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**Akintunde Oyetunde A**  
 Department of Mathematics,  
 Faculty of Science, Federal  
 University Oye-Ekiti, Ekiti  
 State, Nigeria

**OJO Faith Afolabi**  
 Department of Mathematics,  
 Faculty of Science, Federal  
 University Oye-Ekiti, Ekiti  
 State, Nigeria

**Correspondence**  
**Akintunde Oyetunde A**  
 Department of Mathematics,  
 Faculty of Science, Federal  
 University Oye-Ekiti, Ekiti  
 State, Nigeria

## Comparative statistical assessment of air pollution in Okobaba sawmill Nigeria by the use of frechet distribution

**Akintunde Oyetunde A and OJO Faith Afolabi**

### Abstract

The air pollution data in this research study are stratified into three strata which are: Nitrogen (IV) Oxide, Sulphur (IV) Oxide, and Carbon Monoxide. Chi-square test and the analysis of variance were used to analyze the collected data. The data collected for carbon monoxide, Nitrogen (IV) oxide and Sulphur (IV) oxide fit into Frechet distribution because their chi-square tabulated values are less than their chi-square calculated values, which implied that Frechet distribution can be used for analyses and control of air pollution in Nigeria. It was statistically established that requiring probabilistic distributions to better describe each phenomenon or experiment studied and making available data more fit and reliable. This research study therefore recommends that the use of observations in field studies to develop the empirical source-receptor relationships (for instance, using Frechet distribution and statistical receptor models) should be put into consideration to improve the air pollution in developing countries like Nigeria.

**Keywords:** Frechet distribution, air pollution, chi-square, Carbon monoxide, Sulphur IV oxide, Nitrogen IV oxide

### Introduction

Air pollution is an important factor affecting health and life quality. Air pollution is the contamination of the indoor or outdoor air by a range of gasses and solids that modify its natural characteristics. The atmosphere consists of mixture of gases that completely surround the earth. Air is said to be polluted when chemicals, gases, particulate matter and biological materials that can cause harm or discomfort to humans or other living organisms or cause damage to the environment are released into the atmosphere (Akpanisi, 2003). Air pollution is the most insidious of all forms of pollution because we can partially avoid the direct contact of other forms of pollution; we must breathe the air (Dix, 1981; Speeding, 1997).

Air pollution is often not visible to the naked eye as the sizes of the pollutants are smaller than the human eye can detect. They can become visible in some situations for example in the form of sooty smoke from the open burning of crop residues or other waste, as well as from burning wood, coal, petrol and diesel fuels, for cooking and heating, transport or power production. The fact that you cannot see the air pollution does not mean that it does not exist. Air pollution of the urban centers is one of the world's worst pollution problems. The ambient air quality of an area affects the chemistry of its atmosphere and the general wellness of the environment including humans. Air quality reports in most advanced countries are therefore presented regularly to assist the public in the management of their health. Many industrialized countries have air quality standards and guideline to regulate emission into the environment (USEPA, 1993; FGN, 1988). Health harmful air pollutants include particulate matter (PM<sub>2.5</sub> and PM<sub>10</sub>), Carbon monoxide (CO), Ozone (O<sub>3</sub>), Black Carbon (BC), Sulfur dioxide and Nitrogen Oxides (NO).

In pollution science, statistical methods are applied to quantify and evaluate data derived from observations as well as for mathematical models. The basis of control of air pollutants consists essentially of two different approaches, namely:

1. Use observations in field studies to develop the empirical source-receptor relationships (for instance, using statistical receptor models), and
  - Application of various mathematical chemical transport models that relate the concentration of the secondary pollutants formed to the initial primary pollutant concentrations.
- Any statistical analysis depends greatly on the statistical model used to represent the phenomena under study.

Hence, the larger the class of statistical models available to the statistician the easier it is to choose a model. A quick survey of the models in common use reveals the abundance of statistical models in the literature.

However, data of many important and practical problems do not follow any of the probability models available. In such cases a non-parametric model may be recommended. While a two parameter distribution may provide reasonably precision in fitting data, it may be still desirable to extend the flexibility of any distribution to allow for better description of data without having to resort to non-parametric models. The Fréchet distribution has applications ranging from accelerated life testing through earthquakes, floods, rainfall, horse racing, queues, sea currents, wind speeds and track race records. Kotz and Nadarajah (2000) [34] give other applications in their book. Kotz and Nadarajah (2000) [34] also introduced the transmuted Fréchet distribution which stems from the following idea. Shaw & Buckley (2009) considered composite maps of the following two forms: sample transmutation maps (STMs),  $y = G - 1[F_X]$ , and rank transmutation maps (RTMs),  $v = G[F-1 w]$ , where F and G are cumulative distribution functions (CDFs). Gilchrist (2000) refers to STMs and RTMs as Q-transformation and P-transformation, respectively.

Shaw & Buckley (2009) focused on the RTM, which uses as a tool for the construction of new families of non-Gaussian distributions. They used it to modulate a given base distribution for the purposes of modifying the moments, in particular the skew and kurtosis. An attraction of the approach is that if the CDF and inverse CDF (or quantile function (QF)) are tractable for the base distribution, there is a good chance for the transmuted distribution to be so.

## 2. Literature review

Applications of the Frechet distribution in various fields given by Harlow (2002) showed that it is an important distribution for modeling the statistical behavior of materials properties for a variety of engineering applications. Nadarajah and Kotz (2008) [34] discussed the sociological models based on Frechet random variables. Mubarak (2011) studied the Frechet progressive type-II censored data with binomial removals.

The Frechet distribution is a special case of the generalized extreme value distribution. This type-II extreme value distribution (Frechet) case is equivalent to taking the reciprocal of values from a standard Weibull distribution and drive the best linear unbiased estimators of location and scale parameters of the Frechet distribution and obtain the best linear invariant estimations also use maximum likelihood estimators (MLE) to estimate the parameters of the Frechet distribution. Further, Abbas and Tang (2012) studied different estimation methods for Frechet distribution with known shape. Abbas and Tang (2013) discussed the parameters for Frechet Distribution based on type-II censored sample. They consider maximum likelihood estimators and least squares estimators of two-parameter Frechet distribution based on type-II censored sample.

The maximum likelihood estimators (MLE) and least squares estimators (LSE) are developed for estimating the unknown parameters. The observed Fisher information matrix and confidence intervals of the parameters based on asymptotic normality are also derived. An extensive simulation study is carried out to compare the performances of different methods. Nasir and Aslam (2015) studied the Frechet distribution under Bayesian paradigm.

Posterior distributions are derived by using Gumbel Type-II and Levy prior. Quadrature numerical integration technique is utilized to solve posterior distribution. Bayes estimators and their risks have been obtained by using four loss functions.

Extreme value models play an important role in statistics. The generalized extreme value (GEV) distribution (Jenkinson, 1955) and its sub-models are widely used in application involving extreme events. These sub-models are the well-known Weibull, Fréchet and Gumbel distributions. The Fréchet distribution can be seen as the inverse Weibull distribution which gives a probability density function (pdf) such

$$f(t/\lambda, \alpha) = \frac{\alpha}{\lambda} \left(\frac{t}{\lambda}\right)^{-(\alpha+1)} e^{-\left(\frac{t}{\lambda}\right)^\alpha}$$

The survival function is given by:

$$s(t/\lambda, \alpha) = 1 - e^{-\left(\frac{t}{\lambda}\right)^\alpha}$$

Although the GEV distribution is the most used generalization of the Fréchet model, other distributions have been proposed in the literature. De Gusmão (2012) [20] proposed a three parameter generalized inverse Weibull distribution in which includes the Fréchet distribution. Krishna *et al.* (2013) proposed the Marshall-Olkin Fréchet distribution. Barreto-Souza *et al.* (2011) [15] discussed some results for beta Fréchet distribution.

A long-term survival novel proposes a mixture model as introduced by Berkson and Gage (1952), hereafter was called the long-term Fréchet distribution (LF) distribution. Some mathematical properties about the long-term Fréchet distribution (LF) distribution were provided such as moments, survival properties and hazard function.

The maximum likelihood estimators (MLEs) of the parameters and its asymptotic properties are discussed likewise. Similar studies were presented by Roman *et al.* (2012) [25] for the geometric exponential distribution and by Louzada and Ramos (2017) for the weighted Lindley distribution. It performed a numerical simulation towards to examine the performance of the maximum likelihood estimators (MLEs). Finally, the proposed methodology is illustrated in a real data set related to the leukemia free-survival times (in years) for the 50 antillogous transplant patients.

A random variable X is said to have transmuted distribution, according to the quadratic rank transmutation map (QRTM), if its cumulative distribution function is given by  $F_X = 1 + \lambda G_X - \lambda G(X)^2$ ,  $\lambda \leq 1$  (1) where G(x) is the cdf of the base distribution

(Shaw and Buckley (2009)). Note that when  $\lambda=0$ , the base distribution will be obtained. Aryal and Tsokos (2009, 2011) studied the transmuted extreme value distributions and provided the mathematical characterization of transmuted Gumbel and transmuted Weibull distributions and their applications to analyze real data set. Aryal (2013) proposed and studied the transmuted log-logistic distribution. He provided mathematical formulation of this distribution and some of its properties. Ashour and Eltehiwy (2013a, 2013b) studied the transmuted exponentiated modified Weibull and transmuted exponentiated Lomax distributions and discussed some properties of these families. Merovic (2013) introduced the transmuted Rayleigh distribution and provided some properties of this distribution.

### 3. Research methodology

#### 3.1 Integral study of three parameter of frechet distribution

The probability density function (pdf) of the three parameter Frechet distribution with random variable  $x$  is as given below:

$$\text{The pdf : } f(x/\alpha, s, m) = \frac{\alpha}{s} \left(\frac{x-m}{s}\right)^{-1-\alpha} \exp\left\{-\left(\frac{x-m}{s}\right)^{-\alpha}\right\} \tag{1}$$

and its cumulative density function (cdf) is given as:

$$\text{cdf : } F(x/\alpha, s, m) = e^{-\left(\frac{x-m}{s}\right)^{-\alpha}} \tag{2}$$

The expected value of the probability distribution function is obtained as:

$$E(X) = \int_{-\infty}^{\infty} xf(x)dx = \int_0^{\infty} x \frac{\alpha}{s} \left(\frac{x-m}{s}\right)^{-1-\alpha} e^{-\left(\frac{x-m}{s}\right)^{-\alpha}} dx$$

By setting  $t = \left(\frac{x-m}{s}\right)^{-\alpha}$ , thus  $x = m + st^{\frac{1}{\alpha}}$  and  $\frac{dx}{dt} = \frac{-st^{\frac{1}{\alpha}-1}}{\alpha} \Rightarrow dx = \frac{-s}{\alpha} \cdot t^{-1-\frac{1}{\alpha}} dt$

Also,  $\left(\frac{x-m}{s}\right)^{-1-\alpha} = \left(\frac{x-m}{s}\right)^{-1} \left(\frac{x-m}{s}\right)^{-\alpha} = \left(\frac{x-m}{s}\right)^{-\alpha \cdot \frac{1}{\alpha}} \left(\frac{x-m}{s}\right)^{-\alpha} = t^{\frac{1}{\alpha}} \cdot t = t^{1+\frac{1}{\alpha}}$

Thus,  $E(X) = \int_0^{\infty} x \frac{\alpha}{s} t^{1+\frac{1}{\alpha}} e^{-t} \cdot \frac{-s}{\alpha} t^{-1-\frac{1}{\alpha}} dt = \int_0^{\infty} xt^{1+\frac{1}{\alpha}} \cdot t^{-1-\frac{1}{\alpha}} e^{-t} dt = \int_0^{\infty} xt^{1+\frac{1}{\alpha}-1-\frac{1}{\alpha}} e^{-t} dt$

$$= \int_0^{\infty} xe^{-t} dt = \int_0^{\infty} \left(m + st^{\frac{1}{\alpha}}\right) \cdot e^{-t} dt = \int_0^{\infty} me^{-t} dt + \int_0^{\infty} st^{\frac{1}{\alpha}} e^{-t} dt$$

$$= m \int_0^{\infty} e^{-t} dt + s \int_0^{\infty} t^{\frac{1}{\alpha}} e^{-t} dt = m \int_0^{\infty} e^{-t} dt + s \int_0^{\infty} t^{\frac{1}{\alpha}+1-1} e^{-t} dt$$

$$= m \int_0^{\infty} t^{1-1} e^{-t} dt + s \int_0^{\infty} t^{\frac{1}{\alpha}+1-1} e^{-t} dt$$

Recall from gamma function:  $\int_0^{\infty} t^{\alpha-1} e^{-t} dt = \Gamma(\alpha)$  and  $\Gamma(\alpha) = (\alpha - 1)!$

Hence,  $E(X) = m\Gamma(1) + S\Gamma\left(1 + \frac{1}{\alpha}\right) = m + S\Gamma\left(1 + \frac{1}{\alpha}\right)$  for  $\alpha > 1$  (3)

The Equation (3) above is the mean of the distribution in Equation (1)

Also,  $E(X^2) = \int_0^{\infty} x^2 \frac{\alpha}{s} \left(\frac{x-m}{s}\right)^{-1-\alpha} e^{-\left(\frac{x-m}{s}\right)^{-\alpha}} dx = \int_0^{\infty} (m + st^{\frac{1}{\alpha}})^2 \frac{\alpha}{s} t^{1+\frac{1}{\alpha}} e^{-t} \cdot \frac{-s}{\alpha} t^{-1-\frac{1}{\alpha}} dt$

$$= \int_0^{\infty} (m + st^{\frac{1}{\alpha}})^2 e^{-t} dt = \int_0^{\infty} (m^2 + 2mst^{\frac{1}{\alpha}} + s^2 t^{\frac{2}{\alpha}}) e^{-t} dt$$

$$= \int_0^{\infty} m^2 e^{-t} dt + 2ms \int_0^{\infty} t^{\frac{1}{\alpha}} e^{-t} dt + s^2 \int_0^{\infty} t^{\frac{2}{\alpha}} e^{-t} dt$$

$$= m^2 + 2ms\Gamma\left(1 + \frac{1}{\alpha}\right) + s^2\Gamma\left(1 + \frac{2}{\alpha}\right) \tag{4}$$

and the variance is obtained as:

$$\begin{aligned} \text{Var}(x) &= E(X^2) - [E(X)]^2 \\ &= m^2 + 2ms\Gamma\left(1 - \frac{1}{\alpha}\right) + s^2\Gamma\left(1 - \frac{2}{\alpha}\right) - \left[m + s\Gamma\left(1 - \frac{1}{\alpha}\right)\right]^2 \\ &= m^2 + 2ms\Gamma\left(1 - \frac{1}{\alpha}\right) + s^2\Gamma\left(1 - \frac{2}{\alpha}\right) - \left[m^2 + 2ms\Gamma\left(1 - \frac{1}{\alpha}\right) + s^2\left\{\Gamma\left(1 - \frac{1}{\alpha}\right)\right\}^2\right] \\ &= s^2\Gamma\left(1 - \frac{2}{\alpha}\right) - s^2\Gamma\left[1 - \frac{1}{\alpha}\right]^2 = s^2\left[\Gamma\left(1 - \frac{2}{\alpha}\right) - \left\{\Gamma\left(1 - \frac{1}{\alpha}\right)\right\}^2\right] \text{ for } \alpha > 2 \end{aligned} \tag{5}$$

The Equation (5) above is the variance of the distribution in Equation (1)

### 3.2 Integral study of frechet distribution with two parameters

By setting m=0 in the three-parameter Frechet distribution to become two parameter Frechet distribution, thus its pdf and cdf become:

$$\text{pdf: } f(x/\alpha, s) = \frac{\alpha}{s} \left(\frac{x}{s}\right)^{-1-\alpha} \exp\left\{-\left(\frac{x}{s}\right)^{-\alpha}\right\} \tag{6}$$

$$\text{cdf: } f(x/\alpha, s) = \exp\left\{-\left(\frac{x}{s}\right)^{-\alpha}\right\} \tag{7}$$

In order to get its mean, its first-order moment expectation about origin is obtained as:

$$E(X) = \int_{-\infty}^{\infty} xf(x)dx = \int_0^{\infty} x \frac{\alpha}{s} \left(\frac{x}{s}\right)^{-1-\alpha} e^{-\left(\frac{x}{s}\right)^{-\alpha}} dx$$

$$\text{Setting } t = \left(\frac{x}{s}\right)^{-\alpha} \Rightarrow t^{\frac{1}{\alpha}} = \left(\frac{x}{s}\right)^{-\alpha \cdot \frac{1}{\alpha}} \Rightarrow t^{\frac{1}{\alpha}} = \frac{x}{s} \Rightarrow x \equiv st^{\frac{1}{\alpha}}$$

$$\text{Thus, } \frac{dx}{dt} = \frac{-st^{\frac{1}{\alpha}-1}}{\alpha} \Rightarrow dx = \frac{-s}{\alpha} \cdot t^{-1-\frac{1}{\alpha}} dt \text{ and } \left(\frac{x}{s}\right)^{-1-\alpha} = \left(\frac{x}{s}\right)^{-1} \left(\frac{x}{s}\right)^{-\alpha} = \left(\frac{x}{s}\right)^{-\alpha \cdot \frac{1}{\alpha}} \left(\frac{x}{s}\right)^{-\alpha} = t^{\frac{1}{\alpha}} \cdot t = t^{1+\frac{1}{\alpha}}$$

$$\begin{aligned} \text{Thus, } E(x) &= \int_0^{\infty} x \frac{\alpha}{s} t^{1+\frac{1}{\alpha}} e^{-t} \cdot \frac{s}{\alpha} t^{-1-\frac{1}{\alpha}} dt = \int_0^{\infty} x e^{-t} dt = \int_0^{\infty} st^{\frac{1}{\alpha}} e^{-t} dt \\ &= s \int_0^{\infty} t^{\frac{1}{\alpha}} e^{-t} dt = s \int_0^{\infty} t^{\frac{1}{\alpha}+1-1} e^{-t} dt = s\Gamma\left(1 - \frac{1}{\alpha}\right) \text{ for } \alpha > 1 \end{aligned} \tag{8}$$

The Equation (8) above is the mean of the distribution in Equation (6)

$$\begin{aligned} \text{Thus, } E(X^2) &= \int_0^{\infty} X^2 f(x) dx = \int_0^{\infty} x^2 \frac{\alpha}{s} \left(\frac{x}{s}\right)^{-1-\alpha} e^{-\left(\frac{x}{s}\right)^{-\alpha}} dx \\ &= \int_0^{\infty} (st^{\frac{1}{\alpha}})^2 \frac{\alpha}{s} \cdot t^{1+\frac{1}{\alpha}} e^{-t} \cdot \frac{s}{\alpha} t^{-1-\frac{1}{\alpha}} dt = \int_0^{\infty} s^2 t^{\frac{2}{\alpha}} e^{-t} dt \\ &= s^2 \int_0^{\infty} t^{\frac{2}{\alpha}+1-1} e^{-t} dt = s^2\Gamma\left(1 - \frac{2}{\alpha}\right) \end{aligned} \tag{9}$$

$$\begin{aligned} \text{and its variance: } \text{Var}(X) &= E(X^2) - [E(X)]^2 = s^2\Gamma\left(1 - \frac{2}{\alpha}\right) - \left\{s\Gamma\left(1 - \frac{1}{\alpha}\right)\right\}^2 \\ &= s^2\left[\Gamma\left(1 - \frac{2}{\alpha}\right) - \left\{\Gamma\left(1 - \frac{1}{\alpha}\right)\right\}^2\right] \text{ For } \alpha > 2 \end{aligned} \tag{10}$$

Thus the mean and variance of two parameter Frechet distribution are derived to be:

**Mean:**  $E(X) = s\Gamma\left(1 - \frac{1}{\alpha}\right)$  **and Variance:**  $Var(X) = s^2 \left[ \Gamma\left(1 - \frac{2}{\alpha}\right) - \left\{ \Gamma\left(1 - \frac{1}{\alpha}\right) \right\}^2 \right]$

It is noted that variance for two-parameter Frechet distribution is the same as the variance for three-parameter Frechet distribution.

**Mean:**  $E(X) = s\Gamma\left(1 - \frac{1}{\alpha}\right)$  **and Variance:**  $Var(X) =$

It is noted that variance for two-parameter Frechet distribution is the same as the variance for three-parameter Frechet distribution.

**3.3 Relationship of frechet distribution to other distribution**

It had been established that:

- If  $x \sim u(0,1)$  (Uniform distribution (continuous)), then  $m + s(-\log(x))^{-\frac{1}{s}}$  Frechet (a, s, m).
- If  $x \sim frechet(a, s, m)$  then  $kx+b \sim frechet(a, ks, km + b)$
- The cumulative distribution function of the Frechet distribution solves the maximum stability postulate equation
- If  $x \sim frechet(a, s, m = 0)$  then its reciprocal is weibull-distributed:  $x^{-1} \sim weibull(k = a, \lambda = s^{-1})$
- If  $x \sim frechet(a, s, m)$  and  $y = \max\{x_1, \dots, x_n\}$  then  $y \sim frechet(a, n^{\frac{1}{s}}, s, m)$

**4. Frechet distribution analysis of air pollution**

This research work used Frechet distribution to determine the comparative assessment of air pollution at Okababa Sawmill Lagos State. The levels and probabilities of air pollutants in the sawmill were determined and examined.

**Table 1:** Levels of air pollutant in Okobaba Sawmill

PARAMETERS	A	B	C
CO(ppm)	720	30	ND
NO2(ppm)	0.84	0.73	0.04
SO2(ppm)	0.61	0.23	0.08

**Table 4.2:** Summation of Probability levels of CO, **NO<sub>2</sub>** and **SO<sub>2</sub>**

C0	NO2	SO2
0	0	0
0.1098	0.2665	0.0841
0.018	0.114	0.0409
0.0043	0.0647	0.0266
0.0012	0.0424	0.0195
0.0004	0.0303	0.0153

The chi-square tests with values of  $\chi^2 = 1122.9375$ ,  $\chi^2 = 75.086335$  and  $\chi^2 = 180.762395$  at 0.05 level of significance revealed that the data collected at the sawmill for carbon monoxides, Nitrogen (IV) oxide and Sulphur (IV) Oxide respectively fit into Frechet distribution.

The Figure below shows the level of air pollution at Okobaba sawmill from the environment and statistical analyses.

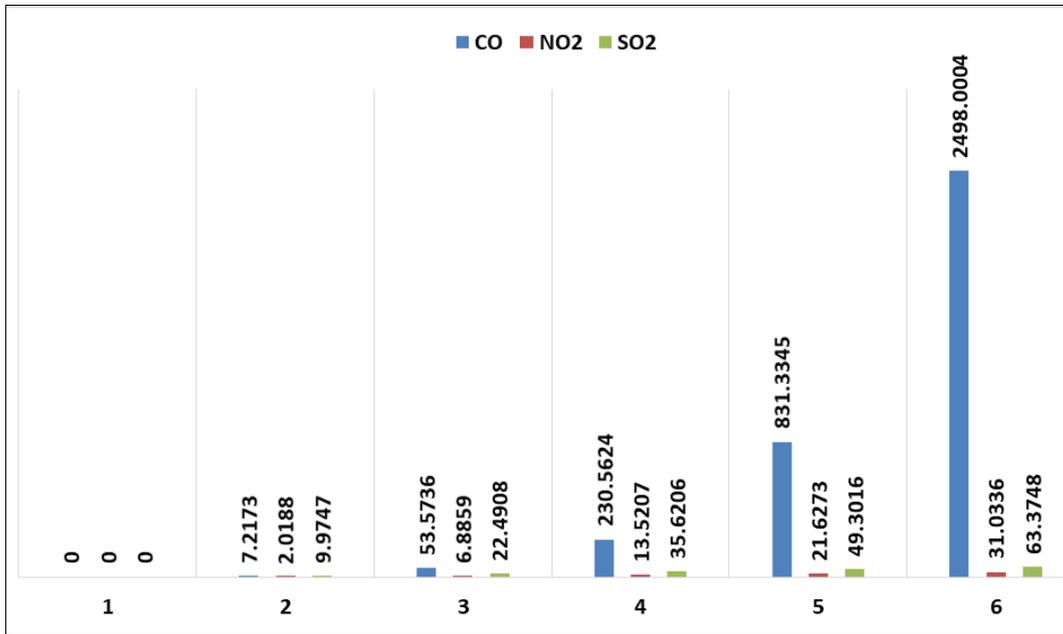


Fig 1: Level of air pollution at okababa sawmill

### 5. Conclusion

Going by the findings of this research study, the following factors identified as air pollution is an important factor affecting health and life quality. Road traffic and biomass burning can produce substantial increases in the concentrations of carbon monoxide (CO), nitrogen oxides ( $NO_2$ ), volatile organic compounds (VOC), particulate matter (PM), sulphur oxides ( $SO_2$ ), ozone ( $O_3$ ). Nitrogen oxides may also cause adverse effects on vegetation and contribute to the formation of secondary inorganic PM and  $O_3$  with associated effects on health, ecosystems and climate. Sulphur dioxide is a precursor in the formation of PM and damages forests and terrestrial ecosystems, affecting the human respiratory system. Carbon monoxide can reduce the oxygen-carrying capacity of blood. In the atmosphere, CO slowly oxidizes into carbon dioxide or ozone. All these factors should be put into consideration to improve the air pollution in the country Nigeria.

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