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Thermal and mechanical interactions in a fractional order microstretch thermoelastic half-space

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Abstract
The present investigation deals with the thermal and mechanical interactions in a fractional order microstretch thermoelastic half-space subjected to inclined mechanical forces acting at the boundary of the surface of the half-space. Integral transform technique (Laplace transform and Fourier transform) has been applied to solve the basic equations mathematically. The mathematical expressions of mechanical stresses, coupled tangential stress, micro stress and the temperature distribution has been obtained some particular results and special cases also have been derived from the present research.

Keywords: Microstretch thermoelastic, integral transform, fractional order, inclined forces, thermal stresses

Introduction
Eringen [1, 2] developed the linear theory of micropolar thermo elasticity, theory of micropolar elastic solids with stretch and derived the basic equations of motion, constitutive relations and boundary conditions for these theories. The later theory was very important for a class of materials which can stretch and contract. In this theory Eringen [2] presented a new model which explained the motion of a certain class of materials Eringen [3] developed the theory of thermo-microstretch elastic materials including micro structural expansion and contractions. The material points of microstretch thermoelastic material are able to stretch and contract independently of their translational and relational motion. Here it is noted that the theory of microstretch thermoelastic solids is a particular case of micromorphic thermo elasticity and is the generalization of micropolar theory of thermo elasticity.
The differential equations including higher-order fractional derivatives play a significant role in mathematical modelling of some complex systems. First of all, Abel presented an application of fractional calculus. Povstenko [4] also developed a quasi-static thermoelastic model for uncoupled equations taking the fractional time derivative in the heat conduction equation. Later Povstenko [5] investigated the thermal stresses in an infinite medium including cylindrical holes by using the fractional heat conduction equation. Sherief *et al.* [6] presented a mathematical model of fractional order theory of thermoelasticity by revising the Cattaneo law and deduced the basic equations, constitutive relations. Youssef [7] constructed another mathematical model for fractional theory of thermoelasticity by taking different value of fractional parameter α. He also discussed an application of this theory. Ezzat [8, 9] proposed another theory of fractional order generalized thermo elasticity. Later Ezzat and Fayik [10] extended this fractional order generalized thermoelasticity by including the thermo-diffusion and presented a new theory named as fractional order thermoelasticity theory with diffusion in elastic medium. They also, derived the uniqueness theory, reciprocity theorem and variational principle. Shaw and Mukhopadhyay [11] extended the Sheriff's theory of fractional thermoelasticity to include the effect of two temperatures for micropolar thermoelastic materials. Sur and Kanoria [12] investigated a one-dimensional problem in fractional thermo elasticity with two-temperatures in the context of LS-theory and GL-theory. Yu *et al.* [13] discussed a problem in electromagnetic anisotropic medium using fractional order theory of thermo elasticity. Sumelka [14] discussed some applications and qualitative aspects in fractional continuum mechanics. A 1-diemensional fractional thermoelastic problem with diffusion in a half-space was discussed by Povstenko and Klekot [15]. Shaw and Mukhopadhyay [16] developed a theory of fractional ordered thermoelastic diffusion. Recently Chirila and Marin [17] worked on dipolar thermoelastic materials Marin *et al.* [18] recently presented a mathematical model of fractional order strain in dipolar thermo elasticity.

In this paper we have used the fractional theory of thermo elasticity developed by Ezzat and Fayik [10] and analyzed the thermo-mechanical interactions in a fractional order microstretch thermoelastic medium. The normal stress, tangential stress, couple tangential stress, micro stress and temperature distribution are computed using the numerical method technique involving Laplace transform and Fourier transform.

Basic equations

Following Eringen [3], Ezzat and Fayik [10] the basic equations for homogeneous, isotropic microstretch generalized thermoelastic solids in the absence of body forces, body couples and stretch forces are given by:
Stress equation of motion:

$$(\lambda+\mu)\nabla(\nabla.\boldsymbol{u})+(\mu+K)\nabla^2\boldsymbol{u}+K\nabla\times\boldsymbol{\phi}+\lambda_0\nabla\phi^*-\beta_1\nabla T=\rho\ddot{\boldsymbol{u}}, \tag{1}$$

Couple stress equation of motion:

$$(\gamma\nabla^2-2K)\boldsymbol{\phi}+(\alpha+\beta)\nabla(\nabla.\boldsymbol{\phi})+K\nabla\times\boldsymbol{u}=\rho j\ddot{\boldsymbol{\phi}}, \tag{2}$$

Equation of balance of stress moments:

$$(\alpha_0\nabla^2-\lambda_1)\phi^*-\lambda_0\nabla.\boldsymbol{u}+\nu_1\left(1+\tau_1\frac{\partial}{\partial t}\right)T=\frac{\rho j_0}{2}\ddot{\phi}^*, \tag{3}$$

Fractional order equation of heat conduction:

$$K^*\nabla^2T=\left(\frac{\partial}{\partial t}+\frac{{\tau_0}^{\alpha}\partial^{\alpha+1}}{\Gamma(\alpha+1)\partial t^{\alpha+1}}\right)(\rho c^*T+\nu_1T_0\varphi^*)+\left(1+\frac{{\tau_0}^{\alpha}\partial^{\alpha}}{\Gamma(\alpha+1)\partial t^{\alpha}}\right)(\beta_1T_0\nabla.\dot{\boldsymbol{u}}-\rho Q), \tag{4}$$

The constitutive relations are:

$$t_{ij}=(\lambda_0\phi^*+\lambda u_{r,r})\delta_{ij}+\mu(u_{i,j}+u_{j,i})+K(u_{j,i}-\epsilon_{ijk}\phi_k)-\beta_1\delta_{ij}T, \tag{5}$$

$$m_{ij}=\alpha\phi_{r,r}\delta_{ij}+\beta\phi_{i,j}+\gamma\phi_{j,i}+b_0\epsilon_{mji}\phi^*_{,m}, \tag{6}$$

$$\lambda_i^*=\alpha_0\phi^*_{,i}+b_0\epsilon_{ijm}\phi_{j,m}, \tag{7}$$

Following Sherief [6], The Caputo fractional derivative in the heat conduction equation (4) can be written as:

$$\frac{d^{\alpha}f(t)}{dt^{\alpha}}=\begin{cases}\dfrac{1}{\Gamma(n-\alpha)}\int_0^t(t-\tau)^{n-\alpha-1}\dfrac{d^nf(\tau)}{d\tau^n}d\tau, & n-1<\alpha<n\\[2ex] \dfrac{d^nf(\tau)}{dt^n}, & \alpha=n\end{cases} \tag{8}$$

In which
$n-1<\alpha<n,\ m\in N=\{1,2.....\}$.

Where symbols have $\lambda,\mu,\alpha,\beta,\gamma,K,\lambda_0,\lambda_1,\alpha_0,b_0$,usual meanings. are ρ is mass density, $u=(u_1,u_2,u_3)$is the displacement vector and $\boldsymbol{\phi}=(\phi_1,\phi_2,\phi_3)$is the microrotation vector, $\phi^*$is the scalar microstretch function, T is temperature and T_0 is the reference temperature of the body chosen,K^* is the coefficient of the thermal conductivity,c^* is the specific heat at constant strain, j is the microinertia,t_{ij} are components of stress, m_{ij} are components of couple stress, λ_i^* is the microstress tensor, δ_{ij} is the Kroneker's delta function.

Formulation of the problem

We consider an isotropic homogeneous fractional microstretch thermoelastic half-space in an intact form at uniform temperatureT_0. The origin of rectangular Cartesian coordinate system is taken on the x_3 axis with x_3-axis pointing vertically downward the medium.

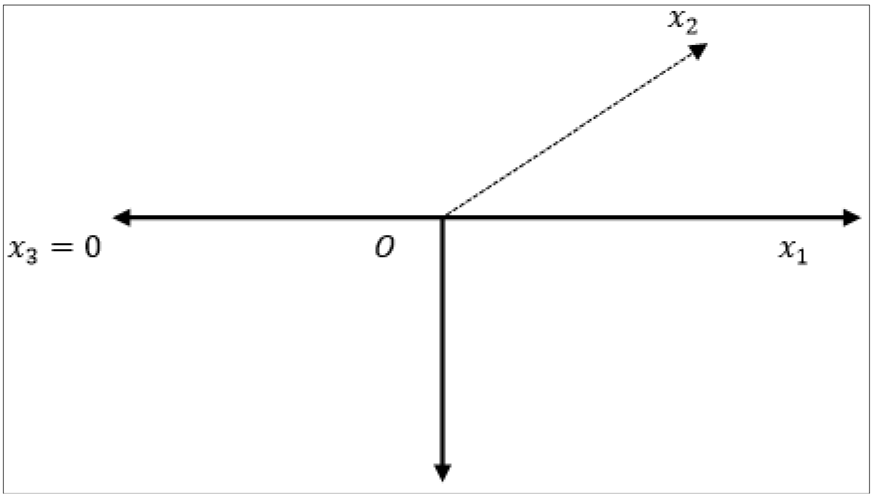

Fig 1: Geometry of the problem

We consider plane strain problem with all the field variables depending on (x_1, x_3, t). For two dimensional problems, we take

$$\boldsymbol{u} = (u_1, 0, u_3), \boldsymbol{\phi} = (0, \phi_2, 0), \tag{9}$$

For further consideration, it is convenient to introduce in equations (1)-(4) the dimensionless quantities defined as:

$$x_i' = \frac{\omega^*}{c_1} x_i, u_i' = \frac{\rho\omega^* c_1}{\beta_1 T_0} u_i, \phi_i' = \frac{\rho c_1^2}{\beta_1 T_0}\phi_i, \phi^{*\prime} = \frac{\rho c_1^2}{\beta_1 T_0}\phi^*, T' = \frac{T}{T_0}, t' = \omega^* t, \tau_1' = \omega^*\tau_1, \tau_0' = \omega^*\tau_0, t_{ij}' = \frac{1}{\beta_1 T_0} t_{ij}, \omega^* = \frac{\rho c^* c_1^2}{K^*}, c_1^2 = \frac{\lambda+2\mu+k}{\rho}, m_{ij}^* = \frac{\omega^*}{c\beta_1 T_0} m_{ij} \tag{10}$$

Utilizing the expressions defined by (10) in the equations (1)-(4) and with the help of equation (9), we reach to the following equations:

$$a_1 \frac{\partial e}{\partial x_1} + a_2 \nabla^2 u_1 - a_3 \frac{\partial \phi_2}{\partial x_3} + a_4 \frac{\partial \phi^*}{\partial x_1} - \left(1 + \tau_1 \frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x_1} = \ddot{u}_1, \tag{11}$$

$$a_1 \frac{\partial e}{\partial x_3} + a_2 \nabla^2 u_3 + a_3 \frac{\partial \phi_2}{\partial x_1} + a_4 \frac{\partial \phi^*}{\partial x_3} - \left(1 + \tau_1 \frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x_3} = \ddot{u}_3, \tag{12}$$

$$\nabla^2 \phi_2 - 2a_6\phi_2 + a_6\left(\frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1}\right) = a_7 \ddot{\phi}_2, \tag{13}$$

$$\nabla^2 \phi^* - a_8\phi^* - a_9 e + a_{10}\left(1 + \tau_1 \frac{\partial}{\partial t}\right) T = a_{12}\ddot{\phi}^*, \tag{14}$$

$$\nabla^2 T - \left(\frac{\partial}{\partial t} + \frac{\tau_0 {\omega^*}^{\alpha-1}\partial^{\alpha+1}}{\partial t^{\alpha+1}}\right)(T - a_{13}\nabla^2\varphi + a_{14}\varphi^*) = Q_0, \tag{15}$$

Here, $a_2 = \frac{\mu+k}{\rho\, {c_1}^2}, a_3 = \frac{k}{\rho\, {c_1}^2}, a_4 = \frac{\lambda_0}{\delta\, {c_1}^2}, a_6 = \frac{2k{c_1}^2}{\gamma\, {\omega^*}^2}, a_7 = \frac{\delta j {c_1}^2}{\gamma}, a_8 = \frac{\lambda_1 {c_1}^2}{\alpha_0 {\omega^*}^2}, a_9 = \frac{\lambda_0\, {c_1}^2}{\alpha_0\, {\omega^*}^2}, a_{10} = \frac{\nu_1\delta {c_1}^4}{\beta_1\alpha_0{\omega^*}^2}, a_{12} = \frac{\delta {c_1}^2 j_0}{2\alpha_0}, a_{13} = -\frac{{\beta_1}^2 T_0}{k^*\omega^*}, a_{14} = \frac{\nu_1\beta_1 T_0}{\rho\omega^* k^*}, a_{15} = s + \tau_0 {\omega^*}^{\alpha-1} s^{\alpha+1}$,.

$\nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_3^2}$, is the Laplacian operator.

Making use of Helmholtz's decomposition theorem i.e. representation of a vector into scaler and vector potentials the displacement components u_1 and u_3 are related to non- dimensional potential functions ϕ and ψ as:

$$u_1 = \frac{\partial\phi}{\partial x_1} - \frac{\partial\psi}{\partial x_3}, u_3 = \frac{\partial\phi}{\partial x_3} + \frac{\partial\psi}{\partial x_1} \tag{16}$$

Substituting the values of u_1 and u_3 from (16) in (11)-(15), we obtain:

$$\nabla^2\phi - \ddot{\phi} + a_4\phi^* - \left(1 + \tau_1 \frac{\partial}{\partial t}\right) T = 0, \tag{17}$$

$$\left(\nabla^2 - a_8 - a_{12}\frac{\partial^2}{\partial t^2}\right)\phi^* - a_9\nabla^2\phi + a_{10}\left(1 + \tau_1\frac{\partial}{\partial t}\right) T = 0, \tag{18}$$

$$\nabla^2 T - \left(\frac{\partial}{\partial t} + \frac{\tau_0 {\omega^*}^{\alpha-1} \partial^{\alpha+1}}{\partial t^{\alpha+1}}\right)(T - a_{13}\nabla^2 \varphi + a_{14}\varphi^*) = Q_0, \quad (19)$$

$$a_2 \nabla^2 \psi - \ddot{\psi} + a_3 \phi_2 = 0, \quad (20)$$

$$\nabla^2 \phi_2 - 2a_6 \phi_2 - a_6 \nabla^2 \psi = a_7 \ddot{\phi}_2, \quad (21)$$

Solution of the problem

We define Laplace transform and Fourier transform respectively as:

$$\bar{f}(s, x_1, x_3) = \int_0^\infty f(t, x_1, x_3) e^{-st} dt, \quad (22)$$

$$\hat{f}(x_3, \xi, s) = \int_{-\infty}^\infty \bar{f}(s, x_1, x_3) e^{\iota \xi x_1} dx_1, \quad (23)$$

Applying Laplace transform defined by (22) on (17)-(21) and then applying Fourier transforms defined by (23) on the resulting quantities, we obtain:

$$\left(\frac{d^2}{dx^2} - \xi_1\right)\hat{\phi} + a_4 \widehat{\phi^*} - \tau_{11}\hat{T} = 0 \quad (24)$$

$$-a_9\left(\frac{d^2}{dx^2} - \xi^2\right)\hat{\phi} + \left(\frac{d^2}{dx^2} - a_{20}\right)\widehat{\phi^*} + a_{21}\hat{T} = 0 \quad (25)$$

$$(D^2 - \xi^2)\hat{T} - \left(s + z_0 {\omega^*}^{\alpha-1} s^{\alpha+1}\right)(\hat{T} - a_{13}(D^2 - \xi^2)\hat{\varphi} + a_{14}\widehat{\varphi^*} = Q_1 \quad (26)$$

$$\left[\frac{d^2}{dx_3^2} - \xi^2 - s^2\right]\hat{\psi} - a_3 \widehat{\phi_2} = 0, \quad (27)$$

$$\left[\frac{d^2}{dx_3^2} - a_{30}\right]\widehat{\phi_2} + a_6\left[\frac{d^2}{dx_3^2} - \xi^2\right]\hat{\psi} = 0, \quad (28)$$

Eliminating$\widehat{\phi^*}$&$\hat{T}$, $\hat{\phi}$&$\hat{T}$ and $\hat{\phi}$&$\widehat{\phi^*}$ respectively from the equations (24)-(26), we obtain:

$$[D^6 - AD^4 - BD^2 + C]\hat{\phi} = f_1 \quad (29a)$$

$$[D^6 - AD^4 - BD^2 + C]\widehat{\phi^*} = f_2 \quad (29b)$$

$$[D^6 - AD^4 - BD^2 + C]\hat{T} = f_3 \quad (29c)$$

Also eliminating $\hat{\phi}_2$ equations (27)-(28) yield:

$$[D^4 + ED^2 + F]\hat{\psi} = 0, \quad (30)$$

Where

$\xi_{11} = \xi^2 + s^2, a_{16} = \xi^2 + a_8 + a_{12}s^2, a_{17} = \xi^2 + a_{15}, a_{18} = a_{13}a_{15}, a_{19} = a_{14}a_{15}, a_{20} = a_2\xi^2 - s^2, a_{21} = \xi^2 + 2a_6 + a_7 s^2,$
$f_1 = -Q_1(\tau_{11}{\gamma^*}^2 + a_{25}),$
$f_2 = -Q_1(a_{26}{\gamma^*}^2 + a_{27}), f_3 = -Q_1({\gamma^*}^4 - a_{28}{\gamma^*}^2 + a_{29}), f_4 = [{\gamma^*}^6 - A{\gamma^*}^4 - B{\gamma^*}^2 + C]$
$A = -a_{16} - a_{17} - \xi_{11} + a_4 a_9 + a_{18}$, $B = a_{10}a_{19} + \xi_{11}a_{16} + \xi_{11}a_{17} - a_4 a_9 \xi^2 - a_4 a_9 a_{17} - a_{10}a_{18} - a_9 a_{19} - a_{18}\xi^2 - a_{16}a_{18}$, $C = -\xi_{11}a_{16}a_{17} - \xi_{11}a_{10}a_{19} - a_4 a_{17}\xi^2 + a_{10}a_{18}\xi^2 + a_{16}a_{18}\xi^2$, $B_2 = (a_3 a_6 - a_2 a_{21} - a_{20})/a_2$, $B_3 = (a_{20}a_{21} - a_3 a_6 \xi^2)/a_2$

The mathematical solutions of the equations (29)-(30) satisfying the radiation conditions that $\left(\hat{\phi}, \widehat{\phi^*}, \hat{T}, \widehat{\phi_2}, \hat{\psi}\right) \to 0$ as $x_3 \to \infty$ are given by:

$$\hat{\phi} = B_1 e^{-m_1 x_3} + B_2 e^{-m_2 x_3} + B_3 e^{-m_3 x_3} + L_1, \quad (31)$$

$$\widehat{\phi^*} = d_1 B_1 e^{-m_1 x_3} + d_2 B_2 e^{-m_2 x_3} + d_3 B_3 e^{-m_3 x_3} + L_2, \quad (32)$$

$$\hat{T} = e_1 B_1 e^{-m_1 x_3} + e_2 B_2 e^{-m_2 x_3} + e_3 B_3 e^{-m_3 x_3} + L_3, \quad (33)$$

$$\hat{\psi} = B_4 e^{-m_4 x_3} + B_5 e^{-m_5 x_3}, \quad (34)$$

$$\widehat{\phi_2} = h_4 B_4 e^{-m_4 x_3} + h_5 B_5 e^{-m_5 x_3}, \quad (35)$$

And
$m_i^2 (i = 1,2,3)$ Are the roots of the characteristic equation given (3.8) and $m_l^2 (l = 4,5)$ are the roots of the characteristic equation of equation (3.11).

Boundary Conditions
We consider concentrated normal force and concentrated thermal source at the boundary surface $x_3 = 0$, mathematically, these can be written as:

$$t_{33} = -F_1\delta(x_1)\delta(t), t_{31} = -F_2\delta(x_1)\delta(t),\ m_{32} = 0,\ \lambda_3^* = 0, \frac{\partial T}{\partial x_3} = 0, \quad (36)$$

Where,
F_1, F_2 are the magnitude of the applied forces.
Substituting the values of $\hat{\phi}, \widehat{\phi^*}, \hat{T}, \hat{\psi}, \widehat{\phi_2}$ from the equations (31)-(35) in the boundary condition (36) and using (5)-(7), (9)-(10), (22)-(23) and solving the resulting equations, we obtain:

$$\widehat{t_{33}} = \sum_{i=1}^{5} G_{1i} e^{-m_i x_3} + M_1 e^{-\gamma^* x_3}, \quad (37)$$

$$\widehat{t_{31}} = \sum_{i=1}^{5} G_{2i} e^{-m_i x_3} + M_2 e^{-\gamma^* x_3}, \quad (38)$$

$$\widehat{m_{32}} = \sum_{i=1}^{5} G_{3i} e^{-m_i x_3} + M_3 e^{-\gamma^* x_3}, \quad (39)$$

$$\widehat{\lambda_3^*} = \sum_{i=1}^{5} G_{4i} e^{-m_i x_3} + M_4 e^{-\gamma^* x_3}, \quad (40)$$

$$\hat{T} = \sum_{i=1}^{5} G_{5i} e^{-m_i x_3} + M_5 e^{-\gamma^* x_3}, \quad (41)$$

Here

$$b_1 = \frac{\lambda_0}{\rho c_1^2}, b_2 = \frac{\lambda}{\rho c_1^2}, b_3 = \frac{2\mu+K}{\rho c_1^2}, b_5 = \frac{\mu+K}{\rho c_1^2}, b_6 = \frac{\mu}{\rho c_1^2}, b_7 = \frac{K}{\rho c_1^2}, b_8 = \frac{\omega^{*2}\gamma}{\rho c_1^4}, b_9 = \frac{\omega^{*2} b_0}{\rho c_1^4}, b_{10} = \frac{\omega^{*2}}{\rho c_1^4}$$

$$G_{mi} = g_{mi} C_i, C_i = \frac{\Delta_i}{\Delta_0}, i = 1,2, \dots, 5$$

Special case
Micro polar Thermoelastic Solid
In absence of microstretch effect in Equations (37) - (41), we obtain the corresponding expressions of stresses, displacements and temperature for micro polar generalized thermoelastic half space.

Inversion of the transform
The transformed displacements, stresses and temperature changes are functions of the parameters of Laplace and Fourier transforms s and ξ respectively and hence these are of the form $f(s, \xi, z)$. To obtain the solution of the problem in the physical domain, we must invert the Laplace and Fourier transform by using the method applied by Kumar [19].

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