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The mental arithmetic of the multiplication

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Abstract
The paper proposes a mental arithmetic which is applicable to the multiplication between infinite and arbitrary two big integers with different digits. It explains a few rules for the use of the mental arithmetic. Through a lot of examples of the practical multiplication, it demonstrates that the present mental arithmetic is a correct and advantageous calculation method of the multiplications.

Keywords: Theorem, infer, arbitrary integers, examples

1. Introduction

The mental arithmetic is an important topic in the research fields of mathematics, if people hold the excellent mental arithmetic, perhaps everybody will become a human computer. Japan and India have been studying and developing the mental arithmetic in the fields of mathematics. There are a lot of studies on this topic which have been published over the world [1-6]. However, their methods are merely applicable to the very limited integers, for example, the calculation of the square of the integer whose last number is 5; the multiplication between two integers with 2 digits, and their first digits are the same, the plus of their second digits is equal to 10 [1], and so on. Therefore, it is very siginificant to find the mental arithmetic which can be applicable to much more even infinite integers. For this purpose, the paper intend to present a new mental arithmetic which is applicable to the multiplication between infinite and arbitrary two big integers.

2. The Mental Arithmetic

At first, we consider the multiplication between two integers of two digits.

1. Theorem: Considering an integer of two digits：$90<A<100$, or $A=90+a$, and $0<a<10$, B is an arbitrary integer of two digits which is less than 100, or $B=10b+c$, and $0<=b<10$, $1<=c<10$, thus, the ultiplication of $A\times B$ is equal to $100\times[B+(A-100)]$ (or $100\times[A+(B-100)]$, then followed by $(A-100)\times(B-100)$ according to their correct digit.

To prove: it is obvious that $A+(B-100)=B+(A-100)$, therefore, $N_1=100\times[A+(B-100)] + (A-100)\times(B-100) =N_2=100\times[B+(A-100)]+ (A-100)\times(B-100)$. Thus, we merely need to prove $M=A\times B=N_1$.

$$M=A\times B=(90+a)\times(10b+c)=900b+90c+10ab+ac, \tag{1}$$

$$N_1=100\times[A+(B-100)+ (A-100)\times(B-100) =100a-1000+1000b+100c+10ab+ac-100a-100b-10c+1000=900b+90c+10ab+ac. \tag{2}$$

Comparing the result of eq.(1) with that of eq.(2), it demonstrates that $M=N_1$, the theorem is proven.

(2). Examples

Example 1: calculating 97×94, in terms the theorem, $97-100=-3$, $94-100=-6$, $7+(-3)=97+(-6)=91$, $(-3)\times(-6)=18$, the result should be 4 digits, therefore,

$$97\times 94=9118. \tag{3}$$

Example 2: calculating 93×47, $93-100=-7$, $47-100=-53$, $47+(-7)=40$, $(-7)\times(-53)=371$, the result should be 4 digits, therefore,

$$93 \times 47 = 4371. \quad (4)$$

It points out that the result of exaple 2 should be 4 digits, so if the result of (A－100)×(B－100) is more than 2 or more digits, the more part is added to the front part, like example 2.
Example 3: calculating 99×16, 99－100=－1, 16－100=－84, 16+(－1)=15, (－1)×(－84)=84, therefore,

$$99 \times 16 = 1584. \quad (5)$$

This mental arithmetic can be infinitely generalized to the multiplication between two integers of 3 digits or 4 digits or more digits.
Infer 1: If A is an integer of 3 digits, and 990< A<1000, B is an arbitrary interger of more than 1 but not more than 3 digits, and 10< B <1000, thus, M=A×B=1000×[B+(A－1000)] (or 1000×[A+(B－1000)], then followed by (A－1000)×(B－1000) according to their correct digit.
Example 1: calculating 997×992, 997－1000=－3, 992－1000=－8, 992+(－3)=989, (－3)×(－8)=24, the result should be 6 digits, therefore,

$$997 \times 992 = 989024. \quad (6)$$

Example 2: calculating 994×786, 994－1000=－6, 786－1000=－214, 786+(－6)=780, (－6)×(－214)=1284, therefore,

$$994 \times 786 = 781284. \quad (7)$$

Example 3: calculating 994×76, 994－1000=－6, 76－1000=－924, 76+(－6)=70, (－6)×(－924)=5544, the result should be 5 digits, therefore,

$$994 \times 76 = 75544. \quad (8)$$

Infer 2: If A is an integer of 4 digits, and 9990< A<10000, B is an arbitrary interger of more than 2 but not more than 4 digits, and 10< B <10000, thus, M=A×B=10000×[B+(A－10000)] (or 10000×[A+(B－10000)], then followed by (A－10000)×(B－10000) according to their correct digit.
Example 1: calculating 9998×9994, because 9998－10000=－2, 9994－10000=－6, 9994+(－2)=9992, (－2)×(－6)=12, the result should be 8 digits, therefore,

$$9998 \times 9994 = 99920012. \quad (9)$$

Example 2: calculating 9998×1234, because 9998－10000=－2, 1234－10000=－8766, 1234+(－2)=1232, (－2)×(－8766)=17532, the result should be 8 digits, therefore,

$$9998 \times 1234 = 12337532. \quad (10)$$

Example 3: calculating 9999×764, because 9999－10000=－1, 764－10000=－9236, 764+(－1)=763, (－1)×(－9236)=9236, the result should be 7 digits, therefore,

$$9999 \times 764 = 7639236. \quad (11)$$

Example 4: calculating 9997×82, because 9997－10000=－3, 82－10000=－9918, 82+(－3)=79, (－3)×(－9918)= 29754, the result should be 6 digits, therefore,

$$9997 \times 82 = 819754. \quad (12)$$

The above examples demonstrate that this mental arithmetic can be infinitely generalized to the multiplication between two arbitrary big integers, such as

$(99990+a)\times(100000b+10000c+1000d+100e+10f+g)$;
$(999990+a)\times(1000000b+100000c+10000d+1000e+100f+10g+h)$; ……., and $1 <= a, b, c, d, e, f, g, h <=9$.

Infer 3: If the multiplicatin occurs between two arbitrary integers all with 2 digits, namely, 10a+b, at first, increasing one of them up to 90+b, 1<=a, b<=9, which can be calculated out by the theorem, then subtracting the additional part, thus, it obtains the result.
Example 1: Calculating 32×27, let 27+70=97, according to the theorem, 97×32=3104, and 32×70=2240, thus

$$32 \times 27 = 3104 - 2240 = 864. \quad (13)$$

Example 2: Calculating 73×47, let 73+20=93, using the theorem, 93×47=4371, 20×47=940, thus,

$73 \times 47 = 6471 - 940 = 3431.$ (14)

Example 3: Calculating 29×69, let 69+30=99, according to the theorem, 99×29=2871, 30×29=870, thus,

$29 \times 69 = 2871 - 870 = 2001.$ (15)

Similarly, this mental arithmetic is also applicable to the multiplication between two arbitrary integers all with more than 2 digits. For example, in consideration of the multiplication between two arbitrary integers all with 3 digits,

Calculating 237×779, according to the infer 3, it is written 999×237－237×200－237×20, according to the theorem, 999×237=236763, therefore,

$237 \times 779 = 236763 - 47400 - 4740 = 184623.$ (16)

Furthermore, if considering the multiplication between two arbitrary integers all with 4 digits, for example,

Calculating 3571×7287, it can be written 9997×3571－3571×2000－3571×700－3571×10, according to theorem, 9997×3571=35699287, therefore,

$3571 \times 7287 = 35699287 - 7142000 - 2499700 - 35710 = 26021877.$ (17)

3. Conclusion

The paper proposed a method of mental arithmetic, although this mental arithmetic is applicable to the multiplication between two arbitrary big integers, in the practical calculations, a few points are emphasized as the following:

1. With respect to the theorem, because A－100=90+a－100 is equal to an integer of one digit, according to the mental arithmetic, in genereal, it is more convenient to choose B+(90+a－100) rather to use A+(10b+c－100) for the mental arithmetic.
2. In the practical calculation, at first, it should realize that how many digits the result will be, for example, in terms of the theorem, if the digit of the result of 100×[A+(B－100)] + (A－100)×(B－100) or 100×[B+(A－100)]+ (A－100)×(B－100) is less than the digit of that the result should be, it can fill "0" between 100×[A+(B－100)] and (A－100)×(B－100); or 100×[B+(A－100)] and (A－100)×(B－100) to make the digits be same as the digits of that the result should be. If the digit of 100×[A+(B－100)+ (A－100)×(B－100) or 100×[B+(A－100)]+ (A－100)×(B－100) is more than the digit of that the result should be, thus, the front digits of (A－100)×(B－100) should be plus to the end digits of 100×[B+(A－100)] or 100×[B+(A－100)] to make the digits of their result be same as the digits of that the result should be.
3. The present mental arithmetic is also applicable to the cases of A×B, and the digit of B is less than that of A.
4. With respect to the multiplication between two arbitrary integers all with 2 digits, in general, it is convenient to increase the bigger integer 10a+b up to 90+b, and 1<=a, b<=9, then subtracting the additional part to get the result.

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