



E-ISSN:2709-9407

P-ISSN:2709-9393

JMPES2022; 3(2): 67-68

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www.mathematicaljournal.com

Received: 21-04-2022

Accepted: 29-05-2022

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A proof of the Collatz conjecture

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Abstract

Using the mathematical induction, the present paper proves that with respect to all of the arbitrary positive natural numbers, if the starting number n is odd, let it become $3n+1$, if it is even, let it become $n \div 2$, successively carry on the calculation in terms of the rule, eventually, the result will always become 1. Therefore, the paper proves the Collatz conjecture.

Keywords: Collatz conjecture, the mathematical induction, odd, even, the rule of calculation

1. Introduction

Lothar Collatz is a German mathematician, he proposed a conjecture in 1937, which is the so-called Collatz conjecture. This is one of the puzzling problems in the world, but it seems very simple and interesting. Japan and some western countries provided huge money as the prize for the researcher who is able to solve this conjecture within the recent 80-100 years. The Collatz conjecture is also known as the Syracuse conjecture; the Hasse conjecture; the $3n+1$ conjecture; the Ulam conjecture or the angle valley conjecture in the fields of mathematics all over the world. People have, especially in the recent years, published a lot of papers to prove the conjecture^[1-3], but no one has been generally approved by the people who are working in this field of mathematics, even someone proposed a suspicion against the Collatz conjecture^[4]. The present paper is trying to propose a simple proof of the Collatz conjecture.

2. The Collatz Conjecture

The Collatz conjecture can be described as the following theorem:

Theorem: With respect to an arbitrary positive natural number as the starting number, if it is even, divide it by 2; if it is odd, multiply it by 3 and add 1. Successively carry on the calculation in terms of the rule, eventually, the starting number will always get struck at 1.

For examples:

- 1) If the starting number is 24, thus, $24(\div 2) \rightarrow 12(\div 2) \rightarrow 6(\div 2) \rightarrow 3(\times 3+1) \rightarrow 10(\div 2) \rightarrow 5(\times 3+1) \rightarrow 16(\div 2) \rightarrow 8(\div 2) \rightarrow 4(\div 2) \rightarrow 2(\div 2) \rightarrow 1$.
- 2) If the starting number is 11, thus, $11(\times 3+1) \rightarrow 34(\div 2) \rightarrow 17(\times 3+1) \rightarrow 52(\div 2) \rightarrow 26(\div 2) \rightarrow 13(\times 3+1) \rightarrow 40(\div 2) \rightarrow 20(\div 2) \rightarrow 10(\div 2) \rightarrow 5(\times 3+1) \rightarrow 16(\div 2) \rightarrow 8(\div 2) \rightarrow 4(\div 2) \rightarrow 2(\div 2) \rightarrow 1$.
- 3) If the starting number is 2021, thus $2021(\times 3+1) \rightarrow 6064(/2) \rightarrow 3032(/2) \rightarrow 1516(/2) \rightarrow 758(/2) \rightarrow 379(\times 3+1) \rightarrow 1138(/2) \rightarrow 569(\times 3+1) \rightarrow 1708(/2) \rightarrow 854(/2) \rightarrow 427(\times 3+1) \rightarrow 1282(/2) \rightarrow 641(\times 3+1) \rightarrow 1924(/2) \rightarrow 962(/2) \rightarrow 481(\times 3+1) \rightarrow 1444(/2) \rightarrow 722(/2) \rightarrow 361(\times 3+1) \rightarrow 1084(/2) \rightarrow 542(/2) \rightarrow 271(\times 3+1) \rightarrow 814(/2) \rightarrow 407(\times 3+1) \rightarrow 1222(/2) \rightarrow 611(\times 3+1) \rightarrow 1834(/2) \rightarrow 917(\times 3+1) \rightarrow 2752(/2) \rightarrow 1376(/2) \rightarrow 688(/2) \rightarrow 344(/2) \rightarrow 172(/2) \rightarrow 86(/2) \rightarrow 43(\times 3+1) \rightarrow 130(/2) \rightarrow 65(\times 3+1) \rightarrow 196(/2) \rightarrow 98(/2) \rightarrow 49(\times 3+1) \rightarrow 148(/2) \rightarrow 74(/2) \rightarrow 37(\times 3+1) \rightarrow 112(/2) \rightarrow 56(/2) \rightarrow 28(/2) \rightarrow 14(/2) \rightarrow 7(\times 3+1) \rightarrow 22(/2) \rightarrow 11(\times 3+1) \rightarrow 34(/2) \rightarrow 17(\times 3+1) \rightarrow 52(/2) \rightarrow 26(/2) \rightarrow 13(\times 3+1) \rightarrow 40(/2) \rightarrow 20(/2) \rightarrow 10(/2) \rightarrow 5(\times 3+1) \rightarrow 16(/2) \rightarrow 8(/2) \rightarrow 4(/2) \rightarrow 2(/2) \rightarrow 1$.
- 4) Considering a big number 9999 as the starting number, in terms of the calculation rule, it obtains: $9999(\times 3+1) \rightarrow 29998(/2) \rightarrow 14999(\times 3+1) \rightarrow 44998(/2) \rightarrow 22499(\times 3+1) \rightarrow 67498(/2) \rightarrow 33749(\times 3+1) \rightarrow 101248(/2) \rightarrow 50624(/2) \rightarrow 25312(/2) \rightarrow 12656(/2) \rightarrow 6328(/2) \rightarrow 3154(/2) \rightarrow 1582(/2) \rightarrow 791(\times 3+1) \rightarrow 2374(/2) \rightarrow 1187(\times 3+1) \rightarrow 3562(/2)$

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$\rightarrow 1781(\times 3+1) \rightarrow 5344(/2) \rightarrow 2672(/2) \rightarrow 1336(/2) \rightarrow 668(/2)$
 $\rightarrow 334(/2) \rightarrow 167(\times 3+1) \rightarrow 502(/2) \rightarrow 251(\times 3+1) \rightarrow 754(/2)$
 $\rightarrow 377(\times 3+1) \rightarrow 1132(/2) \rightarrow 566(/2) \rightarrow 283(\times 3+1) \rightarrow 850(/2)$
 $\rightarrow 425(\times 3+1) \rightarrow 1276(/2) \rightarrow 638(/2) \rightarrow 319(\times 3+1) \rightarrow 958(/2)$
 $\rightarrow 479(\times 3+1) \rightarrow 1438(/2) \rightarrow 719(\times 3+1) \rightarrow 2158(/2) \rightarrow 1079(\times 3+1)$
 $\rightarrow 3238(/2) \rightarrow 1619(\times 3+1) \rightarrow 4858(/2) \rightarrow 2429(\times 3+1) \rightarrow$
 $7288(/2) \rightarrow 3644(/2) \rightarrow 1822(/2) \rightarrow 911(\times 3+1) \rightarrow 2734(/2) \rightarrow$
 $1367(\times 3+1) \rightarrow 4102(/2) \rightarrow 2051(\times 3+1) \rightarrow 6154(/2) \rightarrow 3077(\times 3+1)$
 $\rightarrow 9232(/2) \rightarrow 4616(/2) \rightarrow 2308(/2) \rightarrow 1154(/2) \rightarrow 577(\times 3+1)$
 $\rightarrow 1732(/2) \rightarrow 866(/2) \rightarrow 433(\times 3+1) \rightarrow 1300(/2) \rightarrow 650(/2)$
 $\rightarrow 325(\times 3+1) \rightarrow 976(/2) \rightarrow 488(/2) \rightarrow 244(/2) \rightarrow 122(/2) \rightarrow 61(\times 3+1)$
 $\rightarrow 184(/2) \rightarrow 92(/2) \rightarrow 46(/2) \rightarrow 23(\times 3+1) \rightarrow 70(/2) \rightarrow 35(\times 3+1)$
 $\rightarrow 106(/2) \rightarrow 53(\times 3+1) \rightarrow 160(/2) \rightarrow 80(/2) \rightarrow 40(/2) \rightarrow 20(/2) \rightarrow 10(/2)$
 $\rightarrow 5(\times 3+1) \rightarrow 16(/2) \rightarrow 8(/2) \rightarrow 4(/2) \rightarrow 2(/2) \rightarrow 1.$

Although the above-mentioned examples are favourable to the theorem, it is required that the theorem is also true for all of the arbitrary positive natural numbers.

3. The Proof

In consideration of the problem of infinite positive natural numbers, in general, it is convenient to use the mathematical induction to deal with.

With respect to the Collatz conjecture, at first, considering $n=1$, evidently, $3 \times 1+1=4$, $4 \div 2=2$, $2 \div 2=1$, the conjecture is true.

Supposing when $n=k$ (k is an arbitrary positive natural number), the Collatz conjecture is true, thus, it requires to prove that when $n=k+1$, the Collatz conjecture is also true. There are two cases for us to discuss:

(1). If k is an odd number, thus, in terms of the Collatz conjecture, it will become

$$3k+1=(k+1)+2k \quad (1),$$

according to the proposition, k will become 1 by successively use of the rule of calculation, this demonstrates that with respect to eq.(1), the left of eq.(1) can become 1 by use of the rule of calculation; in the right of eq.(1), according to the proposition, $2k$ will become 1 by use of the rule of calculation, therefore, in the right of eq.(1), $(k+1)$ will also become 1 by use of the rule of calculation, otherwise, the right and therefore the left of eq.(1) can not become 1 by use of the rule of calculation, this is contrary to the proposition. Therefore, when $n=k+1$, the Collatz conjecture is also true.

The Collatz conjecture has been proven for the case of the positive natural number k is odd.

(2). If k is an even number, according to the rule, k will become $k \div 2$, if the result is still even, successively divide it by 2, going on this step, in general, if k is not equal to 2^m (m is a positive integer), it can always become an odd number k' by repeatedly using the rule $k \div 2$. Evidently, that the result become 1 from k' or $k'+1$ is equivalent of that it becomes 1 from k or $k+1$. Thus, this is completely same as the case (1), using the mathematical induction, when the starting number $n=1$, it will become $3 \times 1+1=4$, $4 \div 2=2$, $2 \div 2=1$, the conjecture is true. Evidently, either of k and k' which is odd and arising from k is an arbitrary positive natural number, supposing that when $n=k'$, the conjecture is true, according to the rule of calculation, the odd number k' will become

$$3k'+1=(k'+1)+2k' \quad (2).$$

As the same reason of case (1), using the rule of calculation, the left of eq.(2) will become 1, and according to the supposition, $2k'$ in the right of eq.(2) will become 1, therefore, $k'+1$ in the right of eq.(2) will also become 1 by use of the rule of calculation, namely, when $n=k'+1$, the conjecture is also true. if $k=2^m$, thus, repeatedly using $k \div 2$, it will eventually become $k'=1$, which is also an odd number and in accord with the previous proof for k' and $k'+1$.

Therefore, the Collatz conjecture is true for all of the positive natural numbers.

4. The Conclusion

In the present proof on the Collatz conjecture, the paper considered two cases with the mathematical induction. For the first case, if the arbitrary positive integer k is odd, supposing that when $n=k$ the conjecture is true, it is easy to demonstrate that when $n=k+1$, the conjecture is also true.

But for the second case, if the arbitrary positive integer k is even, it is difficult to directly demonstrate whether the conjecture is also true by use of the rule of calculation when $n=k+1$. Therefore, at first, it is necessary to make k become an odd number k' by use of the rule of calculation, in fact, if k is not justly equal to 2^m (m is a positive integer), k will always become an odd number k' after repeatedly using the rule $k \div 2$ for the even number, then, using the mathematical induction, as the same reason of the first case; supposing that when $n=k'$ the conjecture is true, thus, when $n=k'+1$, the conjecture will be also true. In the proof, because k is an arbitrary positive integer, so k' which is arising from k is also an arbitrary positive integer, and according to the supposition, either of k and k' will become 1 by use of the rule of calculation. Therefore, using the mathematical induction for k and $k+1$ is equivalent of using the mathematical induction for k' and $k'+1$; in other words, the Collatz conjecture is true for all of the arbitrary positive k' is equivalent of that the conjecture is true for all of the arbitrary positive natural number k .

By the way, if k is equal to 2^m (m is a positive integer), repeatedly using $k \div 2$, k or k' will become 1, which is also an odd number and in accord with the previous proof for k' and $k'+1$.

5. References

1. Patrick Wiftricht, Eric Landquist. The Collatz Conjecture and Integers of the form $2^k b - m$ and $3^k - 1$ [J]. Ferman University Electronic Journal of Undergraduate Mathematics. 2013;17:1-5.
2. Abhijit Manohar. Solution to Collatz Conjecture[J]. American Journal of Applied Mathematics and Statistics. 2021;9(3):107-110.
3. Kevin P. Thompson. An Alternative Computational Approach to the Collatz Conjecture [J]. arXiv \rightarrow mathematics \rightarrow number theory, 2011; arXiv:1101.2373.
4. Juan A. Perez. Collatz conjecture: Is It False?[J]. arXiv:1708.04615v1[math.GM]; Aug. 2017.