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A concise proof on the Fermat's last theorem

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Abstract
On the basis of the proof on the Fermat's last theorem in cases n=3k(k=1,2,3,……), the paper further respectively proves that the Fermat's last theorem is also true in cases of n=3k+1(k=1,2,3,……) and n=3k+2(k=1,2,3,……). Because n=3k; 3k+1; and 3k+2(k=1, 2, 3, ……) contain all the integers of n>2, therefore the paper completely proves the Fermat's last theorem.

Keywords: Fermart's last theorem, Diophantine equation, n=3k, n=3k+1, n=3k+2, the disproval method

1. Introduction
Like the Goldbach's conjecture, the Fermat's last theorem is also one of the puzzling problems in the field of the number theory, even in the fields of mathematics. From long time ago, the topic about the Fermat's last theorem has attracted the interesting of many researchers who know the number theory [1-13]. Although in the fields of mathematics , peoples have accepted that Andrew Wiles proved the Fermat's last theorem in 1994 and got the great Abel prize, his proof is very long with more than 100 pages in the Annals of Mathematics in France[8], moreover, his proof is complex and not easy to understand. Therefore, it is very significant to find a rigorous and better proof. This paper is trying to present a concise proof on the Fermat's last theorem.

2. The cases of n=3k (k=1, 2, 3,……)
In consideration of the cases of n=3k (k=1, 2, 3,……), the Diophantine equation is written

$$x^{3k} + y^{3k} = z^{3k}. \tag{1}$$

Using the mathematical induction, when k=1, eq. (1) becomes

$$x^3 + y^3 = z^3, \tag{2}$$

this case has been proven elsewhere [1].
Supposing that when k= k' , there are also no positive integers x, y, and z to fit eq. (1), thus when k= k' +1, eq. (1) becomes

$$x^{3(k'+1)} + y^{3(k'+1)} = z^{3(k'+1)}. \tag{3}$$

Using "the disproval method", if there are positive integers x, y, and z to fit eq.(3), eq.(3) can be written as

$$x^3 \cdot x^{3k'} + y^3 \cdot y^{3k'} = z^3 \cdot z^{3k'}, \tag{4}$$

if only considering eq.(1) in the same domain definition as that of eq.(4), and substituting eq.(1) into eq.(4), it obtains

$$(x^3 - z^3)\cdot x^{3k'} + (y^3 - z^3)\cdot y^{3k'} = 0. \tag{5}$$

Because x > 0, y > 0, z > 0 and $x \neq y \neq z$[13] ; eq.(3) and eq.(4) demonstrate that z > x, z > y, thus, in eq.(5), $(x^3 - z^3)\cdot x^{3k} + (y^3 - z^3)\cdot y^{3k} < 0$, it is in contradiction with eq.(5), therefore, there should be no positive integers x, y, and z to fit eq.(3), the Fermat's last theorem in all cases of n=3k(k=1,2,3,……) are true.

3. The cases of n=3k+1(k=1, 2,…….)

It has been proven above that there is no integer solution (x,y,z) to fit the eq.(1)

In consideration of equation

$$x^{3k+1} + y^{3k+1} = z^{3k+1} \text{ (k=1, 2,……)}, \quad (6)$$

in order to prove that there is no an integer solution (x,y,z) to fit eq.(6), eq.(6) can be written:

$$x \cdot x^{3k} + y \cdot y^{3k} = z \cdot z^{3k}. \quad (7)$$

Using "the disproval method", if there are positive integers x, y, z and k to fit eq.(6), and if merely considering the values of x, y in eq.(1) are belong to the same domain definition as that of x, y in eq.(7), thus, substituting eq.(1) into eq.(7), it arrives

$$(x - z)\cdot x^{3k} + (y - z)\cdot y^{3k} = 0. \quad (8)$$

In eq.(8), because x > 0, y > 0, z > 0 , and according to the theorem 1 of Ref. [13], $x \neq y \neq z$, therefore, z > x, and z > y as well, so the inequality $(x - z)\cdot x^{3k} + (y - z)\cdot y^{3k} < 0$ must be true, eq.(8) is wrong! In terms of "the disproval method", when n=k+1, there are also no positive integers x, y, and z to make them fit eq.(6). The Fermat's last theorem in all of the cases of n=3k+1 (k=1,2,3……) has been proven. By the way, if considering x, y in eq.(1) take the values of decmal, thus, substituting eq.(1) into eq.(7) can't obtain eq.(8).

4. The cases of n=3k+2(k=1,2, ……)

In further, considering the equation

$$x^{3k+2} + y^{3k+2} = z^{3k+2} \quad \text{(k=1,2,……)}, \quad (9)$$

eq. (9) can be written as

$$x^2 \cdot x^{3k} + y^2 \cdot y^{3k} = z^2 \cdot z^{3k}. \quad (10)$$

As the same reason of discussions above, using "the disproval method", if there are positive integers of x, y, z and k to fit eq.(9), and merely considering the values of x, y in eq.(1) belong to the same domain definition as that of eq.(10), thus, substituting eq.(1) into eq.(10), it obtains

$$(x^2 - z^2)\cdot x^{3k} + (y^2 - z^2)\cdot y^{3k} = 0, \quad (11)$$

because x > 0, y > 0, z > 0 , and according to the theorem 1 of Ref. [13], $x \neq y \neq z$, therefore, z > x, and z > y as well, so the inequality $(x^2 - z^2)\cdot x^{3k} + (y^2 - z^2)\cdot y^{3k} < 0$ must be true, eq. (11) is wrong! Therefore, there are no positive integers of x, y, and z to make them fit eq.(9). Eq. (11) holds true only in the same domain definition of x, y in eq.(1) and eq. (10)

In general, when n=3k; 3k+1; and 3k+2 (k=1,2,3,……), there are no positive integers x, y, and z to fit the Diophantine equation $x^n + y^n = z^n$, the Fermat's last theorem has been completely proven.

5. Conclusion

The paper respectively proved that when n=3k; 3k+1; and 3k+2 (k=1, 2, 3,…….), there are no positive integers x, y, and z to fit the equation $x^n + y^n = z^n$. Because n=3k; 3k+1; and 3k+2 (k=1,2,3,……) contain all the integers of n >2, for example, when k=1, 3k=3, 3k+1=4, 3k+2=5; when k=2, 3k=6, 3k+1=7, 3k+2=8; when k=3, 3k=9, 3k+1=10, 3k+2=11;……Therefore, the paper completely proved the Fermat's last theorem. In the proof, the paper used "the disproval method", if there are positive integers x, y, and z to fit eq.(3) , eq.(6) and eq.(9), if merely considering x, y in eq. (1) belong to the same domain definition of that of eq. (4) or eq. (7) or eq.(10), thus, respectively substituting eq. (1) into eq. (4) , eq. (7) and eq.(10), but they result in the wrong results of eq. (5), eq. (8) and eq.(11). This demonstrates that there are no positive integers x, y, and z to fit eq. (3) or eq. (6) or eq.(9). It should point out that if the domain definition of x and y in eq. (1) is different from that of eq. (4) or eq. (7) or eq.(10), thus, eq.(5), eq. (8) and eq.(11) can't be obtained.

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