Journal of Mathematical Problems, Equations and Statistics

E-ISSN: 2709-9407
P-ISSN: 2709-9393
JMPES 2020; 1(2): 36.42
© 2020 JMPES
www.mathematicaljournal.com
Received: 14-02-2020
Accepted: 16-03-2020

Akintunde Oyetunde A
Department of Mathematics, Faculty of Science, Federal University Oye-Ekiti, Ekiti State, Nigeria

OJO Faith Afolabi
Department of Mathematics, Faculty of Science, Federal University Oye-Ekiti, Ekiti State, Nigeria

Correspondence
Akintunde Oyetunde A
Department of Mathematics, Faculty of Science, Federal University Oye-Ekiti, Ekiti State, Nigeria

Comparative statistical assessment of air pollution in Okobaba sawmill Nigeria by the use of frechet distribution

Akintunde Oyetunde A and OJO Faith Afolabi

Abstract
The air pollution data in this research study are stratified into three strata which are: Nitrogen (IV) Oxide, Sulphur (IV) Oxide, and Carbon Monoxide. Chi-square test and the analysis of variance were used to analyze the collected data. The data collected for carbon monoxide, Nitrogen (IV) oxide and Sulphur (IV) oxide fit into Frechet distribution because their chi-square tabulated values are less than their chi-square calculated values, which implied that Frechet distribution can be used for analyses and control of air pollution in Nigeria. It was statistically established that requiring probabilistic distributions to better describe each phenomenon or experiment studied and making available data more fit and reliable. This research study therefore recommends that the use of observations in field studies to develop the empirical source-receptor relationships (for instance, using Frechet distribution and statistical receptor models) should be put into consideration to improve the air pollution in developing countries like Nigeria.

Keywords: Frechet distribution, air pollution, chi-square, Carbon monoxide, Sulphur IV oxide, Nitrogen IV oxide

Introduction

Air pollution is an important factor affecting health and life quality. Air pollution is the contamination of the indoor or outdoor air by a range of gasses and solids that modify its natural characteristics. The atmosphere consists of mixture of gases that completely surround the earth. Air is said to be polluted when chemicals, gases, particulate matter and biological materials that can cause harm or discomfort to humans or other living organisms or cause damage to the environment are released into the atmosphere (Akpanisi, 2003). Air pollution is the most insidious of all forms of pollution because we can partially avoid the direct contact of other forms of pollution; we must breathe the air (Dix, 1981; Speeding, 1997).

Air pollution is often not visible to the naked eye as the sizes of the pollutants are smaller than the human eye can detect. They can become visible in some situations for example in the form of sooty smoke from the open burning of crop residues or other waste, as well as from burning wood, coal, petrol and diesel fuels, for cooking and heating, transport or power production. The fact that you cannot see the air pollution does not mean that it does not exist. Air pollution of the urban centers is one of the world's worst pollution problems. The ambient air quality of an area affects the chemistry of its atmosphere and the general wellness of the environment including humans. Air quality reports in most advanced countries are therefore presented regularly to assist the public in the management of their health. Many industrialized countries have air quality standards and guideline to regulate emission into the environment (USEPA, 1993; FGN, 1988). Health harmful air pollutants include particulate matter (PM2.5 and PM10) 1, Carbon monoxide (CO), Ozone (O_3), Black Carbon (BC), Sulfur dioxide and Nitrogen Oxides (NO).

In pollution science, statistical methods are applied to quantify and evaluate data derived from observations as well as for mathematical models. The basis of control of air pollutants consists essentially of two different approaches, namely:

1. Use observations in field studies to develop the empirical source-receptor relationships (for instance, using statistical receptor models), and

Application of various mathematical chemical transport models that relate the concentration of the secondary pollutants formed to the initial primary pollutant concentrations.

Any statistical analysis depends greatly on the statistical model used to represent the phenomena understudy.

Hence, the larger the class of statistical models available to the statistician the easier it is to choose a model. A quick survey of the models in common use reveals the abundance of statistical models in the literature.
However, data of many important and practical problems do not follow any of the probability models available. In such cases a non-parametric model may be recommended. While a two parameter distribution may provide reasonably precision in fitting data, it may be still desirable to extend the flexibility of any distribution to allow for better description of data without having to resort to non-parametric models. The Fréchet distribution has applications ranging from accelerated life testing through earthquakes, floods, rainfall, horse racing, queues, sea currents, wind speeds and track race records. Kotz and Nadarajah (2000) [34] give other applications in their book. Kotz and Nadarajah (2000) [34] also introduced the transmuted Fréchet distribution which stems from the following idea. Shaw & Buckley (2009) considered composite maps of the following two forms: sample transmutation maps (STMs), $\mathrm{y} = G - 1[F_X]$, and rank transmutation maps (RTMs), v = G[F−1 w], where F and G are cumulative distribution functions (CDFs). Gilchrist (2000) refers to STMs and RTMs as Q-transformation and P-transformation, respectively.
Shaw & Buckley (2009) focused on the RTM, which uses as a tool for the construction of new families of non-Gaussian distributions. They used it to modulate a given base distribution for the purposes of modifying the moments, in particular the skew and kurtosis. An attraction of the approach is that if the CDF and inverse CDF (or quantile function (QF)) are tractable for the base distribution, there is a good chance for the transmuted distribution to be so.

2. Literature review

Applications of the Frechet distribution in various fields given by Harlow (2002) showed that it is an important distribution for modeling the statistical behavior of materials properties for a variety of engineering applications. Nadarajah and Kotz (2008) [34] discussed the sociological models based on Frechet random variables. Mubarak (2011) studied the Frechet progressive type-II censored data with binomial removals.
The Frechet distribution is a special case of the generalized extreme value distribution. This type-II extreme value distribution (Frechet) case is equivalent to taking the reciprocal of values from a standard Weibull distribution and drive the best linear unbiased estimators of location and scale parameters of the Frechet distribution and obtain the best linear invariant estimations also use maximum likelihood estimators (MLE) to estimate the parameters of the Frechet distribution. Further, Abbas and Tang (2012) studied different estimation methods for Frechet distribution with known shape. Abbas and Tang (2013) discussed the parameters for Frechet Distribution based on type-II censored sample. They consider maximum likelihood estimators and least squares estimators of two-parameter Frechet distribution based on type-II censored sample.
The maximum likelihood estimators (MLE) and least squares estimators (LSE) are developed for estimating the unknown parameters. The observed Fisher information matrix and confidence intervals of the parameters based on asymptotic normality are also derived. An extensive simulation study is carried out to compare the performances of different methods. Nasir and Aslam (2015) studied the Frechet distribution under Bayesian paradigm.
Posterior distributions are derived by using Gumbel Type-II and Levy prior. Quadrature numerical integration technique is utilized to solve posterior distribution. Bayes estimators and their risks have been obtained by using four loss functions.
Extreme value models play an important role in statistics. The generalized extreme value (GEV) distribution (Jenkinson, 1955) and its sub-models are widely used in application involving extreme events. These sub-models are the well-known Weibull, Fréchet and Gumbel distributions. The Fréchet distribution can be seen as the inverse Weibull distribution which gives a probability density function (pdf) such

$$f(t/\lambda,\alpha) = \frac{\alpha}{\lambda}\left(\frac{t}{\lambda}\right)^{-(\alpha+1)} e^{-\left(\frac{t}{\lambda}\right)^{\alpha}}$$

The survival function is given by:

$$s(t/\lambda,\alpha) = 1 - e^{-\left(\frac{t}{\lambda}\right)^{\alpha}}$$

Although the GEV distribution is the most used generalization of the Fréchet model, other distributions have been proposed in the literature. De Gusmão (2012) [20] proposed a three parameter generalized inverse Weibull distribution in which includes the Fréchet distribution. Krishna *et al*. (2013) proposed the Marshall-Olkin Fréchet distribution. Barreto-Souza *et al*. (2011) [15] discussed some results for beta Fréchet distribution.
A long-term survival novel proposes a mixture model as introduced by Berkson and Gage (1952), hereafter was called the long-term Fréchet distribution (LF) distribution. Some mathematical properties about the long-term Fréchet distribution (LF) distribution were provided such as moments, survival properties and hazard function.
The maximum likelihood estimators (MLEs) of the parameters and its asymptotic properties are discussed likewise. Similar studies were presented by Roman *et al*. (2012) [25] for the geometric exponential distribution and by Louzada and Ramos (2017) for the weighted Lindley distribution. It performed a numerical simulation towards to examine the performance of the maximum likelihood estimators (MLEs). Finally, the proposed methodology is illustrated in a real data set related to the leukemia free-survival times (in years) for the 50 antilogous transplant patients.
A random variable X is said to have transmuted distribution, according to the quadratic rank transmutation map (QRTM), if its cumulative distribution function is given by $F_X = 1 + \lambda\, G_X - \lambda G(X)^2$, $\lambda \leq 1$ (1) where G(x) is the cdf of the base distribution

(Shaw and Buckley (2009)). Note that when λ=0, the base distribution will be obtained. Aryal and Tsokos (2009, 2011) studied the transmuted extreme value distributions and provided the mathematical characterization of transmuted Gumbel and transmuted Weibull distributions and their applications to analyze real data set. Aryal (2013) proposed and studied the transmuted log-logistic distribution. He provided mathematical formulation of this distribution and some of its properties. Ashour and Eltehiwy (2013a, 2013b) studied the transmuted exponentiated modified Weibull and transmuted exponentiated Lomax distributions and discussed some properties of these families. Merovic (2013) introduced the transmuted Rayleigh distribution and provided some properties of this distribution.

3. Research methodology

3.1 Integral study of three parameter of frechet distribution

The probability density function (pdf) of the three parameter Frechet distribution with random variable x is as given below:

$$The\ pdf : f(x/\propto, s, m) = \frac{\propto}{s}\left(\frac{x-m}{s}\right)^{-1-\propto}\ exp\left\{-\left(\frac{x-m}{s}\right)^{-\propto}\right\} \tag{1}$$

and its cumulative density function (cdf) is given as:

$$\text{cdf} : f(x/\propto, s, m) = e^{-\left(\frac{x-m}{s}\right)^{-\propto}} \tag{2}$$

The expected value of the probability distribution function is obtained as:

$$E(X) = \int_{-\infty}^{\infty} x f(x) dx = \int_0^{\infty} x \frac{\propto}{s}\left(\frac{x-m}{s}\right)^{-1-\propto} e^{-\left(\frac{x-m}{s}\right)^{-\propto}} dx$$

By setting $t = \left(\frac{x-m}{s}\right)^{-\propto}$, $thus\ x = m + st^{-\frac{1}{\propto}}\ and\ \frac{dx}{dt} = \frac{-st^{-\frac{1}{\propto}-1}}{\propto} \Rightarrow dx = \frac{-s}{\propto}.t^{-1-\frac{1}{\propto}}dt$

Also, $\left(\frac{x-m}{s}\right)^{-1-\propto} = \left(\frac{x-m}{s}\right)^{-1}\left(\frac{x-m}{s}\right)^{-\propto} = \left(\frac{x-m}{s}\right)^{-\propto.\frac{1}{\propto}}\left(\frac{x-m}{s}\right)^{-\propto} = t^{\frac{1}{\propto}}.t = t^{1+\frac{1}{\propto}}$

Thus, $E(X) = \int_0^{\infty} x\frac{\propto}{s}t^{1+\frac{1}{\propto}}e^{-t}.\frac{-s}{\propto}t^{-1-\frac{1}{\propto}}dt = \int_0^{\infty} xt^{1+\frac{1}{\propto}}.t^{-1-\frac{1}{\propto}}e^{-t}\,dt = \int_0^{\infty} xt^{1+\frac{1}{\propto}-1-\frac{1}{\propto}}e^{-t}\,dt$

$$= \int_0^{\infty} xe^{-t}\,dt = \int_0^{\infty}\left(m + st^{-\frac{1}{\propto}}\right).e^{-t}\,dt = \int_0^{\infty} me^{-t}\,dt + \int_0^{\infty} st^{\frac{-1}{\propto}}e^{-t}\,dt$$

$$= m\int_0^{\infty} e^{-t}\,dt + s\int_0^{\infty} t^{\frac{-1}{\propto}}e^{-t}\,dt = m\int_0^{\infty} e^{-t}\,dt + s\int_0^{\infty} t^{\frac{-1}{\propto}+1-1}e^{-t}\,dt$$

$$= m\int_0^{\infty} t^{1-1}e^{-t}\,dt + s\int_0^{\infty} t^{\frac{-1}{\propto}+1-1}e^{-t}\,dt$$

Recall from gamma function: $\int_0^{\infty} t^{\propto-1}\,e^{-t}dt = \Gamma(\propto)$ and $\Gamma(\propto) = (\propto-1)!$

$$\text{Hence, } E(X) = \mathrm{m}\Gamma(1) + S\Gamma(1-\tfrac{1}{\propto}) = m + S\Gamma(1-\tfrac{1}{\propto}) \text{ for } \propto > 1 \tag{3}$$

The Equation (3) above is the mean of the distribution in Equation (1)

Also, $E(X^2) = \int_0^{\infty} x^2\frac{\propto}{s}\left(\frac{x-m}{s}\right)^{-1-\propto} e^{-\left(\frac{x-m}{s}\right)^{-\propto}} dx = \int_0^{\infty}(m + st^{-\frac{1}{\propto}})^2\frac{\propto}{s}t^{1+\frac{1}{\propto}}e^{-t}.\frac{-st^{-1-\frac{1}{\propto}}}{\propto}dt$

$$= \int_0^{\infty}(m + st^{\frac{-1}{\propto}})^2 e^{-t}dt = \int_0^{\infty}(m^2 + 2mst^{\frac{-1}{\propto}} + s^2t^{\frac{-2}{\propto}})e^{-t}dt$$

$$= \int_0^{\infty} m^2e^{-t}\,dt + 2ms\int_0^{\infty} t^{-\frac{1}{\propto}}e^{-t}dt + s^2\int_0^{\infty} t^{-\frac{2}{\propto}}e^{-t}dt$$

$$= m^2 + 2ms\Gamma\left(1-\frac{1}{\propto}\right) + s^2\Gamma\left(1-\frac{2}{\propto}\right) \tag{4}$$

and the variance is obtained as:

$$\mathrm{Var(x)} = E(X^2) - [E(X)]^2$$

$$= m^2 + 2ms\Gamma\left(1-\frac{1}{\propto}\right) + s^2\Gamma\left(1-\frac{2}{\propto}\right) - \left[m + s\Gamma\left(1-\frac{1}{\propto}\right)\right]^2$$

$$= m^2 + 2ms\Gamma\left(1-\frac{1}{\propto}\right) + s^2\Gamma\left(1-\frac{2}{\propto}\right) - \left[m^2 + 2ms\Gamma\left(1-\frac{1}{\propto}\right) + s^2\left\{\Gamma\left(1-\frac{1}{\propto}\right)\right\}^2\right]$$

$$= s^2\Gamma\left(1-\frac{2}{\propto}\right) - s^2\Gamma\left[1-\frac{1}{\propto}\right]^2 = s^2\left[\Gamma\left(1-\frac{2}{\propto}\right) - \left\{\Gamma\left(1-\frac{1}{\propto}\right)\right\}^2\right] \; for \; \propto > 2 \quad (5)$$

The Equation (5) above is the variance of the distribution in Equation (1)

3.2 Integral study of frechet distribution with two parameters

By setting m=0 in the three-parameter Frechet distribution to become two parameter Frechet distribution, thus its pdf and cdf become:

pdf: $f(x/\propto, s) = \frac{\propto}{s}\left(\frac{x}{s}\right)^{-1-\propto} exp\left\{-\left(\frac{x}{s}\right)^{-\propto}\right\}$ (6)

cdf: $f(x/\propto, s) = exp\left\{-\left(\frac{x}{s}\right)^{-\propto}\right\}$ (7)

In order to get its mean, its first-order moment expectation about origin is obtained as:

$$\mathrm{E}(X) = \int_{-\infty}^{\infty} xf(x)dx = \int_0^{\infty} x\frac{\propto}{s}\left(\frac{x}{s}\right)^{-1-\propto} e^{-\left(\frac{x}{s}\right)^{-\propto}} dx$$

Setting $t=\left(\frac{x}{s}\right)^{-\propto} \Rightarrow t^{-\frac{1}{\propto}} = \left(\frac{x}{s}\right)^{-\propto . \frac{1}{\propto}} \Rightarrow t^{-\frac{1}{\propto}} = \frac{x}{s} \Rightarrow x \equiv st^{-\frac{1}{\propto}}$

Thus, $\frac{dx}{dt} = \frac{-st^{-\frac{1}{\propto}-1}}{\propto} \Rightarrow dx = \frac{-s}{\propto}.t^{-1-\frac{1}{\propto}}dt$ and $\left(\frac{x}{s}\right)^{-1-\propto} = \left(\frac{x}{s}\right)^{-1}\left(\frac{x}{s}\right)^{-\propto} = \left(\frac{x}{s}\right)^{-\propto . \frac{1}{\propto}}\left(\frac{x}{s}\right)^{-\propto} = t^{\frac{1}{\propto}}.t = t^{1+\frac{1}{\propto}}$

Thus, E(x)= $\int_0^{\infty} x\frac{\propto}{s}t^{1+\frac{1}{\propto}}e^{-t}.\frac{s}{\propto}t^{-1-\frac{1}{\propto}}dt = \int_0^{\infty} xe^{-t}dt = \int_0^{\infty} st^{-\frac{1}{\propto}}e^{-t}dt$

$$= s\int_0^{\infty} t^{-\frac{1}{\propto}}e^{-t}dt = s\int_0^{\infty} t^{-\frac{1}{\propto}+1-1}e^{-t}dt = S\Gamma\left(1-\frac{1}{\propto}\right) \quad for \; \propto > 1 \quad (8)$$

The Equation (8) above is the mean of the distribution in Equation (6)

Thus, $E(X^2) = \int_0^{\infty} X^2 f(x)dx \qquad = \int_0^{\infty} x^2\frac{\propto}{s}\left(\frac{x}{s}\right)^{-1-\propto} e^{-\left(\frac{x}{s}\right)^{-\propto}} dx$

$$= \int_0^{\infty} (st^{-\frac{1}{\propto}})^2\frac{\propto}{s}.t^{1+\frac{1}{\propto}}e^{-t}\frac{s}{\propto}t^{-1-\frac{1}{\propto}}dt = \int_0^{\infty} s^2t^{-\frac{2}{\propto}}e^{-t}dt$$

$$= s^2\int_0^{\infty} t^{-\frac{2}{\propto}+1-1}e^{-t}dt = s^2\Gamma\left(1-\frac{2}{\propto}\right) \quad (9)$$

and its variance: Var(X)= $E(X^2) - [E(X)]^2 = S^2\Gamma\left(1-\frac{2}{\propto}\right) - \left\{S\Gamma\left(1-\frac{1}{\propto}\right)\right\}^2$

$$= S^2\left[\Gamma\left(1-\frac{2}{\propto}\right) - \left\{\Gamma\left(1-\frac{1}{\propto}\right)\right\}^2\right] \; \text{For} \propto > 2 \quad (10)$$

Thus the mean and variance of two parameter Frechet distribution are derived to be:

Mean: E(X)= $S\Gamma\left(1-\frac{1}{\propto}\right)$**and Variance: Var(X)=**$S^2\left[\Gamma\left(1-\frac{2}{\propto}\right)-\left\{\Gamma\left(1-\frac{1}{\propto}\right)\right\}^2\right]$

It is noted that variance for two-parameter Frechet distribution is the same as the variance for three-parameter Frechet distribution.

Mean: E(X)= $S\Gamma\left(1-\frac{1}{\propto}\right)$ **and Variance: Var (X)=**

It is noted that variance for two-parameter Frechet distribution is the same as the variance for three-parameter Frechet distribution.

3.3 Relationship of frechet distribution to other distribution

It had been established that:

- If $x \sim u(0,1)$ (Uniform distribution (continuous)), then $m + s(-\log(x))^{-\frac{1}{x}}$ Frechet (a, s, m).
- If $x \sim frechet(a, s, m)$ then kx+b$\sim frechet(a, ks, km + b)$
- The cumulative distribution function of the Frechet distribution solves the maximum stability postulate equation
- If $x \sim frechet(a, s, m = 0)$ then its reciprocal is weibull-distributed: $x^{-1} \sim weibull(k = a,\ \lambda = s^{-1})$
- If $x \sim frechet(a, s, m)$ and $y = \max\{x_1, \ldots, x_n\}$ then $y \sim frechet(a, n^{\frac{1}{n}}, s, m)$

4. Frechet distribution analysis of air pollution

This research work used Frechet distribution to determine the comparative assessment of air pollution at Okababa Sawmill Lagos State. The levels and probabilities of air pollutants in the sawmill were determined and examined.

Table 1: Levels of air pollutant in Okobaba Sawmill

PARAMETERS	A	B	C
CO(ppm)	720	30	ND
NO2(ppm)	0.84	0.73	0.04
SO2(ppm)	0.61	0.23	0.08

Table 4.2: Summation of Probability levels of CO, NO_2 and SO_2

C0	NO2	SO2
0	0	0
0.1098	0.2665	0.0841
0.018	0.114	0.0409
0.0043	0.0647	0.0266
0.0012	0.0424	0.0195
0.0004	0.0303	0.0153

The chi-square tests with values of $x^2 = 1122.9375$, $x^2 = 75.086335$ and $x^2 = 180.762395$ at 0.05 level of significance revealed that the data collected at the sawmill for carbon monoxides, Nitrogen (IV) oxide and Sulphur (IV) Oxide respectively fit into Frechet distribution.

The Figure below shows the level of air pollution at Okobaba sawmill from the environment and statistical analyses.

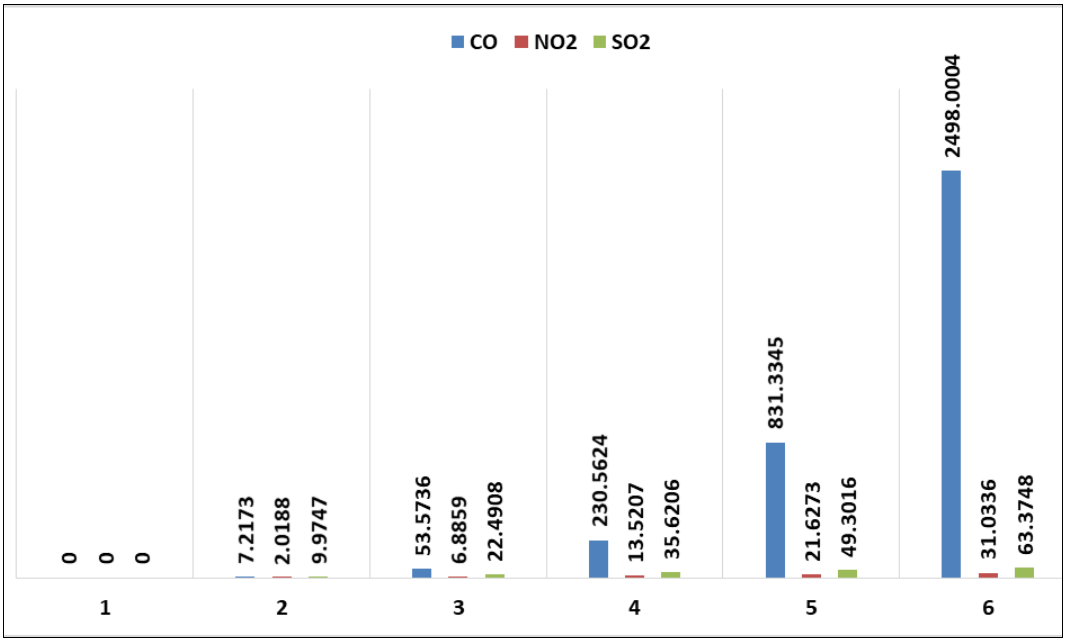

Fig 1: Level of air pollution at okababa sawmill

5. Conclusion

Going by the findings of this research study, the following factors identified as air pollution is an important factor affecting health and life quality. Road traffic and biomass burning can produce substantial increases in the concentrations of carbon monoxide (CO), nitrogen oxides (NO_2), volatile organic compounds (VOC), particulate matter (PM), sulphur oxides (SO_2), ozone (O_3). Nitrogen oxides may also cause adverse effects on vegetation and contribute to the formation of secondary inorganic PM andO_3 with associated effects on health, ecosystems and climate. Sulphur dioxide is a precursor in the formation of PM and damages forests and terrestrial ecosystems, affecting the human respiratory system. Carbon monoxide can reduce the oxygen-carrying capacity of blood. In the atmosphere, CO slowly oxidizes into carbon dioxide or ozone. All these factors should be put into consideration to improve the air pollution in the country Nigeria.

References

1. B Tawfik AL. Steiner Atmospheric Environmental 2013;72:50.
2. Kristó, Csíki Zöld Füzetek, 1994, 7–26.
3. Peters DW, Dockery Muller JE, Mittleman MA. “Increased particulate air pollution and the triggering of myocardial infarction,” Circulation 2001;103(23):2810-2815.
4. Saxon D, Diaz-Sanchez. “Air pollution and allergy: you are what you breathe,” Nature Immunology 2005;6(3):223-226.
5. Abbas and Tang studied different estimation methods for Frechet distribution with 53. Known shape, 2012.
6. Adelagun ROA, Berezi EP, Akintunde OA. Air Pollution in a Sawmill Industry: The Okobaba Experience, Journal of Sustainable Development and Environmental Protection, April-June 2012, Pages published by Institute for Environmental Research and Development, Bells University of Technology Ota 2012;2(2):29-36.
7. Afify AZ, Hamedani GG, Ghosh I, Mead ME. The transmuted Marshall-Olkin Fréchet distribution: properties and applications. International Journal of Statistics and Probability 2015;4:132-184.
8. Afify AZ, Nofal ZM, Butt NS. Transmuted complementary Weibull geometric distribution. Pak. J Stat. Oper. Res 2014;10:435-454.
9. Afify AZ, Yousof HM, Ahmad M, Ahmed AN. The Kumaraswamy Marshall-Olkin Fréchet distribution with applications. Submitted, 2015.
10. Afify AZ, Yousof HM, Cordeiro GM, Ortega EMM, Nofal ZM. The Weibull Fréchet distribution and its applications. Journal of Applied Statistics, forthcoming, 2016.
11. Ahmad A, Ahmad SP, Ahmed A. Transmuted inverse Rayleigh distribution: a generalization of the inverse Rayleigh distribution. Mathematical Theory and Modeling 2014;4:90-98.
12. Brunekreef, Holgate ST. “Air pollution and health. ”The Lancet 2002;360(9341):1233–1242.
13. Kosmider JE, Loader RC, Murphy, Mason RJ. “Apoptosis induced by ozone and oxysterols in human alveolar epithelial cells,” Free Radical Biology and Medicine 2010;48(11):1513-1524.
14. Barlow RE, Proschan F. Mathematical theory of reliability, 17. Siam, 1981.
15. Barreto-Souza WM, Cordeiro GM, Simas AB. Some results for beta Fréchet distribution. Commun. Statist. Theory-Meth 2011;40:798-811.
16. Behavioral maxima of three Extreme value distributions–Gumbel, Frechet and negative Weibull–by Fisher and Tippet 1928.
17. Bjerkedal T. Acquisition of resistance in guinea pigs infected with different doses of virulent tubercle bacilli. Amer. J Hyg 1960;72:130-148.

18. Cordeiro GM, de Castro M. A new family of generalized distributions. Journal of Statistical Computation and Simulation 2011;81:883-898.
19. da Silva RV, de Andrade TA, Maciel D, Campos RP, Cordeiro GM. A new lifetime model: the gamma extended Fréchet distribution. Journal of Statistical Theory and Applications 2013;12:39-54.
20. De Gusmão. Proposed a three parameter generalized inverse Weibull distribution in which includes the Fréchet distribution 2012.
21. de Gusmão FR, Ortega EM, Cordeiro GM. The generalized inverse Weibull distribution. Statistical Papers 2011;52:591-619.
22. Boldo C, Linares Lumbreras J *et al.*, "Health impact assessment of a reduction in ambient PM(2.5) levels in Spain," Environment International, 2011;37(2):342-348, 2011.
23. Wu, Kuo SL. Atmosphere, 2013;4:349.
24. European Commission, Air quality standard, 2017, http://ec.europa.eu/environment/air/quality/standards.htm
25. European Environment Agency. "Romania air pollution country fact sheet — European Environment Agency" 2013.
26. European Environment Agency Human health - Climate Change", 2015.
27. Kelly, Tetley TD. "Nitrogen dioxide depletes uric acid and ascorbic acid but not glutathione from lung lining fluid," Biochemical Journal 1997;325(1):95-99.
28. Barreto Madureira D, Capani F, Aon-Bertolino L, Saraceno E, Alvarez-Giraldez LD. "The role of catechols and free radicals in benzene toxicity: an oxidative DNA damage pathway," Environmental and Molecular Mutagenesis, 2009;50(9):771–780.
29. lorga C, Balaceanu Raicu S. Stefan, Atmospheric Pollution Research 2015;6:824-834.
30. Bayram RJ, Sapsford MM, Abdelaziz, OA Khair (2001). "Effect of ozone and nitrogen dioxide on the release of proinflammatory mediators from bronchial epithelial cells of no atopicno asthmatic subjects and atopic asthmatic patients *in vitro*," Journal of Allergy and Clinical Immunology 2001;107(2):287-294.
31. Song H, Tan W, Zhang X. "Ozone induces inflammation in bronchial epithelial cells," Journal of Asthma 2011;48(1):79–83.
32. Mudway S, Kelly FJ. "Ozone and the lung: a sensitive issue," Molecular Aspects of Medicine 2000;21(1-2):1-48.
33. Bolton L, Trush MA, Penning TM, Dryhurst G, Monks TJ. "Role of quinones in toxicology," Chemical Research in Toxicology 2000;13(3):135-160.
34. Kotz, Nadarajah. extreme value distributions work done by Bernoulli in 1709 (pdf), 2000.
35. Bogdan Niculescu E. "Aspecte climatice specifice ale depresiunilor Giurgiu, Ciuc, Brașov", in *Factori și procese* pedogenetice din zonă temperată, 2004, 3-115.
36. Saide Peralta. "Aerosol predictions and their links to weather forecasts through online interactive atmospheric modeling and data assimilation", Theses Dissertation, 2013.
37. Szép Mátyás L. Carpathian Journal Earth Environmental Science 2014;9:241.
38. Szép Mátyás L, Keresztes R, Ghimpusian M. Revista de Chimie 2016;67:205.
39. Szép R, Keresztes A, Korodi, Sz. Tonk. *Revista de Chimie* 2016;67(4):639.
40. Szép R, Keresztes R, Lucian C. Revista de Chimie-Bucharesti 2016;67(3):408.
41. Szép R, Keresztes R, Sz. Tonk A. Korodi. *Revista de Chimie*, 2016;97(10):1914.
42. Turnock E, Butt W, Richardson TB, Mann GW, Reddington CL, Forster PM *et al*. Environmental Research Letters 2016, 11.
43. Nishanth Ojha N, Kumar MKS, Naja M. Atmospheric Environmental 2011;45:1752–1758.
44. Yang, Omaye ST. "Air pollutants, oxidative stress and human health," Mutation Research 2009;674(1-2):45–54.
45. WHO. "Air quality guidelines: for Europe," World Health Organization Regional Publications 2001;91:1–287.
46. World Health Organization. "Air quality guidelines for Europe", Regional Office for Europe, Copenhagen, 2000
47. World Health Organization. "Air Quality Guidelines: Global Update 2005: Particulate Matter, Ozone, Nitrogen Dioxide, and Sulfur Dioxide", 2006.
48. World Health Organization, "Review of evidence on health aspects of air pollution - REVIHAAP Project", Technical Report, Copenhagen, 2013
49. Wang YX. Journal Geophys. Research, 2004, 109.
50. Zhang YL, Cao F. Environmental Pollution 2015;202:217–219.