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Forecasting performance of hybrid ARIMA-FIGARCH model and hybrid of ARIMA-GARCH model: A comparative study

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Abstract
This paper considers the Comparison of forecasting performance the hybridization between ARIMA Model and Fractionally Integrated Generalized Autoregressive Conditional Heteroscedastic (FIGARCH) processes. With will be used to develop the most appropriate model for forecasting financial Time Series data. The data employed for this study was secondary type in nature for all the variables and it is obtained from the publications of Central Bank of Nigerian bulletin, National Bureau of Statistics and World Bank Statistics Database dated, January, 2005 to Dec, 2019. The result of unit root test shows that all variables are stationary at level and first differences at 5% level of significant. From the Furthermore, the sum of the alpha and beta parameters is close to unity ($\alpha + \beta = 0.9832001$), indicating that the persistence of the NSE return is high. Although the returns volatility appears to have what seems to be long memory: the sum of α and β is significantly less than one While the results indicate that the coefficient gamma is not significant, implying that the sign of the innovation has not significant influence on the volatility of returns and also delta is significant it shows the present of long -memory. Based on the results obtaining in table (11) using information's criteria with shows that AIC, BIC and HQIC of ARIMA-FIGARCH model (14.488, 14.577 and 14.524) are less than for ARIMA-GARCH. With shows that ARIMA-FIGARCH are best model for forecasting National Stock Exchange of Nigeria

Keywords: FIGARCH, GARCH, ARIMA and hybridizations

1. Introduction

Forecasting and modeling of Time Series data are not new terms to stakeholders, both in the economic and business fields respectively. Time Series data for financial market constantly exhibit (volatility) variability and uncertainty in market fluctuations. Volatility when envisaged in defines the measure of fluctuations of currency. Volatility in exchange rate has raised great concern to all economic and business analysts as it's after international trade macroeconomic variables (export and import) and the economic growth of a nations. Volatility in Exchange rate result in international transaction that may leads to the downtrends in international trade and economic welfare (Wong and Lee, 2016). Thus, forecasting and modeling of exchange rate play vital roles in a nation's economy. Nigeria as a nation is not left out in this exchange rate volatility as (exchange rate) is one of the significant indicators that determines nation's economic growth. To expound this point, for instance; the appreciation or depreciation of naira is of interest to financial analysts, policy makers, investors, researchers, to mention but few; and even to the nations Hence, the researcher is interest in proposed an efficient modelling technique that will be appropriate in describing the volatility of time series data in a more lucid manner.

It is vivid that to the fact that volatility, plays a major role in economic and financial applications such as; monetary policy making, investment, and security evaluation. In 1976 Box-Jenkins developed and designed an ARIMA model as a forecasting tool for financial economic variables which was named after the creator as the Box-Jenkins Methodology. The Box- Jenkins Methodology 1976 methodology attempts to find an autoregressive integrated moving average (ARIMA) (p, d, q) model that satisfies the stochastic procedure where the derived sample from ARIMA model can be estimated using the Box-Jenkins approach. The Box-Jenkins method comprises an iterative three- stage modeling approach that includes model identification, parameter estimation, and model checking.

Since a necessary condition for an ARIMA model is stationary, when the observed time series present trend and non- seasonal behavior, then differencing is done to the data series to eliminate the trend. Box Jenkins methodology is one of the well-known methods in time series modelling, and forecasting which is extensively applied in numerous areas of time series analysis. ARIMA model is frequently used by some analysts due to its ability in handling non-stationary data. More so, it is simple to implement, and it generally offer accurate prediction over a short period of time.

Even though ARIMA is a powerful method in forecasting several types of times series data, it cannot handle the volatility that is present in data series. Thus, various families of generalized autoregressive conditional heteroscedasticity (GARCH) models were proposed by researchers in order to cover loopholes of ARIMA models. GARCH model is divided into two categories symmetric, and asymmetric. The difference between asymmetric, and symmetric model is related to their effect of sign on volatilities. In the symmetric model, the conditional variance only depends on the magnitude and not in the sign of the underlying asset, whereas in the asymmetric model, the sign whether positive or negative of the underlying asset having the same magnitude of shocks has different effect on volatility. (Ahmed and Sulliman, 2011). Symmetric model performs better in capturing leptokurtosis and volatility clustering of financial returns, but since they have a symmetric distribution, they fail in modelling leverage effect (Narsoo, 2015). This limitation of symmetric model has led to the development of several asymmetric models that were able to capture the asymmetric relationship including Exponential GARCH (EGARCH), Threshold GARCH (TGARCH), and Power GARCH (PGARCH) models.

The primary purpose of introducing FIGARCH model was to develop a more flexible class of processes for the conditional variance, that are capable of explaining and representing the observed temporal dependencies in financial market volatility. In particular, the FIGARCH model allows only a slow hyperbolic rate of decay for the lagged squared or absolute innovations in the conditional variance function. This model can accommodate the time dependence of the variance and a leptokurtic unconditional distribution for the returns with a long memory behavior for the conditional variances. The primary purpose of introducing FIGARCH model was to develop a more flexible class of processes for the conditional variance, that are capable of explaining and representing the observed temporal dependencies in financial market volatility. In particular, the FIGARCH model allows only a slow hyperbolic rate of decay for the lagged squared or absolute innovations in the conditional variance function. This model can accommodate the time dependence of the variance and a leptokurtic unconditional distribution for the returns with a long memory behavior for the conditional variances.

1.1 Statement of the problem

In recent times, stake holders, policy makers, financial economist, academic researchers to mention but few – have picked interest in movement, and fluctuations in financial Time Series. In an attempt to bring the situation under control, several types of case studies, and approaches have been applied to the data in order to handle some characteristics that exist in the series Generalized Autoregressive Conditional Heteroscedasticity (GARCH) family model a more famous and frequently useful method, particularly known in handling volatility of data series.

For instant to model the long-term persistence, Engle and Bollerslev (1986) developed the Integrated GARCH (IGARCH) model as an extension to the original GARCH model. It is argued that IGARCH models have a property called "persistent variance" since any shocks to the conditional variance, either happened to day or in the past, will persist indefinitely in to the future. However, Nelson (1990) showed that the IGARCH process without drift would definitely converge to zero with probability one, infinite steps. Hence, IGARCH models are generally considered as short memory models by researchers (Davidson 2004; Granger and Ding 1996a). (1996) generalized the IGARCH model to a new class named Fractionally Integrated GARCH (FIGARCH) models, with the purpose of explicitly describing the long memory behavior of the conditional variances of financial time series This model can accommodate the time dependence of the variance and a leptokurtic unconditional distribution for the returns with a long memory behavior for the conditional variances the. FIGARCH model has its short-run dynamics described by the conventional GARCH parameters (αis and βjs). The combination of ARIMA model with nonlinear FIGARCH model is essential so that the conditional mean, and conditional heteroscedasticity of the series can be captured in order to have an effective way to overcome the weaknesses of each component and be able to improve the accuracy of forecasting. To investigate the ability of the ARIMA-FIGARCH model with Nigerian stock exchange to predict persistence of long memory volatilities. The performance of these model is then compared with ARIMA-GARCH as a benchmark.

1.2 Aim and objectives

The main aim of this research is to develop a hybrid model for modelling the mean and long memory in volatility will achieved through the following objectives

1. To develop a hybrid of ARIMA-FIGARCH
2. To develop a code in R for implementing the ARIMA-FIGARCH
3. To investigate the ability of the ARIMA-FIGARCH Model with Nigerian stock exchange to predict persistence of long memory volatilities. The performance of these model is then compared with ARIMA-GARCH.

2. Literature Review

Financial Time Series well-known to show certain features which referred to as stylized facts. The term stylized facts were introduced by an economist, Nicholas Kaldor in (1961), in his work about economic development theory. (Sewell, 2011), defined stylized facts as a" term in economics used to refer to the empirical results that are so steady across market and recognized as truth". In financial time series, there are two existing stylized structures which are leverage effect, and volatility clustering. Stylized fact attributed asymmetry to that volatility is higher after negative shocks occurred. This characteristic is referred as leverage effect, (Black, 1976). He recognized that volatility tends to increase in response to bad news and decrease in response to good news as stock returns are negatively correlated to variations in returns volatility clustering has been shown to be existing in a wide variety of financial assets comprising, exchange rates and market indices securities, interest rate (Bollerslev, 1986). As stated by (Mandelbrot, 1963), volatility clustering's can be defined as large variations that tend to be monitored by" large fluctuations, of either sign, or small variations that tend to be followed by small fluctuations. In other word, when volatility is high, it will possibly be remaining for certain periods of time, and it may be short for other times. In financial market, fluctuations of shock stock exchange return either positive or negative would determine volatility.

2.1. Time series model of Nigerian stock exchange

Kuznets (1971) defines a country's economic growth as a long – term rise in the capacity of supply leading to increase in the production of goods for the population accompanied by advancing technology and the institutional and ideological adjustment that it demands. It therefore encompasses growth, structural and institutional changes and the essential elements that make up life such as education, health, nutrition and a better environment i.e., human and development indices. Ekundayo (2002) argues that a nation requires a lot of foreign investment to attain sustainable economic growth and development, the capital market provides means through which that is made possible. Several policies and programs have been consciously created to promote the growth of the Nigerian economy overtime. Some of these policies include the enterprises promotion decree, the privatization of government enterprises (2000), which were quoted on the Nigerian stock exchange. There were also the bank recapitalization directives (2004), by the CBN, in which banks were directed to recapitalize to a minimum of twenty-five billion naira. For this many banks accessed the capital market (through the primary public offers) for their financing needs, the government also introduced the pension reform Act of 2004. This act provides that part of the pension fund should be invested in the capital market by pension fund administrators. However, the impact of the capital market on the growth and development of the economy has not been significantly positively felt (Babalola, 2007). This may be due to low market capitalization, delay in delivery of share certificate problem of manual call, slow growth of securities market, double taxation, problem of macro-economic instability among others. Also, most Nigerians are not aware of the benefit derivable from the market operations. Furthermore, there is a problem of reluctance of Nigerian businessmen to go to the public for fear of losing control of their business. More so, the Nigerian stock exchange market, over the years has undergone reforms due to the declining effect of global financial crisis. While capital market has the potentials of stimulating economic growth and development through effective resource allocation, the expected high economic growth that comes with capital market development has not been experienced in Nigeria (Popoola, 2014).

2.2. Review of hybridize models

Babu and Reddy (2015) develop a linear hybrid model by means of partitioning-interpolation based ARIMA-GARCH. The hybrid ARIMA-GARCH model attained to reveal the accuracy of forecast while stabilizing data trend through the forecast limit. Volatility clustering and fat tail distribution are overall features of extremely volatile Time Series data. Because to these features, filter decomposition and moving average is constructed. Also, a sole partitioning-interpolation procedure is established to construct the partitioning-interpolation based ARIMA-GARCH model. The model developed by Babu and Reddy (2015) was useful on the selected India data sets to obtain multi-ahead forecast which is, related to the ability with wavelet-ARIMA, ARIMA- GARCH, and ANN models. The finding attained were used to forecast criteria such as Mean Absolute Error (MAE), Mean Absolute Percent Error (MAPE) and Root Mean Square Error (RMSE). From the thought, the developed model achieves improved in terms of forecast accuracy
and stabilizing data trend related to the others classic

Thorlie et. al. (2014), they develop hybrid GJR- GARCH and ARMA –GARCH, models with normal, distributions and skewed distribution on Sierra Leone exchange rate against U.S. Dollar (Leones/USD). The monthly data from January 2004 till December 2013 was collected from the Central Bank of Sierra Leone and was used in the study. The predicting shows was evaluated using three different criteria which are Mean Absolute Error (MAE), Mean Square Error (MSE), and Adjusted Mean Absolute Percentage Error (AMAPE). The finding of the result revealed that the predicting performance of non-linear GARCH was superior to GARCH model when fat-tailed asymmetric conditional distributions are occupied into concern of the volatility. The general results proposed that hybrid ARMA-GJR-GARCH with skewed distribution is the greatest model for predicting Sierra Leone against US Dollars exchange rate.

3. Methodology

3.1 Introduction

This chapter discusses the techniques that will be employed by the researcher when conducting the study on modelling and predicting financial Time Series data. The hybridization between Autoregressive Integrated Moving Average (ARIMA) and Fractionally Integrated Generalized Autoregressive Conditional Heteroscedastic (FIGARCH) process. will be used to develop the most appropriate model for forecasting financial Time Series data.

3.2. Volatility measurement

Assume the return can be used in calculating the volatility of any given asset return. Thus, we may use as a measure of volatility which could be written as

The Mean equation:

$$y_t = \mu + z_t \tag{1}$$

$$\varepsilon_{t=}h_t^{\frac{1}{2}}(y_t-\mu) \tag{2}$$

$$\frac{\varepsilon_{i,t}}{\phi_{i-1,t}} \sim \text{N (0, 1)} \ \forall_i = 1 \ldots N_t$$

Where $y_t -$ is the return on day i, $\mu -$is the average return and { Z_t } is a mean-zero serially uncorrelated process h_t-the variance is used as a volatility measure.

3.3. Proposed Modifications Hybrid Model

Develop model is a predicting procedure that combines two or more individual models. In this study, hybridization of ARIMA and FIGARCH model will be done in two phase's procedure. In the first phase, the best of ARIMA model is applying to model the linear data of time series. In the second phase, FIGARCH approach is applying to model the nonlinear designs of the residuals sequence from the fitted ARIMA model. In this procedure, the error term of the ARIMA model will follow FIGARCH processes of order (*p, d q)*. The primary steps in construction hybrid models are the same with ARIMA methodology which it contains of model identification, parameter estimation, diagnostic checking and forecasting.

3.4. General Equation of Hybrid Models

The general equation of hybrid ARIMA (*p*, *d*, *q*) - FIGARCH (*p*, d, q), model can be written as followed where it contains of two equations. Mean equation comes from ARIMA model while variance equation comes from FIGARCH model. We combine the ARIMA and FIGARCH- which are in equations below together, the hybrid ARIMA (p, d, q)- FIGARCH (p, d, q) model can be specified as

$\varphi_p(L)(1-L)^d(y_t-\mu) = \theta_q(L)\varepsilon_t$ Were

$\varphi(L) = 1-\varphi_1 L-\varphi_2 L^2 \ldots -\varphi_p L^P$ And $\theta(L) = 1+\theta_1 L+\theta_2 L^2+\cdots+\theta_q L^q$

$$(1-\varphi_1 L-\varphi_2 L^2 \ldots -\varphi_p L^P)(1-L)^d(y_t-\mu) = (1+\theta_1 L+\theta_2 L^2+\cdots+\theta_q L^q)\varepsilon_t \tag{3}$$

$$(1-L)^d(y_t-\mu)-\varphi_1(1-L)^d y_{t-1}-\varphi_2(1-L)^d y_{t-2} \ldots \varphi_p(1-L)^d y_{t-p} = \varepsilon_t+\theta_1\varepsilon_{t-1}+\theta_2\varepsilon_{t-2}+\cdots+\theta_q\varepsilon_{t-q} \tag{4}$$

$$(1-L)^d((y_t-\mu) = \textstyle\sum_{i=1}^{p}\varphi_i(1-L)^d y_{t-i}+\sum_{j=1}^{q}\theta_j\varepsilon_{t-j}+\varepsilon_t \sim ARIMA(p,d,q) \tag{5}$$

$$Where\ the\ \varepsilon_t = h_t^{\frac{1}{2}}z_t \tag{6}$$

The Mean equation

$$then\ y_t = \mu+z_t \tag{7}$$

$$y_t-\mu = z_t \tag{8}$$

{ Z_t } is a mean-zero serially uncorrelated process μ = average returns ε_t = residual returns, defined as by EQN 2

$\varepsilon_{t=}h_t^{\frac{1}{2}}(y_t-\mu)$ *subtitude eqn* (2) *in* (5)

The hybrid of ARIMA (p, d, q)-FIGARCH (p, d, q) is given below

$$(1-L)^d((y_t-\mu) = \sum_{i=1}^{p}\varphi_i(1-L)^d y_{t-i}+\sum_{j=1}^{q}\theta_j\varepsilon_{t-j}+\varepsilon_t$$

Where $\varepsilon_{t=}h_t^{\frac{1}{2}}(y_t-\mu)$

$$(1-L)^d((y_t-\mu)=\sum_{i=1}^{p}\varphi_i(1-L)^d y_{t-i}+\sum_{j=1}^{q}\theta_j\varepsilon_{t-j}+h_t^{\frac{1}{2}}(y_t-\mu) \quad (9)$$

Where

$$h_t^{\frac{1}{2}}=\alpha_0(1-\beta(1))^{-1}+\gamma(L)\varepsilon_t^2 \quad (10)$$

$$(1-L)^d((y_t-\mu)=\sum_{i=1}^{p}\varphi_i(1-L)^d y_{t-i}+\sum_{j=1}^{q}\theta_j\varepsilon_{t-j}+(y_t-\mu)\alpha_0(1-\beta(1))^{-1}+\gamma(L)\varepsilon_t^2 \quad (11)$$

$if\mu=0$ then

$$\Delta^d y_t=\sum_{i=1}^{p}\varphi_i(1-L)^d y_{t-i}+\sum_{j=1}^{q}\theta_j\varepsilon_{t-j}+y_t\alpha_0(1-\beta(1))^{-1}+\gamma(L)\varepsilon_t^2$$
$$\sim ARIMA(p,d,q)-FIGARCH(p,d,q) \quad (12)$$

Everywhere $y_t -$ represent NSE time series and ε_t is random error. Then $\varphi_1,\varphi_{2\ldots-\varphi_p}$ are the autoregressive coefficients that attempt to predict an output of a system based on the previous outputs and $\theta_1,\theta_2+\cdots+\theta_q$ are the moving averages coefficients.

Where

γ(L) = γ L + γL_2 +.... of course, for the FIGARCH (p, d, q), must be non-negative, i.e., $\lambda_k \geq 0$ for k = 1, 2
$(1-L)^d$-Fractional differencing operator. d-is a fraction 0<d<1 and
$\varepsilon_t^2 - is\ a\ persistant\ shocks\ of\ long-memory$ where $(\varphi,\theta,\alpha,\beta,\gamma)$ are unknown parameter need to estimated.
where h (.) is a nonnegative function of its arguments and $\alpha = (\alpha_{0,}\alpha_1,\ldots,\alpha_q)'$ is a vector of unknown parameters.

3.5.0 Unit Root Test

Recently, testing for unit roots has already become a standard procedure in time series studies and the application of unit root test such as DF- Test and KPSS. Unit root test is important because the absence of unit root indicates that the series has some variances are not being determined by time and that the effects of shocks dissolve over time. Besides that, the existence of non-stationary variables will cause a spurious regression which has high R^2 and t-statistic is significant but the results would not consist of any economic meaning

3.5.1 The augmented Dickey Fuller test (ADF)

The Dickey Fuller (DF) test is used for testing the presence of a unit root. The approach involves testing the null hypothesis that a series does contain a unit root (i.e., it is non-stationary) against the alternative of stationarity. The ADF test is comparable with the simple Dickey Fuller test, but it involves adding an unknown number of lagged first differences of the dependent variable to capture autocorrelated omitted variables that would otherwise, by default, enter the error term *et al*. In this way, one can validly apply unit root tests when the underlying data generating process is quite general. However, it is also very important to select the appropriate lag length; too few lags may result in over-rejecting the null hypothesis when it is true (i.e. adversely affecting the size of the test), while too many lags may reduce the power of the test (since unnecessary nuisance parameters reduce the effective number of observations available).

3.5.2 Kwiatkowski–Phillips–Schmidt–Shin (KPSS) test

The work of Kwiatkowski *et al*. (1992) was motivated by the fact that unit root tests developed by Dickey and Fuller (1979), Dickey and Fuller (1981), and Said and Dickey (1984) indicated that most aggregate economic series had a unit root. In these tests, the null hypothesis is that the series has a unit root. Since such tests have low power in samples of sizes occurring in many applications, Kwiatkowski *et al*. (1992) proposed that trend stationarity should be considered as the null hypothesis, and the unit root should be the alternative. Rejection of the null of trend stationarity could then be viewed as convincing evidence in favor of a unit root. It was soon realized that the KPSS test of Kwiatkowski *et al*. (1992) has a much broader utility. For example, Lee and Schmidt (1996) and Giraitis *et al*. (2003) [7] used it to detect long memory, with short memory as the null hypothesis; de Jong *et al*. (1997) developed a robust version of the KPSS test. The work of Lo (1991) is crucial because he observed that under temporal dependence, to obtain parameter free limit null distributions, statistics similar to the KPSS statistic must be normalized by the long run variance rather than by the sample variance. Likewise, there have been dozens of contributions that enhance unit root testing, see e.g., Cavaliere and Xu (2014) and Chambers *et al*. (2014).

3.6. Diagnostics test

Diagnostic tests for serial auto correlations, normality and heteroskedasticity will carry out for the estimated model.

3.6.1 Breusch-Godfrey LM test for autocorrelation

Autocorrelation problem is the current and the past error term has relationship among each other and this most likely to occur in time series data. Compare to Durbin-Watson (DW) test and Durbin's h test, we choose the Breusch-Godfrey LM test because the DW test will provides inconclusive results and cannot take higher r orders of series correlation into account and the Durbin's h test is unable to use the lagged dependent variable. In the test, there is no autocorrelation problem for null hypothesis. The null hypothesis will be rejected if P-value of F-statistics is lower than the level of significance, α. We need to determine its optimal lag length via the minimum AIC and SIC based on the number of lagged residuals when conducting the auxiliary model. This test is used to check if the error ε_t (in ARCH (p) model) is truly a skedastic function. The regression is given thus:

$H_o : \alpha_1 = \alpha_2 = ... = \alpha_p = 0$ There are no ARCH effects in the residuals under the null the LM statistic is distributed asymptotically as $x^2(p)$ statistic.

3.6.2 Breusch-Pagan-Godfrey heteroskedasticity test

In order to ensure that the residuals are randomly dispersed throughout the range of the dependent variable, we are going to use heteroskedasticity test. The variance of the error should therefore be constant for all values of the dependent variable. In the presence of heteroskedasticity, the distributions of parameters are no longer normal. The decision rule is to reject the null hypothesis if the probability of the F-statistic and observed R^2 are less than 0.05, meaning heteroskedasticity is present. On the other hand, if the probability of the F-statistic and observed R^2 are greater than 0.05, we do not reject the null hypothesis, implying that there is no heteroskedasticity. As such, errors are homoscedastic. $H_o : \rho_{u,1} = ... = \rho_{u,h} = 0$ I.e. all lags' correlations are zero

$H_A : \rho_{u,i} \neq 0$ I.e. for at least one i= 1…h is tested i.e. at least one lag with non- zero correlations.

The test statistic is given as

If $the, \hat{u}_t$ are residuals from an estimated ARMA (p, q) model, the test statistics have an approximate asymptotic $x^2(h-p-q)$ distribution if the null hypothesis holds. We reject H_o if p-value is less than the significance level (jargue, 1987).

3.6.3 Akaike information criteria Akaike

[11], developed the Information Criterion (AIC), for selecting models so as to achieve the most accurate out-of-sample forecasts. This criterion determines the size of the errors by evaluating the log-likelihood, but also penalizes over fitting of models by including a penalty term (usually twice the number of parameters used). While including extra (but possibly unnecessary parameters in the model will reduce the size of the errors, the penalty function ensures these unnecessary terms will be less attractive when using the criterion. The model that have the lowest AIC is always selected. Akaike proposed the following equation as a useful way to select models:

AIC = nLog (SSE) + 2k (8)

Where k = Numbers of parameters that are fitted in the model
Log = Natural logarithm
N = Number of observations in the series
SSE = Sum of the squared error

4. Results and Discussions

4.1 NSE Time Plot

The plot of NSE series gives a good picture of its main characteristics. The typical behaviour of the NSE series is that it fluctuates over time in which during some period of the year the amount are low and in other periods are high. This characteristic indicates that the daily NSE vary throughout the period and the variability is not constant.

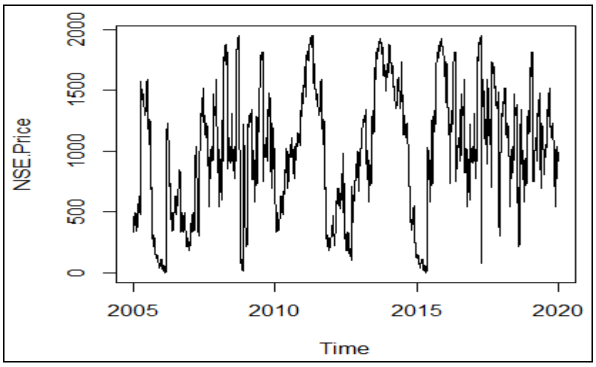

Fig 1: NSE Price

4.1.1 ACF and Pacf Plots

The ACF plot dies down slowly. The autocorrelation function provides a measure of temporal correlation between data points with different time lags. For a purely random event, all autocorrelation coefficients *r* are zero, apart from *r* (0) which is equal to 1. The ACF of a stationary ARMA processes falls exponentially with a rising time-lag. However, this NSE time series variables are strongly correlated. A time series with this attribute is referred to as series with long range dependence and is generated by stochastic processes called processes with a long memory. It is usually characterised by a slowly decaying ACF values with an increasing time lag, which is hyperbolic rather than exponential decay.

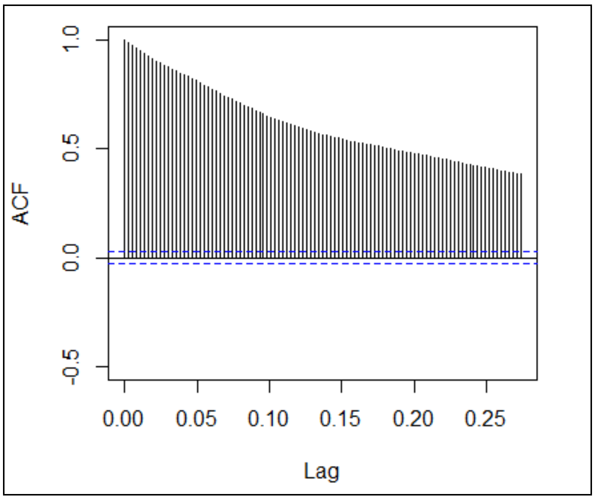

Fig 2: ACF NSE

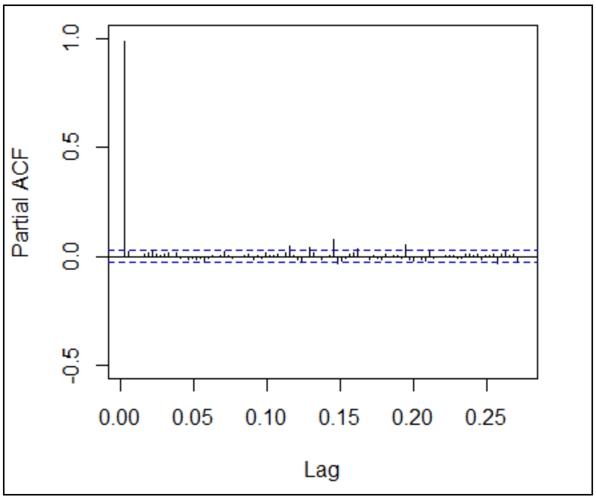

Fig 3: PACF NSE

4.2 Stationarity Tests

Table 1: ADF test with constant only

Variable	Level	5% crit.value	Stationary status
NSE	-6.204491	-2.861873	I (0) at level

Note that indicate significant at 5% level.**

The outcome of the unit root test for ADF-test. The result of unit root test shows that the variable is stationary at level of 5% level of significant. That means the variables are integrated of order zero i.e., 1(0).

Table 2: KPSS – test with constant only

Variable	Level	5% crit.value	Stationary status
NSE	0.881749	0.463000	Is not stationary
	0.011158	0.463000	1(1) is stationary

The KPSS test is 0.881749 is greater than 0.463000 with show that is not significant at level is not stationary while after taking first difference KPSS is 0.011158 is less than 0.463000 that means is stationary 1(1).

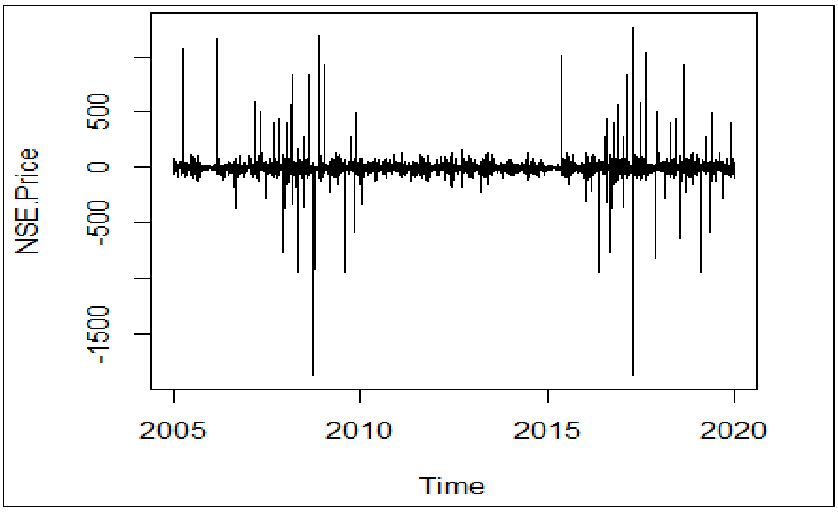

Fig 4: Difference NSE

Table 3: Results for estimated model parameters: ARIMA (p, d, q) for daily NSE

Models	Coefficient	Estimate Error	Log-like hood	AIC
Arima (1, 0, 0)	ar1	0.9868	-31868.02	63742.03
Arima (1, 0, 1)	ar1 ma1	0.9875 -0.0243	-31866.43	63740.86
Arima (2, 0, 1)	ar1 ar2 ma1	0.5260 0.4553 0.4413	-31866.67	63743.35

Based on the table3 above ARIMA (1, 0, 1) is the best one among five model because it has least AIC (63740.86) value we can used it to fit the remaining GARCH-Family Model.

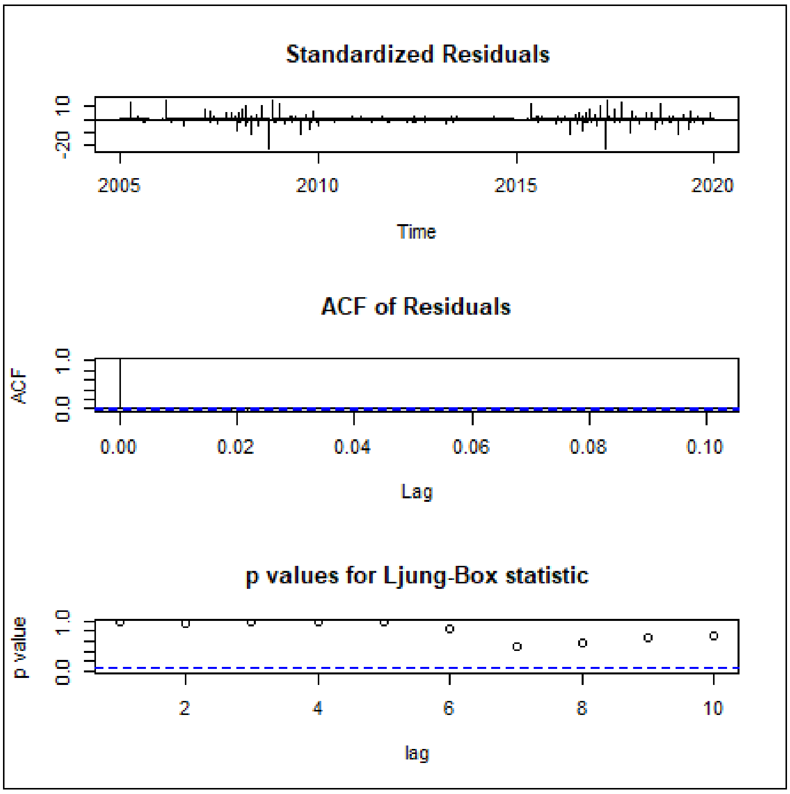

Fig 5: Time Series Diagram of Arima (1, 0, 1)

Table 4: Forecasting performance of ARIMA (1,0, 1) For NSE

ME	RMSE	MAE	MAPE	MAPE	ACF
0.07916218	81.36584	30.96563	-4.561633	7.5309	1.047342

Table 4 confirms the adequacy of the models. This shows that the ARIMA models can adequately capture the behaviour of the data generating process for the daily series(s) Since ACF value 0.01653176 is less than 0.05 level of significant

Table 5: Heteroskedasticity Test: ARCH

F-statistic	5.786775	Prob. F (4,5468)	0.0001
Obs*R-squared	23.07060	Prob. Chi-Square (4)	0.0001

Both the F-statistics and R-square are very significant, suggesting the presence of ARCH effects in the NSE series. Thus, it is necessary to proceed with the estimation of the FIGARCH process

Table 6: The results of the estimated ARIMA-GARCH model

Models	Coefficient	estimate	S. E	t-value	Pr ($\|t\|$)	Log-like hood	AIC
Arima (1,0,1)	ar1 ma1	0.9875 -0.0243	0.0021 0.0136	84.3564		-31866.43	63740.86
Arima-Garch	μ ω α β	3.58e+01 1.436e+04 1.000e-08 9.832e-01	6.245e+01 4.127e+03 2.543e+02 6.126e-03	0.573 3.364 3.583 160.4	0.566551 0.000769* 0.003421* < 2e-16 *	-1459.547	16.35248

The results above indicate that the estimated parameters of mu is not significant that is the mean equations and (alpha and beta) in the equation for the conditional variance are significant. Furthermore, the sum of the alpha and beta parameters is close to unity (α + β = 0.9832001), indicating that the persistence of the NSE return is high. Although the returns volatility appears to have what seems to be long memory, it is still mean reverting: the sum of α and β is significantly less than one, implying that although it takes some time, the volatility process does return to its mean. Furthermore, the results indicate that the coefficient gamma is significant, implying that the sign of the innovation has significant influence on the volatility of returns.

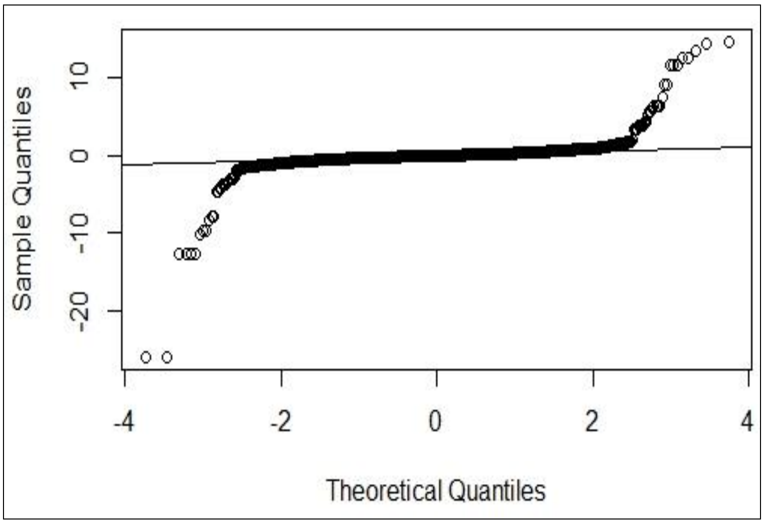

Fig 6: ARIMA-Gargh residuals

Table 7: The results of the estimated ARIMA-FIGARCH model

Models	**Coefficient**	**estimate**	**S. E**	**t-value**	**Pr ($\|t\|$)**	**Log-like hood**	**AIC**
Arima (1,0,1)	**ar1** **ma1**	0.9875 -0.0243	0.0021 0.0136	84.3564		-31866.43	63740.86
Arima-FiGARCH	μ ω α_1 β_1 δ **Shape**	3.56743 719.012 0.00000 0.51159 0.91241 2.994836	24.2515 5686.48 1.23551 0.15409 0.31478 9.53e-02	0.14710 0.12645 0.00011 3.32014 2.89853 31.38160	0.883052 0.899380 1.000000 0.000900*** 0.003749*** 0.000000***	-40123.4	14.488

The results above indicate that the estimated parameters of mu is not significant that is the mean equations and (alpha and beta) in the equation for the conditional variance are significant. Furthermore, the sum of the alpha and beta parameters is not close to unity (α + β = 0.51159), indicating that the persistence of the NSE return is low. Although the returns volatility appears to have what seems to be long memory, it is still mean reverting: the sum of α and β is significantly less than one, implying that although it takes some time, the volatility process returns to its mean. Furthermore, the results indicate that the coefficient gamma is not significant, implying that the sign of the innovation has not significant influence on the volatility of returns and also delta is significant it shows the present of long -memory

Table 8: Weighted Ljung-Box Test on Standardized Residuals of ARIMA-FIGARCH

Lags	Statistic	p-value
Lag 1	0.02378	0.8775
Lag 2	0.07426	0.9381
Lag 4	0.20162	0.9925

H0: No serial correlation

However, the autocorrelation function of the squares of the residuals for the daily series(s) shows autocorrelation. The Ljung Box Q-test given in Table 11 also confirms the results. This is an indication of ARCH effect in the daily series. Formally, McLeod Li test and a test based on the Lagrange multiplier (LM) principle were applied to the square residuals of the fitted models. The results of the McLeod-Li test for ARCH effect is given in table below clear evidence to reject the null hypothesis of no ARCH effect was established for the daily series(s).

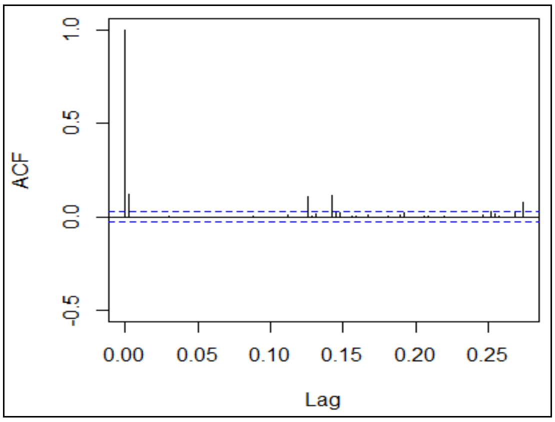

Fig 7: ACF Squared residuals

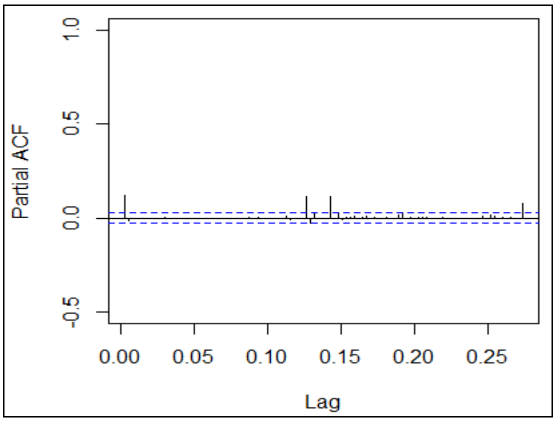

Fig 8: PACF Squared residuals

Table 9: Weighted Arch LM Tests for Arima-Figarch

Lags	Statistic	p-value
ARCH Lag 3	0.006552	0.9355
ARCH Lag 5	0.014414	0.9992
ARCH lag 7	0.021277	1.0000

According to the results, all models were specified correctly. Also, McLeod-Li test for ARCH effect was applied to the residuals of the fitted ARIMA-FIGARCH models visual inspection clearly shows that there is no any heteroskedastic effect left in both series, therefore, the models fit the daily NSE data sets well. The ARCH LM test results in Table 3 also reject the hypothesis of no ARCH effect. Hence indicates that FIGARCH modelling is necessary.

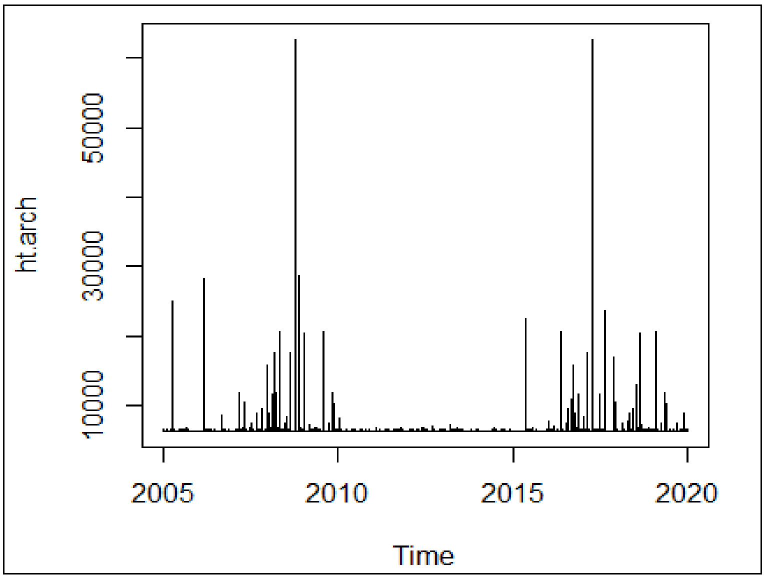

Fig 8: Conditional variances

Table 10: Information criteria values for ARIMA (1,0,1)-GARCH (1,1), ARIMA (1,0,1)- EGARCH (1,1) and ARIMA (1,0,1)-FIGARCH (1,1) models

Modes	AIC	BIC	SIC	HQIC
Arima-Garch	16.35108	16.42370	16.35151	16.38136
Arima-Figarch	14.488	14.577	14.489	14.524

Based on the results obtaining in table (18) using information's criteria with shows that AIC, BIC and HQIC of ARIMA-FIGARCH model (14.488, 14.577 and 14.524) are less than for ARIMA-GARCH and ARIMA-EGARCH model. With shows that ARIMA-FIGARCH are best model are three hybrid model for forecasting National Stock Exchange of Nigeria.

Conclusions

The result of unit root test shows that NSE are stationary at level for ADF test at 5% level of significant. While is stationary after first difference for KPSS test That means the variables are integrated of order one and zero i.e., 1(1) and 1(0). Result shows that in table (11) ARIMA-FIGARCH Model outperform ARIMA-GARCH. So finally, we concluded that ARIMA-FI GARCH model outperform ARIMA-GARCH of Nigeria stock market volatility by used of information criteria.

Reference

1. Phillips PCB, Durlauf SN. Multiple Time Series Regressions with Integrated Processes. Review of Economic Studies LIII 1986, 473-495.
2. Phillips PCB, Loretan M. Estimating Long-Run Economic Equilibria. Review of Economic Studies 1991;58:407-436.
3. Phillips PCB, Park JY. Asymptotic Equivalence of Ordinary Least Squares and Generalized Least Squares in Regressions with Integrated Regressors. Journal of the American Statistical Association1988;83:111-115.
4. Sargan JD. Wages and Prices in the United Kingdom: A Study in Econometric Methodology. In Econometric Analysis for National Economic Planning (R.E. Hart, G. Mills, J.K. Whittaker, eds.), 1964, 25-54.
5. Butterworths, London. Stock JH. Asymptotic Properties of Least-Squares Estimators of Cointegration Vectors. Econometrica 1987;55:1035-1056.
6. Wickens MR, Breusch TS. Dynamic Specification, the Long-Run and the Estimation of Transformed Regression Models. The Economic Journal 1988.
7. Giraitis L, Koul HL, Surgailis D. Large Sample Inference for Long Memory Processes, Imperial College Press 2012.