure that original signal v= (6,4,5,1)



Average and half difference

First average $\frac{v_1+v_2}{2} = 5$

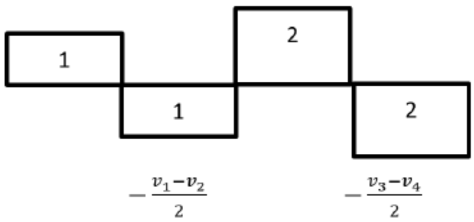
$$\frac{v_3+v_4}{2} = 3$$



First half difference

$$C_3 = \frac{v_1 - v_2}{2} = 1$$

$$C_4 = \frac{v_3 - v_4}{2} = 2$$



Again, we can compute averages & half differences & obtain coefficients

$c_1 = 4 , c_2 = 1$

It can be noticed that original signal v can be reconstructed from the two signals

$$v_1 = \frac{v_1+v_2}{2} + \frac{v_1-v_2}{2} = 5 + 1 = 6 \quad (1)$$

$$v_2 = \frac{v_1+v_2}{2} - \frac{v_1-v_2}{2} = 5 - 1 = 4 \quad (2)$$

$$v_3 = \frac{v_3+v_4}{2} + \frac{v_3-v_4}{2} = 3 + 2 = 5 \quad (3)$$

$$v_4 = \frac{v_3+v_4}{2} - \frac{v_3-v_4}{2} = 3 - 2 = 1 \quad (4)$$

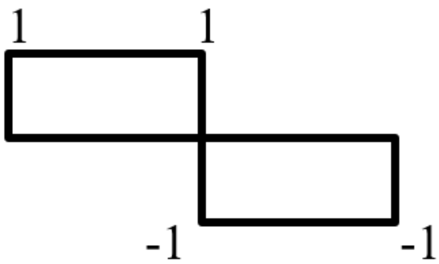
Now second average and second half difference

$$c_1 = \frac{V1+V2+V3+V4}{4}$$

$= \frac{6+4+5+1}{4} = 4$

4	4	4	4

$$c_2 = \frac{(v_1 + v_2) - (v_3 + v_4)}{4} = 1$$



Conclusion

As we have discussed this method for signal of length 2^n , where n=2, it can generalized to signal of any length 2^n for $n \in N$ for n = 3 Haar Basis will contain $2^3 = 8$ vectors $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7, \alpha_8$ given by matrix

$$W = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & -1 & 0 & 0 & 0 \\ 1 & 1 & -1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & -1 & 0 & 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & -1 & 0 & 1 & 0 & 0 & -1 & 0 \\ 1 & -1 & 0 & -1 & 0 & 0 & 0 & 1 \\ 1 & -1 & 0 & -1 & 0 & 0 & 0 & -1 \end{bmatrix}$$

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