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A study of conditional volatility of hybrid Arima, and Figarch model

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Abstract
This study is to discuss the techniques that will be employed by the researcher's when conducting the study on modelling and predicting financial Time Series data. The hybridization between ARIMA Model and Fractionally Integrated Generalized Autoregressive Conditional Heteroscedastic (FIGARCH) processes. With will be used to develop the most appropriate model for forecasting financial Time Series data. However, same as the main weakness of the ARIMA cannot handle volatility clustering with the persister of long -memory. Unfortunately, FIGARCH can handle. Many studies have suggested that structural breaks should be combined into the long memory models to properly fit financial return volatility (Baillie and Morana, 2009; Belkhouja and Boutahary, 2011).

Keywords: FIGARCH, ARIMA hybridizations and conditional volatility

1. Introduction

Forecasting and modeling of Time Series data are not new terms to stakeholders, both in the economic and business fields respectively. Time Series data for financial market constantly exhibit (volatility) variability and uncertainty in market fluctuations. Volatility when envisaged in defines the measure of fluctuations of currency. Volatility in exchange rate has raised great concern to all economic and business analysts as it's after international trade macroeconomic variables (export and import) and the economic growth of a nations. Volatility in Exchange rate result in international transaction that may leads to the downtrends in international trade and economic welfare (Wong and Lee, 2016). Thus, forecasting and modeling of exchange rate play vital roles in a nation's economy. Nigeria as a nation is not left out in this exchange rate volatility. As (exchange rate) is one of the significant indicators that determines nation's economic growth. To expound this point, for instance; the appreciation or depreciation of naira is of interest to financial analysts, policy makers, investors, researchers, to mention but few; and even to the nations Hence, the researcher is interest in proposed an efficient modelling technique that will be appropriate in describing the volatility of time series data in a more lucid manner.

It is vivid that to the fact that volatility, plays a major role in economic and financial applications such as; monetary policy making, investment, and security evaluation. In 1976 Box-Jenkins developed and designed an ARIMA model as a forecasting tool for financial economic variables which was named after the creator as the Box-Jenkins Methodology. The Box- Jenkins Methodology 1976 methodology attempts to find an autoregressive integrated moving average (ARIMA) (p, d, q) model that satisfies the stochastic procedure where the derived sample from ARIMA model can be estimated using the Box-Jenkins approach. The Box-Jenkins method comprises an iterative three- stage modeling approach that includes model identification, parameter estimation, and model checking. Since a necessary condition for an ARIMA model is stationary, when the observed time series present trend and non-seasonal behavior, then differencing is done to the data series to eliminate the trend. Box Jenkins methodology is one of the well-known methods in time series modelling, and forecasting which is extensively applied in numerous areas of time series analysis. ARIMA model is frequently used by some analysts due to its ability in handling non-stationary data. More so, it is simple to implement, and it generally offer accurate prediction over a short period of time. Even though ARIMA is a powerful method in forecasting several types of times series data, it cannot handle the volatility that is present in data series.

Thus, various families of generalized autoregressive conditional heteroscedasticity (GARCH) models were proposed by researchers in order to cover loopholes of ARIMA models. GARCH model is divided into two categories symmetric, and asymmetric. The difference between asymmetric, and symmetric model is related to their effect of sign on volatilities. In the symmetric model, the conditional variance only depends on the magnitude and not in the sign of the underlying asset, whereas in the asymmetric model, the sign whether positive or negative of the underlying asset having the same magnitude of shocks has different effect on volatility. (Ahmed and Sulliman, 2011). Symmetric model performs better in capturing leptokurtosis and volatility clustering of financial returns, but since they have a symmetric distribution, they fail in modelling leverage effect (Narsoo, 2015) [8]. This limitation of symmetric model has led to the development of several asymmetric models that were able to capture the asymmetric relationship including Exponential GARCH (EGARCH), Threshold GARCH (TGARCH), and Power GARCH (PGARCH) models.
The primary purpose of introducing FIGARCH model was to develop a more flexible class of processes for the conditional variance, that are capable of explaining and representing the observed temporal dependencies in financial market volatility. In particular, the FIGARCH model allows only a slow hyperbolic rate of decay for the lagged squared or absolute innovations in the conditional variance function. This model can accommodate the time dependence of the variance and a leptokurtic unconditional distribution for the returns with a long memory behavior for the conditional variances. The primary purpose of introducing FIGARCH model was to develop a more flexible class of processes for the conditional variance, that are capable of explaining and representing the observed temporal dependencies in financial market volatility. In particular, the FIGARCH model allows only a slow hyperbolic rate of decay for the lagged squared or absolute innovations in the conditional variance function. This model can accommodate the time dependence of the variance and a leptokurtic unconditional distribution for the returns with a long memory behavior for the conditional variances.

1.2 Statement of the problem

In recent times, stake holders, policy makers, financial economist, academic researchers to mention but few – have picked interest in movement, and fluctuations in financial Time Series. In an attempt to bring the situation under control, several types of case studies, and approaches have been applied to the data in order to handle some characteristics that exist in the series Generalized Autoregressive Conditional Heteroscedasticity (GARCH) family model a more famous and frequently useful method, particularly known in handling volatility of data series.
For instant to model the long-term persistence, Engle and Bollerslev (1986) developed the Integrated GARCH (IGARCH) model as an extension to the original GARCH model. It is argued that IGARCH models have a property called "persistent variance" since any shocks to the conditional variance, either happened to day or in the past, will persist indefinitely in to the future. However, Nelson (1990) [10-11] showed that the IGARCH process without drift would definitely converge to zero with probability one, infinite steps. Hence, IGARCH models are generally considered as short memory models by researchers (Davidson 2004; Granger and Ding 1996a). (1996) generalized the IGARCH model to a new class named Fractionally Integrated GARCH (FIGARCH) models, with the purpose of explicitly describing the long memory behavior of the conditional variances of financial time series.
The primary purpose of introducing FIGARCH model was to develop a more flexible class of processes for the conditional variance, that are capable of explaining and representing the observed temporal dependencies in financial market volatility. In particular, the FIGARCH model allows only a slow hyperbolic rate of decay for the lagged squared or absolute innovations in the conditional variance function. This model can accommodate the time dependence of the variance and a leptokurtic unconditional distribution for the returns with a long memory behavior for the conditional variances the. FIGARCH model has its short-run dynamics described by the conventional GARCH parameters (αis and βjs). The combination of ARIMA model with nonlinear FIGARCH model is essential so that the conditional mean, and conditional heteroscedasticity of the series can be captured in order to have an effective way to overcome the weaknesses of each component and be able to improve the accuracy of forecasting. To investigate the ability of the ARIMA-FIGARCH model with Nigerian stock exchange to predict persistence of long memory volatilities. The performance of these model is then compared with ARIMA-GARCH, ARIMA-EGARCH and ARIMA-GJR-GARCH as a benchmark.

1.3 Aim and Objectives

The main aim of this research is to develop a hybrid model for modelling the mean and long memory in volatility of mixed data sampling will achieved through the following objectives
To develop a hybrid of ARIMA-FIGARCH

2. Literature Review

Financial Time Series well-known to show certain features which referred to as stylized facts. The term stylized facts were introduced by an economist, Nicholas Kaldor in (1961), in his work about economic development theory. (Sewell, 2011), defined stylized facts as a" term in economics used to refer to the empirical results that are so steady across market and recognized as truth". In financial time series, there are two existing stylized structures which are leverage effect, and volatility clustering. Stylized fact attributed asymmetry to that volatility is higher after negative shocks occurred. This characteristic is referred as leverage effect, (Black, 1976). He recognized that volatility tends to increase in response to bad news and decrease in response to good news as stock returns are negatively correlated to variations in returns volatility clustering has been shown to be existing in a wide variety of financial assets comprising, exchange rates and market indices securities, interest rate (Bollerslev, 1986). As stated by (Mandelbrot, 1963) [6], volatility clustering's can be defined as large variations that tend to be monitored by" large fluctuations, of either sign, or small variations that tend to be followed by small fluctuations. In other word, when volatility is high, it will possibly be remaining for certain periods of time, and it may be short for other times. In financial market, fluctuations of shock stock exchange return either positive or negative would determine volatility.

2.1 Time series model of Nigerian stock exchange

Kuznets (1971) defines a country's economic growth as a long – term rise in the capacity of supply leading to increase in the production of goods for the population accompanied by advancing technology and the institutional and ideological adjustment that it demands. It therefore encompasses growth, structural and institutional changes and the essential elements that make up life such as education, health, nutrition and a better environment i.e., human and development indices. Ekundayo (2002) argues that a nation requires a lot of foreign investment to attain sustainable economic growth and development, the capital market provides means through which that is made possible. Several policies and programs have been consciously created to promote the growth of the Nigerian economy overtime. Some of these policies include the enterprises promotion decree, the privatization of government enterprises (2000), which were quoted on the Nigerian stock exchange. There were also the bank recapitalization directives (2004), by the CBN, in which banks were directed to recapitalize to a minimum of twenty-five billion naira. For this many banks accessed the capital market (through the primary public offers) for their financing needs, the government also introduced the pension reform Act of 2004. This act provides that part of the pension fund should be invested in the capital market by pension fund administrators. However, the impact of the capital market on the growth and development of the economy has not been significantly positively felt (Babalola, 2007). This may be due to low market capitalization, delay in delivery of share certificate problem of manual call, slow growth of securities market, double taxation, problem of macro-economic instability among others. Also, most Nigerians are not aware of the benefit derivable from the market operations. Furthermore, there is a problem of reluctance of Nigerian businessmen to go to the public for fear of losing control of their business. More so, the Nigerian stock exchange market, over the years has undergone reforms due to the declining effect of global financial crisis. While capital market has the potentials of stimulating economic growth and development through effective resource allocation, the expected high economic growth that comes with capital market development has not been experienced in Nigeria (Popoola, 2014)

Odhiambo (2010) examined the causal relationship between stock market development and economic growth in South Africa. The study used annual time series data for the period 1971 – 2007 and the autoregressive distributed lag (ARDL) – Bounds testing method was employed. Three constituents of stock market performance, namely stock market capitalization, stock market traded value and stock market turnover were used against real GDP per capita, a constituent for economic activity. Empirical results showed that the causal relationship between stock market performance and economy activity is susceptible to the components used for measuring stock market development. When market capitalization was used as a measure of stock market performance, economic growth was seen to Granger – cause stock market development. Moreover, when stock market turnover was used, stock market development seemed to Granger – cause economic growth. The results were valid both in the short and long run.

Enisan & Olufisayo (2009) investigated the long run and causal relationship between stock market performance and economic growth from seven countries in Sub – Saharan Africa using the autoregressive distributed lag (ARDL) test. Stock market capitalization was used as proxy to stock market development while GDP was used as an economic activity proxy. The Co – integration test result revealed that stock market development and economic growth were cointegrated in Egypt and South Africa. Results showed that stock market development has a positive and significant long run relationship with economic activity. Causality tests using the Granger tests on vector error correction model (VECM) showed that stock market performance Granger cause economic activity in Egypt and South Africa. Granger causality in the context of VAR showed evidence of bi – directional causality between stock market performance and economic growth for Cote D' Ivoire, Kenya, Morocco, and Zimbabwe. Based on results, the study argues that stock markets can help promote the growth of Africa.

2.2 Review of literature on FIGARCH model

Davidson (2004) had given some insight on the memory properties of the FIGARCH. According to Davidson (2004), the degree of persistence of the FIGARCH model operates in the opposite direction as that of ARFIMA, as the d parameter gets closer to zero, the memory of the process increases. This is due to the inverse relationship between the integration coefficient and the conditional variance. The memory parameter acts directly on the squared errors, not on the t, this particular behavior may also influence the stationarity properties of the process (Davidson, 2004). These observations are strictly related to the impulse response analysis on the effects of a shock on a system driven by a FIGARCH process. In such a system, a shock v_t at time t, should be interpreted as the difference between the squared mean-residuals ε_t^2 at time t and the one-step-ahead forecast to the variance h_t, made at time t−1. This shock is exactly the innovation in the ARMA representation of the FIGARCH process and also it shows had shown that a FIGARCH model possesses more memory than a GARCH or IGARCH model. Review of Some Applications There is a large collection of research papers where FIGARCH models are found to be performing better than many of the other conditional Heteroscedastic models.

Jin and Frechette (2004) estimate FIGARCH volatility models for 14 agricultural futures series and find that FIGARCH fits the data significantly better than a traditional GARCH volatility model. While these studies have provided valuable information on the long memory properties of commodity futures price volatilities, much more work remains to be done. While Jin and Frechette (2004) argue in favor of the FIGARCH model over the GARCH model for commodity futures volatilities, they did not undertake a formal statistical test comparing the two models. Here we undertake a robust Wald test, which formally compares the fit of the GARCH and FIGARCH models. Second, in addition to the standard quasi-maximum likelihood estimator (QMLE), we also apply the semi parametric Local Whittle estimator of the long memory parameter. This provides additional information on the robustness of long-memory inferences concerning daily commodity price volatilities. Third, in addition to daily returns we study high frequency returns on futures contracts using intraday tick data. This study is the first to systematically examine volatility using high frequency commodity futures data.1 we find that estimated models at different sampling frequencies are consistent with the theory that commodity futures returns are "self-similar" processes, and hence have long memory parameters that are invariant to the sampling frequency; see Beran (1994). The "self-similarity" of the estimates of the long memory volatility parameter across relatively short spans of high frequency data strongly suggests that the long memory property is an intrinsic feature of the system rather than being due to exogenous shocks or regime shifts.

3. Methodology

3.1 Introduction

This chapter discusses the techniques that will be employed by the researcher when conducting the study on modelling and predicting financial Time Series data. The hybridization between Autoregressive Integrated Moving Average (ARIMA) and Fractionally Integrated Generalized Autoregressive Conditional Heteroscedastic (FIGARCH) process. Will be used to develop the most appropriate model for forecasting financial Time Series data.

3.2 Existing Methodology

3.2.1 Box-Jenkins Method

A model usually applied in time series analysis forecasting introduced by Box and Jenkins (1976). This model is typically applied to time series analysis, predicting. According to (Bowerman *et al*., 2005), there are four main steps to construct a Box-Jenkins procedure which are identification, parameter estimation, diagnostic checking and forecasting. This technique is useful to both seasonal and non-seasonal data. In this model, data that are select are non-seasonal data.

3.2.2 Stationary time series model

Autoregressive Moving Average (ARMA) model is the combination of autoregressive, AR (*p*) model and moving average, MA (*q*) model. It can be represented by ARMA (*p*, *q*) where *p* is the order of autoregressive part while *q* is the order of moving average part. ARMA (*p*, *q*) model is written as

$$yt = \emptyset 1yt{-}1 + \emptyset 2yt{-}2 + \cdots + \emptyset pyt{-}p - \theta 1\ t{-}1 - \cdots - \theta q\ t{-}q + \varepsilon t \quad (1)$$

Where ϕi are the parameters of the autoregressive part of the model, θ *i* are the parameters of the moving average part, and εt: $N(0,\delta)$

3.2.3 FIGARCH Process

From (4) we see that a GARCH (p, q) process may also be expressed as an ARMA (m, p) process in ε_t^2, by writing

$$(1-\alpha(L)-\beta(L))\varepsilon_t^2 = \alpha_0 + (1-\beta(L))V_t$$

Where m = max{p,q} and $v_t = \varepsilon_t^2 - h_t$. The { v_t } process can be interpreted as the "innovations" for the conditional variance, as it is a zero-mean martingale. Therefore, an integrated GARCH (p, q) process can be written as

$$(1-\alpha(L)-\beta(L))(1-L)\varepsilon_t^2 = \alpha_0 + (1-\beta(L))V_t \quad (2)$$

The fractionally integrated GARCH or FIGARCH class of models is obtained by replacing the first difference operator (1 − L) in (5) with the fractional differencing operator $(1-L)^d$, where d is a fraction 0 < d < 1. Thus, the FIGARCH class of models can be obtained by considering.

$$(1-\alpha(L)-\beta(L))(1-L)^d \varepsilon_t^2 = \alpha_0 + (1-\beta(L))V_t \quad (3)$$

Such an approach can develop a more flexible class of processes for the conditional variance that are capable of explaining and representing the observed temporal dependencies of the financial market volatility in a much better way than other types of GARCH models (Davidson, 2004).

It may be noted that the fractional differencing operator $(1-L)^d$ can be written in terms of

hyper geometric function. $(1-L)^d$ =F(-d,1,1L)= $\sum_{K=0}^{\infty} \Gamma(k-d)\Gamma(k+1)^{-1}\Gamma(-d)^{-1}L^k$

The ARFIMA (p, d, q) class of models for the discrete time real-valued process { y_t } introduced by Granger and Joyeux (1980); Granger (1980, 1981) and Hosking (1981) is defined by

$$\alpha(L)(1-L)^d y_t = b(L)z_t \quad (4)$$

Where α (L) and b(L) are polynomials in the lag operator of orders p and q respectively, and { z_t } is a mean-zero serially uncorrelated process. For the ARFIMA models, the fractional parameter d liesbetween−1/2 and1/2, (Hosking, 1981). The ARFIMA model is nothing but the fractionally integrated ARMA for the mean process. Analogous to the ARFIMA (p, d, q) process defined in (7) for the mean, the FIGARCH (p, d, q) process for { ε_t^2 } can be defined as

$$\phi(L)(1-L)^d \varepsilon_t^2 = \alpha_0 + (1-\beta(L))V_t \tag{5}$$

Where 0 < d < 1, and all the roots of ϕ(L) and [1−β(L)] lie outside the unit circle. In the case of ARFIMA model, the long memory operator is applied to unconditional mean μ of y_t which is constant. But this is not true in the case of FIGARCH model, where it is not applied to α_0, but on squared errors.

4. Proposed modifications hybrid model

Develop model is a predicting procedure that combines two or more individual models. In this study, hybridization of ARIMA and FIGARCH model will be done in two phases procedure. In the first phase, the best of ARIMA model is applying to model the linear data of time series. In the second phase, GARCH family approach is applying to model the nonlinear designs of the residuals sequence from the fitted ARIMA model. In this procedure, the error term of the ARIMA model will follow GARCH family processes of order *p* and *q*. The primary steps in construction hybrid models is the same with ARIMA methodology which it contains of model identification, parameter estimation, diagnostic checking and forecasting.

4.1 General equation of hybrid models

The general equation of hybrid ARIMA (*p*, *d*, *q*) - FIGARCH (*p*, d, q), model can be written as followed where it contains of two equations. Mean equation comes from ARIMA model while variance equation comes from FIGARCH model. We combine the ARIMA and FIGARCH- which are in equations below together, the hybrid ARIMA (p, d, q)- FIGARCH (p, d, q) model can be specified as

$$\varphi_p(L)(1-L)^d(y_t-\mu) = \theta_q(L)\varepsilon_t$$

Were

$\varphi_p(L) = 1 - \sum_{i=1}^{p} \varphi_i L^i$ and $\theta_q(L) = 1 - \sum_{j=1}^{q} \theta_j L^j$

are polynomials in terms of L of degree p and q. y_t is the time series, and ε_t is the random error with μ is the mean of the model. L is the difference operator defined as

$$\Delta y_t = y_t - y_{t-1} = (1-L)y_t.$$

Also, d- is the order of the difference operator.

$$\varphi_1, \varphi_2, \ldots . \varphi_p \;\; and \;\; \theta_1, \theta_2, \ldots \ldots \theta_q$$

are the parameter of autoregressive and moving average terms with order p and q respectively. FIGARCH (p, d, q) model for the conditional heteroskedasticity has the following form: The conditional variance dynamics of the component h_t is a FIGARCH process is given by

$$(1-\beta(L))h_t = \alpha_0 + (1-\beta(L) - \phi(L)(1-L)^d \varepsilon_t^2 \tag{6}$$

$$h_t = \frac{\alpha_0}{(1-\beta(L))} + \frac{(1-\beta(L)-\phi(L)(1-L)^d)}{(1-\beta(L))}\varepsilon_{t,i}^2 \tag{7}$$

If $\beta(L)=\beta(1)$ at lag, where L=1 then

$$h_t = \frac{\alpha_0(1-\beta(L))}{(1-\beta(L))} - \frac{\phi(L)(1-L)^d}{(1-\beta(L))}\varepsilon_{t,i}^2$$

$$h_t = \alpha_0(1-\beta(1))^{-1} + (1-(1-\beta(L))^{-1}\phi(L)(1-L)^d \varepsilon_{t,i}^2 \tag{8}$$

$$h_t = \alpha_0(1-\beta(1))^{-1} + \lambda(L)\varepsilon_{t,i}^2 \tag{9}$$

To hybridized the ARIMA and FIGARCH the $y_t = \mu_t + z_t$ Where

μ_t *is conditional mean of* y_t and z_t *is the shock at time t.*

Then the returns of the stock can be derived as follow to find the value of ε_t

$$z_t = h_t \varepsilon_t$$
$$y_t - \mu_t = z_t$$

Substitute

$$y_t - \mu_t = z_t$$

$$y_t - \mu_t = h_t \varepsilon_t$$

$$\varepsilon_t = \frac{(y_t - \mu_t)}{h_t} \quad \text{where} \quad h_t = \alpha_0 (1 - \beta(1))^{-1} + \lambda(L)\varepsilon_{t,i}^2 \tag{10}$$

The ARIMA (p, d, q) is defined as

$$\varphi_p(L)(1-L)^d(y_t - \mu) = \theta_q(L)\varepsilon_t \tag{11}$$

Then the error term is defined as

$$\varepsilon_t = \frac{(y_t - \mu_t)}{h_t}$$

The ARIMA (p,d, q)-FIGARCH(p,d,q) is given below:

$$\varphi_p(L)(1-L)^d(y_t - \mu_t) = \theta_q(L)(y_t - \mu_t)(h_t)^{-1} \tag{12}$$

$$\varphi_p(L)(1-L)^d = \theta_q(L)(\alpha_0(1-\beta(1))^{-1} + \gamma(L)\varepsilon_t^2 \tag{13}$$

Where $\lambda(L) = \lambda L + \lambda_2 L$ +.... of course, for the FIGARCH (p, d, q), must be non-negative,

i.e., $\lambda_k \geq 0$ for k = 1, 2

References

1. Jarque CM, Bera AK. A Test for Normality of Observations and Regression Residuals. International Statistical Review 1987;55(2):163-172.
2. Kawakatsu H. Matrix exponential GARCH. J Economy 2006;134:95-128.
3. Khashei M, Bijari M, Raissi Ardali GA. Improvement of Auto-Regressive Integrated Moving Average models using Fuzzy logic and Artificial Neural Networks (ANNs). Neuro computing 2009;72:956-967.
4. Kuzin V, Marcellino M, Schumacher C. MIDAS vs. mixed-frequency VAR: Now casting GDP in the euro area. International Journal of Forecasting, Elsevier 2011;27(2):529-542.
5. Liu H, Shi J, Qu X. Empirical investigation on using wind speed volatility to estimate the operation probability and power output of wind turbines. Energy Converts. Manage 2013;67:8-17.
6. Mandelbrot B. The Variation of Certain Speculative Prices. The Journal of Business 1963;36(4):394-419.
7. Miswan NH, Ngatiman NA, Hamzah K, Zamzamin ZZ. Comparative performance of ARIMA and GARCH models in modelling and forecasting volatility of Malaysia market properties and shares. Applied Mathematical Sciences 2014;8:7001-7012. Doi:10.12988/ams.2014.47548.
8. Narsoo J. Forecasting USD/MUR Exchange Rate Dynamics: An Application of Asymmetric Volatility Models. International Journal of Statistics and Applications 2015;5(5):247-256. Doi: 10.5923/j.statistics.20150505.09
9. Nelson DB. Conditional heteroscedasticity in asset returns: A new approach. Econometric: Journal of the Econometric Society 1991, 347-370.
10. Nelson DB. ARCH models as diffusion approximations. J Economy 1990;45:7-38.
11. Nelson DB. Conditional heteroscedasticity in asset returns: A new approach. Econometrical 1990;59:347-370.
12. Osborne JW. Improving your data transformations: Applying the Box-Cox transformation. Practical Assessment, Research, and Evaluation 2010;15:1-9.
13. Pahlavani M, Roshan R. The Comparison among ARIMA and hybrid, 2015.
14. Peng WY, Chu CW. A comparison of univariate methods for forecasting container throughput volumes. Mathematical and Computer Modeling 2009;50:1045-1057.