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Bilateral generating functions for the two-parameter one-variable Srivastava polynomials

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Abstract

In this paper we prove a general theorem on generating functions involving the two-parameter one-variable Srivastava polynomials, Legendre Polynomials of pseudo two variables and Lagrange polynomials of two variables. Some applications of these theorems lead us to derive several bilateral generating functions involving some well-known classical polynomials of one variable which are contained by the two-parameter one-variable Srivastava polynomials.

Keywords: Generating functions, Srivastava polynomials, legendre polynomials, lagrange polynomials

1. Introduction

In (Srivastava, 1972) [6] introduced the following family of polynomials:

$$S_n^N(x) = \sum_{k=0}^{\lfloor \frac{n}{N} \rfloor} \frac{(-n)_{Nk}}{k!} A_{n,k} x^k \quad (n \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}; N \in \mathbb{N}), \quad (1)$$

Where

\mathbb{N} is the set of positive integers,

$\{A_{n,k}\}_{n,k=0}^{\infty}$ is a bounded double sequence of real or complex numbers, $[a]$ denotes the greatest integer in $a \in \mathbb{R}$ and $(\lambda)_n$ denotes the Pochhammer symbol defined by (Srivastava and Manocha, 1984):

$$(\lambda)_n = \frac{\Gamma(\lambda + n)}{\Gamma(\lambda)}, \quad \lambda \neq 0, -1, -2, \dots \quad (2)$$

Afterwards (Gonzalez *et al.* 2001) [2] extended the Srivastava polynomials $S_n^N(x)$ as follows:

$$S_{n,m}^N(x) = \sum_{k=0}^{\lfloor \frac{n}{N} \rfloor} \frac{(-n)_{Nk}}{k!} A_{n+m,k} x^k \quad (n, m \in \mathbb{N}_0; N \in \mathbb{N}). \quad (3)$$

In (Kaanoglu and Ozarslan 2013) [3] introduced the following family of two-parameter one-variable Srivastava polynomials:

$$S_n^{p,q}(x) = \sum_{k=0}^n \frac{(-n)_k}{k!} A_{p+q+n, q+k} x^k \quad (p, q, n, k \in \mathbb{N}_0), \quad (4)$$

where $\{A_{n,k}\}$ is a bounded double sequence of real or complex numbers.

The Legendre Polynomials $P_n(x, y)$ of pseudo two variables are defined by (Khan *et al.*, 2010) [4]

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$$P_n(x, y) = \sum_{r=0}^{\lfloor \frac{n}{2} \rfloor} \frac{n!(x^2 - y - 1)^r x^{n-2r}}{2^{2r} (r!)^2 (n - 2r)!} \tag{5}$$

And satisfy the following generating relation (Khan et al., 2010) [4]:

$$\sum_{n=0}^{\infty} \frac{(c)_n P_n(x, y) t^n}{n!} = (1 - xt)^{-c} {}_2F_1 \left[\begin{matrix} \frac{c}{2}, \frac{c}{2} + \frac{1}{2} \\ 1 \end{matrix}; \frac{t^2(x^2 - y - 1)}{(1 - xt)^2} \right] \tag{6}$$

where ${}_2F_1$ is the Gaussian hypergeometric function defined by (Srivastava and Manocha, 1984):

$${}_2F_1 \left[\begin{matrix} a, b \\ c \end{matrix}; x \right] = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n} \frac{x^n}{n!}, \quad c \neq 0, -1, -2, \dots \tag{7}$$

The Lagrange polynomials $g_n^{(\alpha_1, \Lambda, \alpha_r)}(x_1, \Lambda, x_r)$ of r-variables, which are known as Chan-Chyan-Srivastava polynomials, are generated by the following generating functions (Chan, et al., 2001) [1]:

$$\sum_{n=0}^{\infty} \frac{(\lambda)_n}{(\mu)_n} g_n^{(\alpha_1, \Lambda, \alpha_r)}(x_1, \Lambda, x_r) t^n = F_D^{(r)}[\lambda, \alpha_1, \Lambda, \alpha_r; \mu; x_1 t, \Lambda, x_r t], \tag{8}$$

where $F_D^{(r)}$ is the Lauricella's function of the fourth kind of several variables defined by (Srivastava and Manocha, 1984) [7]

$$F_D^{(r)}(a, b_1, \Lambda, b_r; c; x_1, \Lambda, x_r) = \sum_{m_1, \Lambda, m_r=0}^{\infty} \frac{(a)_{m_1+\Lambda+m_r} (b_1)_{m_1} \Lambda (b_r)_{m_r}}{(c)_{m_1+\Lambda+m_r}} \frac{x_1^{m_1}}{m_1!} \Lambda \frac{x_r^{m_r}}{m_r!}, \tag{9}$$

$$\max\{|x_1|, \Lambda, |x_r|\} < 1.$$

The special case of (8) when $r = 2$ and $\mu = 1$ gives the following result:

$$\sum_{n=0}^{\infty} \frac{(\lambda)_n}{n!} g_n^{(\alpha_1, \alpha_2)}(x_1, x_2) t^n = F_1[\lambda, \alpha_1, \alpha_2; 1; x_1 t, x_2 t], \tag{10}$$

where F_1 is Appell double hypergeometric functions (Srivastava and Manocha, 1984) [7]

$$F_1(a, b_1, b_2; c; x, y) = \sum_{m, n=0}^{\infty} \frac{(a)_{m+n} (b_1)_m (b_2)_n}{(c)_{m+n}} \frac{x^m}{m!} \frac{y^n}{n!} \tag{11}$$

Main Results

Theorem 2.1 The following family of bilateral generating functions holds true:

$$\begin{aligned} & \sum_{p, q, n=0}^{\infty} P_{p+q+n}(x, y) S_n^{p, q}(z) \frac{u^p}{p!} \frac{v^q}{q!} \frac{t^n}{n!} \\ &= \sum_{p, q=0}^{\infty} P_{p+q}(x, y) A_{p+q, q} \frac{(u+t)^p}{p!} \frac{(v-zt)^q}{q!}. \end{aligned} \tag{12}$$

Proof: Denoting the left hand side of (12) by S , expressing $S_n^{p,q}(z)$ as in (4) and using the result (Srivastava and Manocha, 1984) [7]:

$$(-n)_k = \frac{(-1)^k n!}{(n-k)!}, \quad 0 \leq k \leq n,$$

we obtain

$$S = \sum_{p,q,n=0}^{\infty} P_{p+q+n}(x,y) \sum_{k=0}^n \frac{(-z)^k}{k!} A_{p+q+n,q+k} \frac{u^p}{p!} \frac{v^q}{q!} \frac{t^n}{(n-k)!}.$$

Using the following result (Srivastava and Manocha, 1984) [7]:

$$\sum_{n=0}^{\infty} \sum_{k=0}^n A(k,n) = \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} A(k,n+k),$$

we get

$$S = \sum_{p,q,n,k=0}^{\infty} P_{p+q+n,k}(x,y) A_{p+q+n+k,q+k} \frac{u^p}{p!} \frac{v^q}{q!} \frac{t^n}{n!} \frac{(-zt)^k}{k!}.$$

Now, by using the following results (Srivastava and Manocha, 1984) [7]:

$$\sum_{n=0}^{\infty} \sum_{k=0}^{\infty} A(k,n) = \sum_{n=0}^{\infty} \sum_{k=0}^n A(k,n-k) \quad \text{and} \quad \sum_{n=0}^{\infty} (\lambda)_n \frac{x^n}{n!} = (1-x)^{-\lambda},$$

we have

$$\begin{aligned} S &= \sum_{p,q,k=0}^{\infty} P_{p+q+k}(x,y) A_{p+q+k,q+k} \frac{(u+t)^p}{p!} \frac{v^q}{q!} \frac{(-zt)^k}{k!} \\ &= \sum_{p,q=0}^{\infty} P_{p+q}(x,y) A_{p+q,q} \frac{(u+t)^p}{p!} \frac{v^q}{q!} \sum_{k=0}^q \frac{(-q)_k (zt/v)^k}{k!} \\ &= \sum_{p,q=0}^{\infty} P_{p+q}(x,y) A_{p+q,q} \frac{(u+t)^p}{p!} \frac{(v-zt)^q}{q!}. \end{aligned}$$

This completes the proof of Theorem 2.1.

In a similar manner, we also get the following result immediately.

Theorem 2.2 The following family of bilateral generating functions holds true:

$$\begin{aligned} &\sum_{p,q,n=0}^{\infty} g_{p+q+n}^{(\gamma,\delta)}(x,y) S_n^{p,q}(z) \frac{u^p}{p!} \frac{v^q}{q!} \frac{t^n}{n!} \\ &= \sum_{p,q=0}^{\infty} g_{p+q}^{(\gamma,\delta)}(x,y) A_{p+q,q} \frac{(u+t)^p}{p!} \frac{(v-zt)^q}{q!} \end{aligned} \tag{13}$$

Remark 2.1 On taking $u = -t$ in (12) and (13), we deduce the following interesting corollaries:

Corollary 2.1.

$$\sum_{p,q,n=0}^{\infty} P_{p+q+n}(x, y) S_n^{p,q}(z) \frac{(-t)^p}{p!} \frac{v^q}{q!} \frac{t^n}{n!} = \sum_{q=0}^{\infty} P_q(x, y) A_{q,q} \frac{(v-zt)^q}{q!} \tag{14}$$

Corollary 2.2.

$$\sum_{p,q,n=0}^{\infty} g_{p+q+n}^{(\gamma,\delta)}(x, y) S_n^{p,q}(z) \frac{(-t)^p}{p!} \frac{v^q}{q!} \frac{t^n}{n!} = \sum_{q=0}^{\infty} g_q^{(\gamma,\delta)}(x, y) A_{q,q} \frac{(v-zt)^q}{q!} . \tag{15}$$

Remark 2.2 On taking $v = 0$ in (12) and (13) and using the relation $S_n^{p,0}(z) = S_{n,p}^1(z)$, we deduce the following interesting corollaries:

Corollary 2.3.

$$\sum_{p,n=0}^{\infty} P_{p+n}(x, y) S_{n,p}^1(z) \frac{u^p}{p!} \frac{t^n}{n!} = \sum_{p,q=0}^{\infty} P_{p+q}(x, y) A_{p+q,q} \frac{(u+t)^p}{p!} \frac{(-zt)^q}{q!} \tag{16}$$

Corollary 2.4.

$$\sum_{p,n=0}^{\infty} g_{p+n}^{(\gamma,\delta)}(x, y) S_{n,p}^1(z) \frac{u^p}{p!} \frac{t^n}{n!} = \sum_{p,q=0}^{\infty} g_{p+q}^{(\gamma,\delta)}(x, y) A_{p+q,q} \frac{(u+t)^p}{p!} \frac{(-zt)^q}{q!} \tag{17}$$

Where $S_{n,m}^N(z)$ is the extended Srivastava polynomials (3).

3. Applications

In order to obtain our results of this section, we require the following remarks (Kaanoglu et al., 2013) [3]:

Remark 3.1 Choosing $A_{m,n} = (-\alpha - m)_n$ ($m, n \in \mathbb{N}_0$) in (4), we get

$$S_n^{p,q}\left(\frac{-1}{x}\right) = (-1)^q (\alpha + p + n + 1)_q \frac{n!}{(-x)^n} L_n^{(\alpha+p)}(x), \tag{18}$$

Where $L_n^{(\alpha)}(x)$ is the classical Laguerre polynomials (Srivastava and Manocha, 1984) [7]

$$L_n^{(\alpha)}(x) = \frac{(-x)^n}{n!} {}_2F_0\left[-n, -\alpha - n; -; -\frac{1}{x}\right]. \tag{19}$$

Remark 3.2 Choosing $A_{m,n} = \frac{(\alpha + \beta + 1)_{2m} (-\beta - m)_n}{(\alpha + \beta + 1)_m (-\alpha - \beta - 2m)_n}$ ($m, n \in \mathbb{N}_0$) in (4), we get

$$S_n^{p,q}\left(\frac{2}{1+x}\right) = \frac{(\alpha + \beta + 1)_{2p+2q+2n} (-\beta - p - q - n)_q (1 + \alpha + \beta + 2p + q)_n}{(\alpha + \beta + 1)_{p+q+n} (-\alpha - \beta - 2p - 2q - 2n)_q (1 + \alpha + \beta + 2p + q)_{2n}} \times n! \left(\frac{2}{1+x}\right)^n P_n^{(\alpha+p+q, \beta+p)}(x), \tag{20}$$

Where $P_n^{(\alpha, \beta)}(x)$ is the classical Jacobi polynomials (Rainville, 1971) [5].

$$P_n^{(\alpha, \beta)}(x) = \binom{\alpha + \beta + 1}{n} \left(\frac{1+x}{2}\right)^n {}_2F_1\left[-n, -\beta - n; -\alpha - \beta - 2n; \frac{2}{1+x}\right]. \tag{21}$$

Further, we add the following remark:

Remark 3.3 Choosing $A_{m,n} = (-\alpha - m)_n$ ($m, n \in \mathbb{N}_0$) in (4), we get

$$S_n^{p,q}\left(\frac{-x}{\beta}\right) = (-\alpha - p - q - n)_q y_n(x, 1 - \alpha - p - 2n, \beta) \quad (\beta \neq 0), \quad (22)$$

where $y_n(x, \alpha, \beta)$ is the Bessel polynomials (Srivastava and Manocha, 1984) [7]

$$y_n(x, \alpha, \beta) = {}_2F_0\left[-n, \alpha + n - 1; -; \frac{-x}{\beta}\right]. \quad (23)$$

I. In (14) and (15) choosing $A_{m,n} = (-\alpha - m)_n$ and using (18), we get

$$\begin{aligned} & \sum_{p,q,n=0}^{\infty} (1 + \alpha + p + n)_q P_{p+q+n}(x, y) L_n^{(\alpha+p)}(z) \frac{t^p}{p!} \frac{v^q}{q!} \left(\frac{t}{z}\right)^n \\ &= \sum_{q=0}^{\infty} (\alpha + 1)_q P_q(x, y) \frac{(v + t/z)^q}{q!} \end{aligned} \quad (24)$$

and

$$\begin{aligned} & \sum_{p,q,n=0}^{\infty} (1 + \alpha + p + n)_q g_{p+q+n}^{(\gamma,\delta)}(x, y) L_n^{(\alpha+p)}(z) \frac{t^p}{p!} \frac{v^q}{q!} \left(\frac{t}{z}\right)^n \\ &= \sum_{q=0}^{\infty} (\alpha + 1)_q g_q^{(\gamma,\delta)}(x, y) \frac{(v + t/z)^q}{q!}. \end{aligned} \quad (25)$$

Now, by using (6) and (10) in (24) and (25) respectively, we get

$$\begin{aligned} & \sum_{p,q,n=0}^{\infty} (1 + \alpha + p + n)_q P_{p+q+n}(x, y) L_n^{(\alpha+p)}(z) \frac{t^p}{p!} \frac{v^q}{q!} \left(\frac{t}{z}\right)^n \\ &= (1 - x(v + t/z))^{-\alpha-1} {}_2F_1\left[\frac{\alpha+1}{2}, \frac{\alpha+2}{2}; (v + t/z)^2(x^2 - y - 1); \frac{(v + t/z)^2(x^2 - y - 1)}{(1 - x(v + t/z))^2}\right] \end{aligned} \quad (26)$$

and

$$\begin{aligned} & \sum_{p,q,n=0}^{\infty} (1 + \alpha + p + n)_q g_{p+q+n}^{(\gamma,\delta)}(x, y) L_n^{(\alpha+p)}(z) \frac{t^p}{p!} \frac{v^q}{q!} \left(\frac{t}{z}\right)^n \\ &= F_1\left[\alpha + 1, \gamma, \delta; 1; x\left(v + \frac{t}{z}\right), y\left(v + \frac{t}{z}\right)\right]. \end{aligned} \quad (27)$$

Further, if we take $v = 0$ in (26) and (27) respectively, we get

$$\sum_{p,n=0}^{\infty} P_{p+n}(x, y) L_n^{(\alpha+p)}(z) \frac{t^p}{p!} \left(\frac{t}{z}\right)^n$$

$$= \left(1 - \frac{xt}{z}\right)^{-\alpha-1} {}_2F_1 \left[\begin{matrix} \frac{\alpha+1}{2}, \frac{\alpha+2}{2}; \\ 1 \end{matrix}; \frac{t^2(x^2 - y - 1)}{(z - xt)^2} \right] \tag{28}$$

and

$$\sum_{p,n=0}^{\infty} g_{p+n}^{(\gamma,\delta)}(x, y) L_n^{(\alpha+p)}(z) \frac{t^p}{p!} \left(\frac{t}{z}\right)^n = F_1 \left[\alpha + 1, \gamma, \delta; 1; \frac{xt}{z}, \frac{yt}{z} \right]. \tag{29}$$

II. In (14) and (15) choosing $A_{m,n} = \frac{(\alpha + \beta + 1)_{2m} (-\beta - m)_n}{(\alpha + \beta + 1)_m (-\alpha - \beta - 2m)_n}$ and using (20), we get

$$\begin{aligned} & \sum_{p,q,n=0}^{\infty} \frac{(\alpha + \beta + 1 + p + q + n)_{p+q+n} (-\beta - p - q - n)_q}{(1 + \alpha + \beta + 2p + q + n)_n (-\alpha - \beta - 2p - 2q - 2n)_q} P_{p+q+n}(x, y) \\ & \times P_n^{(\alpha+p+q, \beta+p)}(z) \frac{(-t)^p}{p!} \frac{v^q}{q!} \left(\frac{2t}{1+z}\right)^n = \sum_{q=0}^{\infty} (\beta + 1)_q P_q(x, y) \frac{(v - 2t/(1+z))^q}{q!} \end{aligned} \tag{30}$$

and

$$\begin{aligned} & \sum_{p,q,n=0}^{\infty} \frac{(\alpha + \beta + 1 + p + q + n)_{p+q+n} (-\beta - p - q - n)_q}{(1 + \alpha + \beta + 2p + q + n)_n (-\alpha - \beta - 2p - 2q - 2n)_q} g_{p+q+n}^{(\gamma,\delta)}(x, y) \\ & \times P_n^{(\alpha+p+q, \beta+p)}(z) \frac{(-t)^p}{p!} \frac{v^q}{q!} \left(\frac{2t}{1+z}\right)^n = \sum_{q=0}^{\infty} (\beta + 1)_q g_q^{(\gamma,\delta)}(x, y) \frac{(v - 2t/(1+z))^q}{q!}. \end{aligned} \tag{31}$$

Now, using (6) and (10) in (30) and (31) respectively, we get

$$\begin{aligned} & \sum_{p,q,n=0}^{\infty} \frac{(\alpha + \beta + 1 + p + q + n)_{p+q+n} (-\beta - p - q - n)_q}{(1 + \alpha + \beta + 2p + q + n)_n (-\alpha - \beta - 2p - 2q - 2n)_q} \\ & \times P_{p+q+n}(x, y) P_n^{(\alpha+p+q, \beta+p)}(z) \frac{(-t)^p}{p!} \frac{v^q}{q!} \left(\frac{2t}{1+z}\right)^n \\ & = (1 - x(v - 2t/(1+z)))^{-\beta-1} {}_2F_1 \left[\begin{matrix} \frac{\beta}{2}, \frac{\beta}{2} + \frac{1}{2}; \\ 1 \end{matrix}; \frac{(v - 2t/(1+z))^2 (x^2 - y - 1)}{(1 - x(v - 2t/(1+z)))^2} \right] \end{aligned} \tag{32}$$

and

$$\begin{aligned} & \sum_{p,q,n=0}^{\infty} \frac{(\alpha + \beta + 1 + p + q + n)_{p+q+n} (-\beta - p - q - n)_q}{(1 + \alpha + \beta + 2p + q + n)_n (-\alpha - \beta - 2p - 2q - 2n)_q} \\ & \times g_{p+q+n}^{(\gamma,\delta)}(x, y) P_n^{(\alpha+p+q, \beta+p)}(z) \frac{(-t)^p}{p!} \frac{v^q}{q!} \left(\frac{2t}{1+z}\right)^n \\ & = F_1 \left[\beta + 1, \gamma, \delta; 1; x \left(v - \frac{2t}{1+z} \right), y \left(v - \frac{2t}{1+z} \right) \right]. \end{aligned} \tag{33}$$

Further, if we take $\nu = 0$ in (32) and (33) respectively, we get

$$\sum_{p,n=0}^{\infty} (1 + \alpha + \beta + p + n)_p P_{p+n}(x, y) P_n^{(\alpha+p, \beta+p)}(z) \frac{(-t)^p}{p!} \left(\frac{2t}{1+z}\right)^n$$

$$= \left(1 + \frac{2xt}{1+z}\right)^{-\beta-1} {}_2F_1 \left[\begin{matrix} \frac{\beta+1}{2}, \frac{\beta+2}{2}; \\ 1 \end{matrix}; \frac{4t^2(x^2 - y - 1)}{(1+z + 2xt)^2} \right] \quad (34)$$

and

$$\sum_{p,n=0}^{\infty} (1 + \alpha + \beta + p + n)_p g_{p+n}^{(\gamma, \delta)}(x, y) P_n^{(\alpha+p, \beta+p)}(z) \frac{(-t)^p}{p!} \left(\frac{2t}{1+z}\right)^n$$

$$= F_1 \left[\beta + 1, \gamma, \delta; 1; \frac{-2xt}{1+z}, \frac{-2yt}{1+z} \right]. \quad (35)$$

III. In (14) and (15) choosing $A_{m,n} = (-\alpha - m)_n$ and using (22), we get

$$\sum_{p,q,n=0}^{\infty} (-\alpha - p - q - n)_q P_{p+q+n}(x, y) y_n(z, 1 - \alpha - p - 2n, \beta) \frac{(-t)^p}{p!} \frac{\nu^q t^n}{q! n!}$$

$$= \sum_{q=0}^{\infty} (\alpha + 1)_q P_q(x, y) \frac{(-\nu - zt / \beta)^q}{q!} \quad (36)$$

and

$$\sum_{p,q,n=0}^{\infty} (-\alpha - p - q - n)_q g_{p+q+n}^{(\gamma, \delta)}(x, y) y_n(z, 1 - \alpha - p - 2n, \beta) \frac{(-t)^p}{p!} \frac{\nu^q t^n}{q! n!}$$

$$= \sum_{q=0}^{\infty} (\alpha + 1)_q g_q^{(\gamma, \delta)}(x, y) \frac{(-\nu - zt / \beta)^q}{q!}. \quad (37)$$

Now, by using (6) and (10) in (36) and (37) respectively, we get

$$\sum_{p,q,n=0}^{\infty} (-\alpha - p - q - n)_q P_{p+q+n}(x, y) y_n(z, 1 - \alpha - p - 2n, \beta) \frac{(-t)^p}{p!} \frac{\nu^q t^n}{q! n!}$$

$$= (1 - x(-\nu - zt / \beta))^{-\alpha-1} {}_2F_1 \left[\begin{matrix} \frac{\alpha+1}{2}, \frac{\alpha+2}{2}; \\ 1 \end{matrix}; \frac{(-\nu - zt / \beta)^2 (x^2 - y - 1)}{(1 - x(-\nu - zt / \beta))^2} \right] \quad (38)$$

and

$$\sum_{p,q,n=0}^{\infty} (-\alpha - p - q - n)_q g_{p+q+n}^{(\gamma, \delta)}(x, y) y_n(z, 1 - \alpha - p - 2n, \beta) \frac{(-t)^p}{p!} \frac{\nu^q t^n}{q! n!}$$

$$= F_1 \left[\alpha + 1, \gamma, \delta; 1; -x \left(v + \frac{zt}{\beta} \right), -y \left(v + \frac{zt}{\beta} \right) \right]. \quad (39)$$

Finally , if we take $v = 0$ in (38) and (39) respectively, we get

$$\sum_{p,n=0}^{\infty} P_{p+n}(x, y) y_n(z, 1 - \alpha - p - 2n, \beta) \frac{(-t)^p}{p!} \frac{t^n}{n!}$$

$$= \left(1 + \frac{xzt}{\beta} \right)^{-\alpha-1} {}_2F_1 \left[\begin{matrix} \frac{\alpha+1}{2}, \frac{\alpha+2}{2} \\ 1 \end{matrix}; \frac{z^2 t^2 (x^2 - y - 1)}{(\beta + xzt)^2} \right] \quad (40)$$

and

$$\sum_{p,n=0}^{\infty} g_{p+n}^{(\gamma, \delta)}(x, y) y_n(z, 1 - \alpha - p - 2n, \beta) \frac{(-t)^p}{p!} \frac{t^n}{n!}$$

$$= F_1 \left[\alpha + 1, \gamma, \delta; 1; \frac{-xzt}{\beta}, \frac{-yzt}{\beta} \right]. \quad (41)$$

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