Journal of Mathematical Problems, Equations and Statistics

E-ISSN:2709-9407
P-ISSN:2709-9393
JMPES2022; 3(2): 67-68

www.mathematicaljournal.com
Received: 21-04-2022
Accepted: 29-05-2022

Zhang Yue
Department of Physics, Hunan Normal University, ChangSha, China

A proof of the Collatz conjecture

Zhang Yue

Abstract
Using the mathematical induction, the present paper proves that with respect to all of the arbitrary positive natural numbers, if the starting number n is odd, let it become 3n+1, if it is even, let it become n ÷2, successively carry on the calculation in terms of the rule, eventually, the result will always become 1. Therefore, the paper proves the Collatz conjecture.

Keywords: Collatz conjecture, the mathematical induction, odd, even, the rule of calculation

1. Introduction
Lothar Collatz is a German mathematician, he proposed a conjecture in 1937, which is the so-called Collatz conjecture. This is one of the puzzling problems in the world, but it seems very simple and interesting. Japan and some western countries provided huge money as the prize for the researcher who is able to solve this conjecture within the recent 80-100 years. The Collatz conjecture is also known as the Syracuse conjecture; the Hasse conjecture; the 3n+1 conjecture; the Ulam conjecture or the angle valley conjecture in the fields of mathematics all over the world. People have, especially in the recent years, published a lot of papers to prove the conjecture[1-3], but no one has been generally approved by the people who are working in this field of mathematics, even someone proposed a suspicion against the Collatz conjecture[4]. The present paper is trying to propose a simple proof of the Collatz conjecture.

2. The Collatz Conjecture
The Collatz conjecture can be described as the following theorem:
Theorem: With respect to an arbitrary positive natural number as the starting number, if it is even, divide it by 2; if it is odd, multiply it by 3 and add 1. Successively carry on the calculation in terms of the rule, eventually, the starting number will always get struck at 1.
For examples:

1) If the starting number is 24, thus, 24(÷2)→12(÷2)→6(÷2)→3(×3+1)→10(÷2)→5(×3+1)→16(÷2)→8(÷2)→4(÷2)→2(÷2)→1.
2) If the starting number is 11, thus, 11(×3+1)→34(÷2)→17(×3+1)→52(÷2)→26(÷2)→13(×3+1)→40(÷2)→20(÷2)→10(÷2)→5(×3+1)→16(÷2)→8(÷2)→4(÷2)→2(÷2)→1.
3) If the starting number is 2021, thus 2021(×3+1)→6064(/2)→3032(/2)→1516(/2)→758(/2)→379(×3+1)→1138(/2)→569(×3+1)→1708(/2)→854(/2)→427(×3+1)→1282(/2)→641(×3+1)→1924(/2)→962(/2) →481(×3+1)→1444(/2)→722(/2)→361(×3+1)→1084(/2)→542(/2) →271(×3+1)→814(/2)→407(×3+1)→1222(/2)→611(×3+1)→1834(/2)→917(×3+1)→2752(/2)→1376(/2) →688(/2) →344(/2) →172(/2) →86(/2)→43(×3+1)→130(/2) →65(×3+1)→196(/2)→98(/2) →49(×3+1)→148(/2)→74(/2)→37(×3+1)→112(/2)→56(/2)→28(/2)→14(/2) →7(×3+1)→22(/2)→11(×3+1)→34(/2)→17(×3+1)→52(/2) →26(/2) →13(×3+1)→40(/2)→20(/2) →10(/2) →5(×3+1)→16(/2) →8(/2)→4(/2) →2(/2) →1.
4) Considering a big number 9999 as the starting number, in terms of the calculation rule, it obtains:9999(×3+1)→29998(/2)→14999(×3+1)→44998(/2)→22499(×3+1)→67498(/2)→33749(×3+1)→101248(/2)→50624(/2)→25312(/2)→12656(/2)→6328(/2)→3154(/2)→1582(/2)→791(×3+1) →2374(/2) →1187(×3+1) →3562(/2)

Corresponding Author:
Zhang Yue
Department of Physics, Hunan Normal University, ChangSha, China

→1781(×3+1) →5344(/2) →2672(/2) →1336(/2)→668(/2) →334(/2)→167(×3+1) →502(/2) →251(×3+1) →754(/2) →377(×3+1)→1132(/2) →566(/2)→283(×3+1)→850(/2) →425(×3+1)→1276(/2)→638(/2)→319(×3+1)→958(/2) →479(×3+1)→1438(/2)→719(×3+1)→2158(/2)→1079(×3+1)→3238(/2)→1619(×3+1)→4858(/2)→2429(×3+1)→7288(/2)→3644(/2)→1822(/2)→911(×3+1)→2734(/2)→1367(×3+1)→4102(/2) →2051(×3+1)→6154(/2) →3077(×3+1)→9232(/2)→4616(/2)→2308(/2)→1154(/2)→577(×3+1)→1732(/2) →866(/2)→433(×3+1)→1300(/2)→650(/2) →325(×3+1)→976(/2) →488(/2)→244(/2) →122(/2)→61(×3+1)→184(/2)→92(/2)→46(/2)→23(×3+1)→70(/2) →35(×3+1)→106(/2) →53(×3+1)→160(/2) →80(/2) →40(/2) →20(/2) →10(/2) →5(×3+1)→16(/2) →8(/2) →4(/2) →2(/2) →1.

Although the above-mentioned examples are favourable to the theorm, it is required that the theorem is also true for all of the arbitrary positive natural numbers.

3. The Proof

In considration of the problem of infinite positive natural numbers, in general, it is convenient to use the mathematical induction to deal with.

With respect to the Collatz conjecture, at first, considering n=1, evidently, 3×1+1=4, 4÷2=2, 2÷2=1, the conjecture is true.

Supposing when n=k (k is an arbitrary positive natural number), the Collatz conjecture is true, thus, it requires to prove that when n=k+1, the Collatz conjecture is also true. There are two cases for us to discuss:

(1). If k is an odd number, thus, in terms of the Collatz conjecture, it will become

$$3k+1=(k+1)+2k \qquad (1),$$

according to the proposition, k will become 1 by successively use of the rule of calculation, this demonstrates that with respect to eq.(1), the left of eq.(1) can become 1 by use of the rule of calculation; in the right of eq.(1), according to the proposition, 2k will become 1 by use of the rule of calculation, therefore, in the right of eq.(1), (k+1) will also become 1 by use of the rule of calculation, otherwise, the right and therefore the left of eq.(1) can not become 1 by use of the rule of calculation, this is contrary to the proposition. Therefore, when n=k+1, the Collatz conjecture is also true.

The Collatz conjecture has been proven for the case of the positive natural number k is odd.

(2). If k is an even number, according to the rule, k will become k÷2, if the result is still even, successively devide it by 2, going on this step, in general, if k is not equal to 2^m (m is a positive integer), it can always become an odd number k’ by repeatedly using the rule k÷2. Evidently, that the result become 1 from k’ or k’+1 is equivalent of that it becomes 1 from k or k+1. Thus, this is completely same as the case (1), using the mathematical induction, when the starting number n=1, it will become 3×1+1=4, 4÷2=2, 2÷2=1, the conjecture is true. Evidently, either of k and k’ which is odd and arising from k is an arbitrary positive natural number, supposing that when n=k’, the conjecture is true, according to the rule of calculation, the odd number k’ will become

$$3k'+1= (k'+1)+2k' \qquad (2).$$

As the same reason of case (1), using the rule of calculation, the left of eq.(2) will become 1, and according to the supposition, 2k’ in the right of eq.(2) will become 1, therefore, k’+1 in the right of eq.(2) will also become 1 by use of the rule of calculation, namely, when n=k’+1, the conjecture is also true. if $k=2^m$, thus, repeately using k÷2, it will eventually become k’=1, which is also an odd number and in accord with the previous proof for k’ and k’+1.

Therefore, the Collatz conjecture is true for all of the positive natural numbers.

4. The Conclusion

In the present proof on the Collatz conjecture, the paper considered two cases with the mathematical induction. For the first case, if the arbitrary positive integer k is odd, supposing that when n=k the conjecture is true, it is easy to demonstrate that when n=k+1, the conjecture is also ture.

But for the second case, if the arbitrary positive integer k is even, it is difficult to directly demonstrate whether the conjecture is also true by use of the rule of calculation when n=k+1. Therefore, at first, it is necessary to make k become an odd number k’ by use of the rule of calculation, in fact, if k is not justly equal to 2^m (m is a positive integer), k will always become an odd number k’ after repeately using the rule k÷2 for the even number, then, using the mathematical induction, as the same reason of the first case; supposing that when n=k’ the conjecture is true, thus, when n=k’+1, the conjecture will be also true. In the proof, because k is an arbitrary positive integer, so k’ which is arising from k is also an arbitrary positive integer, and according to the supposition, either of k and k’ will become 1 by use of the rule of calculation. Therefore, using the mathematical induction for k and k+1 is equivalent of using the mathematical induction for k’ and k’+1; in other words, the Collatz conjecture is true for all of the arbitrary positive k’ is equivalent of that the conjecture is true for all of the arbitrary positive natural number k.

By the way, if k is equal to 2^m (m is a positive integer), repeately using k÷2, k or k’ will become 1, which is also an odd number and in accord with the previous proof for k’ and k’+1.

5. References

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