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The Vedas as the original source of the Hindu Indian ancient and modern mathematical sciences: A survey article

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Abstract
In [1], it was studied Indian Influence in Mathematics of Abu Ja'far Muhammad ibn Musa Al-Khwarizmi, in 2022. Also, contributions of Indian mathematicians can be seen in [3] in 2021. This survey will serve as an impetus to stimulate the interests in basic Indian Mathematicians to record and recognize them as pioneers not only in the fields of mathematics but also in science, in applied and natural science, in engineering and social science for further teaching as well as research in their respective subject matters with mainly the choicest blessings of the absolute Supreme Creator, Founder God of the Universe, like that in Islam too, Allah has 99 names emphasizing on education leading to both basic as well as higher level mathematics like ideals, quasi-ideals and bi-ideals in semihypergroups, ordered semihypergroups, Γ-semigroups, ordered Γ-semigroups, Γ-near ring with the spirited hope for future perspective in teaching and in research.

Keywords: The temple, the Vedas of god, the six Hindu Indian philosophy, god, Allah, Kuttaka, Indian knowledge system, Indian mathematics

1. Introduction

The unique God and Allah sent vaious Rishis, Sages and Avatars like Dalit Lord Krishna Dvaipayana, Lord Rama and Lord Lakshmana, Lord Narayana, Lord Vishnu of Hinduism; Prophets like the last one Prophet PBUH and his pious companions of Islam; the 10 human Vahe Gurus, the 11th present and future for eternity of the Sikh on different areas of the world to teach, to inspire and to give wisdom and knowledge for the making of the betterment of humanity in every sphere of lives.

The Dalit visionary sage Veda Vyasa as a Shaktyavesha Avatar of Vishnu is considered the one who arranged the mantras of the Vedas and authored the Brahma Sutras. Vyasa put all the Vedic knowledge in this universe in written words that earlier were in the spoken form to make it available to everyone, the positive impact of which is felt even today. He provided humanity the categorized and documented version of the sacred Vedas that is our book of knowledge and wisdom. He also produced the Bhagavad Gita, the sacred text of Hinduism. It is said that Ved Vyasa is immortal and he never died. In this Survey Article, we study Indian Mathematics inspired and learned from and based on the Hindu sacred Vedas, and collaborate with mathematicians of India.

Mathematicians and scientists studied the Vedic literature to receive knowledge into mathematical, scientific, social science, grammar, spiritual, psychological, geographical, research and development as well as behavioral knowledge and wisdom.

The classical divine surviving text of humanity is the sacred text Vedas. Krishna Dvaipayana or Vyasa or Veda Vyasa classified the sacred Vedas [Satyanarayana *et al.* [17] and [Delhi Sanskrit Academy [19]. The four Vedas are as follows:

1. The Rig Veda,
2. The Sama Veda,
3. The Yajur Veda, and
4. The Atharva Veda.

Each of the four Vedas is subdivided into four distinct subsections:
(i) Mantras,
(ii) Brahmanas,

(iii) Aranyakas, and
(iv) Upanishads.

In the Ramayana, the Mahabharata, and the various Upanishads, there are abundance of knowledge about qualitative managements to make principles of education and learning modern oriented. The ancient Gurukul education system has been honoured throughout the world for its multifaceted lives and scientific management temperament in learning with discipline.

2. Indian Civilizations and Indian Philosophy Based on the Vedas as the Magnet of Mathematical Sciences

The ancient Indian civilization is the Indus Valley or Harappan civilisation, and at Mohenjo Daro. Harappan had adopted a uniform system of weights and measures.

The Hindu religious scripture the Vedas, composed in the Sanskrit language, contains hymns, spells, and ritual observations. The basic Indian mathematics of significance on the Indian subcontinent was based on these religious texts. The Vedas contains fundamental rules to construct altars. They consist quite an amount of geometrical knowledge but the basic Indian mathematics was being developed alongside. The Brahmi numerals began to appear before the end of the period of the Sulbasutras around the middle of the third century. Astrology played a crucial role since it was that ”science” which with accurate required information about the planetary bodies and other heavenly bodies encourage the development of mathematics.

The main Sulbasutras were written by Baudhayana, Manava, Apastamba and Katyayana. Another scholar Panini obtained outstanding results in Sanskrit Grammar which has applications in modern computer science for mathematicians or computer scientists working with formal language theory.

The Sulba Sfitras are concerned with mathematical rules for complying with the requirements of Hindu rituals, specifically with measuring and constructing sacrificial pits and altars. These requirements of religious rituals led to squaring the circle called ’circling the square’.

For Six Systems of Indian Philosophy, readers can refer to [20, 27] for exploring further knowledge. The Vedas are the most sacred text of Hinduism. For mathematical concepts in ancient Sanskrit works, that is considered the store house of knowledge and a treasure of ancient and modern Mathematics, readers can refer to the survey article from the sacred Vedas by Satyanarayana, and Satyasri [17].

The six Hindu philosophic systems are as follows:

1. Samkhya founded by Sage Kapila,
2. Yoga founded by Sage Patanjali,
3. Nyaya founded by Sage Gouthama,
4. Vaisesika founded by Sage Kanada,
5. Mimamsa propounded by Sage Jaimini, and
6. Vedanta founded by Sage Vyasa.

Mithila became a knowledgeable center of learning during the Vedic era and this place of learning various subjects like Mathematics, Science, Social Science and Languages transformed into the university of Mithila [18, 36]. It was later on developed by Gangesa Upadhyay. It has been known as an important place for learning and cultures [36, 34]. Maithili language, considered the sweetest language in the world, is mainly spoken in this region in particular and in Bihar [35] in general. Researchers used to come to this place to study about this language [37].

For comprehensive biography on some of the notable figures of mathematics and theoretical computer science from ancient to modern history, one can refer to R. P. Agarwal and S. K. Sen [28]. For further reading in Indian Mathematics, readers can refer to the selected articles by Kripa Shankar Shukla [33] who worked like sages, for Hindu Astronomy from Vedic Period to the Emergence of Siddhantas, Hindu Mathematics, Bhaskara I to Narayana Pandita, Aryabhatiya (I), Aryabhatiya (II), Aryabhatiya (III), Sridhara, Acarya Jayadeva, the Pancasiddhantika of Varahamihira (1), the Pancasid dhantika of Varahamihira (2), Hindu trigonometry, Hindu geometry, use of calculus in Hindu mathematics, use of permutations and combinations in India, magic squares in India, use of series in India, surds in Hindu Mathematics, approximate value of surds in Hindu Mathematics, early Hindu methods in spherical astronomy, use of hypotenuse in the computation of the equation of the centre under the epicyclic theory in the school of Aryabhata I, Hindu astronomer Vatesvara and his works, the evection and the deficit of the equation of the centre of the Moon in Hindu astronomy, phases of the Moon, rising and setting of planets and stars and their conjunctions, Rajamrganka of Bhojaraja, Karanaratna of Devacarya, Raymond P. Mercier’s review of Karanaratna of Devacarya, The yuga of the Yavanajataka: David Pingree’s text and translation review, and The yuga of the Yavanajataka: David Pingree’s text and translation review.

Cosmology, astrology, space, research, planets and galaxies, medicinal science and surgery, nuclear theory, thermodynamics, energy concepts, and many discoveries are parts of the Vedic literature. These basics of Indian mathematics are the core concepts that has enlightened the whole world of not only mathematics but its fruits are enjoyed in Physics, Chemistry, Biology, Biochemistry, Biomedical, Computer Science, Economics, Statistics, Basic and Interdisciplinary Science, Operation Research, Mathematical Modelling, Natural and Applied Science, and Humanity, Engineering, Music and Song, and so on in teaching and in research. It is fundamentally required as building blocks to preserve, to construct and to produce more and more basic results in mathematics teaching pure and applied mathematics and research. These basics of general pure Indian mathematics has culminated and grown into modern mathematics applicable universally in all domains of science, technology and development.

The standard arithmetic algorithms actually originated in India, where they were known by various names such as Patiganita, i.e., Slate Arithmetic. The Vedas are the valid and original source of all knowledge and fountain of wisdom. The Vedas are the source of Architectural Concepts, basis for Science and mathematics, medicine, Space and military, and Chemical Sciences.

3. Basic Indian Mathematics

Recently, Basar, Satyanarayana, Kumar, and Mohammad Yahya Abbasi [3] enlightened the Indian contributors on basic top levels Indian mathematicians motivated from the sacred Hindu Scripture the Vedas. The Indian Mathematics has been so clear and lucid that it was adopted applicable throughout the world immediately in the universe. A bibliography on Sanskrit works on astronomy and mathematics was done by S. N. Sen, A. K. Bag and R. S. Sarma [29]. For geometry in ancient and medieval India, one can refer to Saraswati Amma [44]. The text versions with modern commentaries were brought out by Raghunath P. Kulkarni [40], in Sen and Bag [30]. For insightful coverage of five key centers in non-western

mathematics: Egypt, Mesopotamia, China, India, and Islam, one can refer to [43]. For the Kerala works in mathematics at a considerably advanced level than the earlier works, one can refer to [42]. For a recent version with English and Kannada translations together with the original text, one can see in Padmavathamma [41]. For an overview works in the Siddhanta, one can see in Agathe Keller [39]. For all the folios of the Bakhshali manuscript in Indian mathematics, one can see in [38] by G. R. Kaye in 1927.

The Kerala School mathematicians developed the series expansions for trigonometric functions two centuries before the introduction of Calculus in Europe. However, they did not construct a systematic study of theory of differentiantion and integration.

The theory of trigonometry was introduced in India. This knowledge transferred from India to the middle-east(Arabia) and from thereon to Europe. Expansion of sin $(x + y)$, sines and cosines of various angles of measurement in degree, are produced in India. The sine of an angle, and the sine function depits one of the main contributions of the Sanskrit astronomical works namely Siddhantas which is the predecessor of the modern mathematics. Bhaskara I provided formulae to obtain the values of sine functions. A 16th century Malayalam treatise Yuktibhasa was authored by the Indian astronomer Jyesthadeva of the Kerala school of mathematics around 1530.

Mahavira in his Sanskrit treatise "Ganitasara Sangraha"(Compendium of the Essence of Mathematics) with 9 Chapters and 1131 verses [21] cleared this earlier recognized concept that square root of a negative number does not exist in real number system. Thereafter, an Indian Mathematician Bhaskara also stated that there is no square root of a negative number. The Jain mathematician Mahavira is probably the world's pioneer mathematician who gave the general formula for Permutation and Combinations. Sushruta in his "Sushruta Samhita" the medicinal work gave assertions that 63 combinations is possible out of 6 distinct tastes, considered one at a time, two at a time, and so on. Furthermore, a Sanskrit Scholar Pingala provided formulation of finding the number of combinations of an assigned number of letters, considered one at a time, two at a time, and so on. Bhaskaracharya with the name Anka Pasha, in his work Lilavati, studied the subject matter of permutations and combinations. It was Mahavira who gave the general formulae for combination ${}^{n}C_{r}$ and permutation ${}^{n}P_{r}$. Bhaskaracharya provided significant results concerning this subject matter.

Indians first found integral solutions of Diophantine equations in a systematic manner that can be traced back to Sulbasutras. The solutions of Diophantine equations in rational numbers was studied by Diophantus. But, in general, determining integer solutions to an equation is much more difficult one than that of finding rational solutions. The Sulbasutras also discusses many examples of "Pythagorean Triples". A Pythagorean triples is positive integers (x, y, z) such that $x^2 + y^2 = z^2$. Garhapatyagni altar consists of five layers of bricks with each layer having 21 bricks. These Vedic altar gives interesting insights into specific idea of simultaneous indeterminate equations.

The "Kuttaka" method called by Bhaskara I, was later on, studied and referred extensively by many Indian Mathematicians like Brahmagupta, Mahavira, Aryabhata II, Sripati, Bhaskara II, and Narayana. Brahmagupta called the whole subject of Algebra by Kuttaka-Ganita or Kuttaka. The Kuttaka algorithm was used by the Indians for solving problems in astronomy and in making Indian calendar.

Among the quadratic equations, the most known is the famus Pell equation $x^2 - Dy^2 = 1$ by Pell. Indians efficiently devised algorithm to solve Pell's equation. Consider a, b, c to be positive integers. Then a linear Diophantine equation is of the standard form as below:

$$ay - bx = \pm c, \text{ or } ay + bx = \pm c.$$

Indian authors observed that the equation has integral solution only if c is divisible by the GCD(Greatest Common Divisor) of a and b. The equation $ay - bx = c$ possesses infinitely many integral solutions if the constant c is divisible by the GCD of a and b. The original algorithm of Aryabhata [22] led to Kuttaka. Mahavira, Aryabhata II, Sripati, and Bhaskara II used similar methods for the solutions of simultaneous linear Dipohantine equations of the type:

$$b_1y_1 = a_1x \pm c_1,\ b_2y_2 = a_2x \pm c_2,\ b_3y_3 = a_3x \pm c_3.$$

The Vedic Mathematics by Bharati Krishna Tirtha [16] is a collection of Techniques or Sutras to solve mathematical arithmetics in easy, efficient, and quicker way. It consists of 16 Sutras called Formulae and 13 sub sutras, i.e., sub formulae which can be used for problems involved in Arithmetic, Algebra, Geometry, Calculus, and Conics.

4. Conclusion

The knowledge of Mathematical Sciences, the finest mathematical facts, figures and theories have been achieved by the continuous practices and researches of hundreds of mathematicians for the centuries. Many people had contributed to the development of Mathematical sciences which mathematicians, scientists and social scientists benefit today. The contribution of Indian mathematicians is immense and remarkable in this sense.

It is the need of the today's era to promote ahead the heritage of mathematicians so as to encourage and cherish the magnificent traditional roots of the country in mathematics.

In 2020, one can see in Basar [7] about introducing and development of ideals by Kummer, Dedekind, Hilbert, a woman mathematician Emmy Noether, Artin, McCoy, Good and Hughes, Lajos, and Steinfeld as follows:

"The concepts of ideal and prime ideal were created by Dedekind as a generalization of the concept of ideal numbers as a special subset of a ring defined by Kummer. Thereafter, these notions were extended by Hilbert and Noether. Noether and Artin gave classical definitions of basic notions such as one sided ideal, ideal and other algebraic notions. The concept of prime ideals in rings was defined and studied by McCoy. The notion of a bi-ideal in semigroup was introduced by Good and Hughes. Then, the notion of bi-ideal and generalized bi-ideal in semigroups was introduced and studied by Lajos. Thereafter, the notion of a quasi-ideal was introduced by Steinfeld in rings and semigroups."

In 2021, Basar, Satyanarayana, Syam Prasad Kuncham, Kumar and Mohammad Yahya [4] introduced relative Γ-ideals in abstract affine Γ-nearrings. In 2014, Satyanarayana, Mohammad Yahya Abbasi, Basar and Syam Prasad Kuncham [31] introduced Γ-ideals in abstract affine Γ-nearrings.

In 2020, Basar [7] introduced relative hyperideals, relative bi-hyperideals, relative quasi-hyperideals, relative prime hyperideals, relative weakly prime hyperideals, relative semiprime hyperideals, relative prime and relative semiprime

bi-hyperideals, and hyper relative regularity in involution ordered hypersemigroups. In 2020, Basar and Kumar [6] studied normal bihyperideals in hypersemigroups.

In 2013 and 2014, some nice aspects of ideals has been studied in [15, 24]. In 2015, various types of ideals has been studied in [14, 25, 26]. In 2016, Basar *et al*. [23] studied ordered bi-Γ-ideals in intra-regular ordered Γ-semigroups. In 2017, Basar *et al*. [13] studied ordered bi-Γ-ideals in ordered Γ-semigroups. In 2018, fuzzy ideals has been studied in [12]. In 2019, Basar and Mohammad Yahya Abbasi [8] studied normal Γ-ideals in normal Γ-semigroups. Furthermore, in 2019, various aspects of ideals has been studied in [9, 10, 11] in LA-Γ-semigroups, LA-Γ-semihypergroups. In 2022, Basar, Ali, Satyanarayana and Kumar [2] studied basic results on relative ordered Γ-ideals in ordered LA-Γ-semigroups.

Furthermore, in 2019, Satyanarayana, Srinivasulu Devanaboina, Mallikarjuna Bhavanari and Abul Basar [32] studied power chains in a divisor graph. In 2021, Basar, Satyanarayana and Kumar [5] studied relative act hyperideals and relative hyperideals of hypersemigroups.

It is hoped that the future rests on the development of mankind along the righteous paths which the modern science and mathematics strives for and the Vedas stand for. However, researchers are to study and explore the Vedic Knowledge System such as Mathematics, and Science to put forth the vast glorious Vedic era to the whole world once again. The creative method of expressing every possible number using a set of 10 symbols first appeared in India. It facilitates calculation and places arithmetic foremost amongst useful mathematical concepts. This invention was ahead of the two greatest men, Archimedes and Apollonius.

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