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Matrix domains of Maddox-type Paranormed sequence spaces induced by Norlund matrices from combinatorial sequences

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Abstract
We study matrix domains of paranormed sequence spaces obtained by applying lower triangular (triangle) matrices to Maddox-type variable-exponent spaces. A flexible family of such triangles is produced by N¨orlund matrices generated from admissible weight sequences, including Fibonacci, Pell, Motzkin, and Catalan numbers. For a base paranormed FK-space $X \subseteq \omega$ and a triangle A, the associated matrix domain $X_A = \{x \in \omega: Ax \in X\}$ is equipped with the transported paranorm $q_A(x) = q_X(Ax)$. We prove that completeness and the FK-structure pass from X to X_A, that X_A is (isometrically) isomorphic to X via the map $x \to Ax$, and that Schauder bases are preserved under passage to matrix domains. Finally, we describe the K¨othe-Toeplitz α-, β-, and γ-duals of X_A in terms of the corresponding duals of X and the inverse triangle A^{-1}, and we specialize the general results to N¨orlund matrices arising from classical integer sequences.

Keywords: Paranormed sequence spaces, variable exponents; matrix domains, triangles, Norlund matrices; Schauder bases, Kothe-Toeplitz duals

1. Introduction
Matrix transformations and matrix domains are a standard and powerful tool in sequence space theory and summability. Given a sequence space $X \subseteq \omega$ and an infinite matrix $A = (a_{nk})$, one may encode additional analytic structure by passing from X to the *matrix domain*

$X_A := \{x \in \omega : Ax \in X\}$.

When A is a triangle (lower triangular with nonzero diagonal entries), the map $x \to Ax$ is invertible on ω and often transports structural features of X (completeness, bases, dual descriptions) to the new space X_A.

In this paper we focus on paranormed sequence spaces of Maddox type (with variable exponents) and on triangles generated by admissible weight sequences through a Norlund construction. Maddox introduced and developed several variable-exponent sequence spaces and their basic properties [1, 2]. Since then, many authors studied matrix domains and related duality/basis questions for paranormed (and non-absolute type) sequence spaces; see for instance [3-7] and references therein.

Our goals are:
- to present a unified construction of Norlund-type triangles from admissible sequences (including Fibonacci, Pell, Motzkin, Catalan weights);
- to develop topological and linear-structural results for matrix domains X_A of paranormed FK-spaces X under triangles A;
- to prove preservation of Schauder bases under triangular matrix domains;
- to give systematic descriptions of Kothe-Toeplitz duals of X_A via the inverse matrix A^{-1}.

1.1 Preliminaries and notation
Throughout, $\omega = \omega$ denotes the space of all complex sequences $x = (x_k)_{k \geq 0}$, and all linear spaces are over C.

1.2 Triangles and matrix transformations
An infinite matrix $A = (a_{nk})_{n,k \geq 0}$ acts (formally) on $x \in \omega$ by

$(Ax)_n = \sum_{k=0}^{\infty} a_{nk} x_k, n \geq 0,$

whenever each row sum is meaningful. A matrix is called a *triangle* if $a_{nk} = 0$ for $k > n$ and $a_{nn} \neq 0$ for all $n \geq 0$. For triangles, each row sum is finite, so Ax is defined for all $x \in \omega$.

2. Paranormed FK-spaces

We use the following standard terminology.

Definition 2.1: (Paranorm, FK-space)**.** Let X be a linear space. A map $q: X \to [0,\infty)$ is a *paranorm* if:

- $q(x) = 0 \Longleftrightarrow x = 0$,
- $q(-x) = q(x)$ for all $x \in X$,
- $q(x + y) \leq q(x) + q(y)$ for all $x,y \in X$,
- if $\alpha_n \to 0$ in C, then $q(\alpha_n x) \to 0$ for each fixed $x \in X$.

A paranormed space (X,q) is an *FK-space* (a Fr´echet coordinate space) if it is complete with respect to the metric $d(x,y) = q(x-y)$ and each coordinate functional $p_k(x) = x_k$ is continuous on X (when $X \subseteq \omega$).
Köthe-Toeplitz α-, β-, γ- duals

Definition 2.2: (Kothe-Toeplitz duals)**.** Let $X \subseteq \omega$ be a sequence space and define $\langle x,y\rangle$: $= \sum_{k=0}^{\infty} x_k y_k$ whenever the series converges.

(i) The *α-dual* is

$$X^{\alpha} := \left\{ y \in \omega : \sum_{k=0}^{\infty} |x_k y_k| \cdot \right.$$

$<\infty$ for all $x \in X$.

(ii) The *β-dual* is

$$X^{\beta} := \left\{ y \in \omega : \sum_{k=0}^{\infty} x_k y_k \right.$$

converges for all $x \in X$.

(iii) The *γ-dual* is

$$X^{\gamma} := \left\{ y \in \omega : \left(\sum_{k=0}^{n} x_k y_k \right)_{n \geq 0} \right.$$

is bounded for all $x \in X$.
Remark 2.3. Always $X^{\alpha} \subseteq X^{\beta} \subseteq X^{\gamma}$.

3. Admissible sequences and Norlund matrices

Admissible sequences

Definition 3.1: (Admissible sequence)**.** Let $u = (u_n)_{n\geq 0}$ be a sequence of positive real numbers and define

$U_n := \sum_{k=0}^{n} u_k \quad n \geq 0.$

We call u *admissible* if $u_n > 0$ for all n and $U_n \to \infty$ as $n \to \infty$.

Example 3.2: (Fibonacci, Pell, Motzkin, Catalan)**.** Each of the following yields an admissible sequence u:

- Fibonacci weights: $u_n = F_{n+1}$ where $F_0 = 0$, $F_1 = 1$, $F_{n+2} = F_{n+1} + F_n$.
- Pell weights: $u_n = P_{n+1}$ where $P_0 = 0$, $P_1 = 1$, $P_{n+2} = 2P_{n+1} + P_n$.
- Motzkin weights: $u_n = M_n$ where $M_0 = 1$, $M_1 = 1$ and $M_{n+1} = M_n + \sum_{k=0}^{n-1} M_k M_{n-1-k}$ for $n \geq 1$.
- Catalan weights: $u_n = C_n = \frac{1}{n+1} \binom{2n}{n}$.

Nörlund matrices

Definition 3.3: (Nörlund matrix)**.** Let u be admissible and $U_n = {}^{\mathrm{P}n}{}_{k=0}\, u_k$. The *Nörlund matrix* $A^{(u)} = (a_{nk}^{(u)})_{n,k\geq 0}$ generated by u is defined by

$$a_{nk}^{(u)} := \begin{cases} \frac{u_{n-k}}{U_n}, & 0 \le k \le n, \\ 0, & k > n. \end{cases}$$

For $x \in \omega$ we have

$$(A^{(u)}x)_n = \sum_{k=0}^{n} a_{nk}^{(u)} x_k = \frac{1}{U_n} \sum_{k=0}^{n} u_{n-k} x_k,$$

a weighted mean of $x_0,\dots,x_n$.

Proposition 3.4: *Let u be admissible and $A^{(u)}$ be as in Definition 3.3. Then:*

(i) *$A^{(u)}$ is a triangle with strictly positive diagonal entries* $a_{nn}^{(u)} = u_0/U_n > 0$. *Hence it is invertible on ω and its inverse $B^{(u)} = (A^{(u)})^{-1}$ is again a triangle. (ii) Each row sums to 1: for all $n \geq 0$,*

n
X (u)
$a_{nk} = 1.$
$k = 0$

Proof. (i) By definition $a_{nk}^{(u)} = 0$ for $k > n$, and $a_{nn}^{(u)} = u_0/U_n > 0$. A triangle with nonzero diagonal is invertible on ω, with inverse obtained recursively from $A^{(u)}B^{(u)} = I$. (ii) For fixed n,

$$\sum_{k=0}^{n} a_{nk}^{(u)} = \frac{1}{U_n} \sum_{k=0}^{n} u_{n-k} = \frac{1}{U_n} \sum_{j=0}^{n} u_j = 1.$$

Proposition 3.5: *Let u be admissible. Then:*

(i) *$A^{(u)}$ defines a bounded operator on $(\ell_\infty, \|\cdot\|_\infty)$ with operator norm* 1.
(ii) *$A^{(u)}$ maps c into c and c_0 into c_0.*

Proof. (i) Let $x \in \ell_\infty$ and $M = \|x\|_\infty$. Then

$$|(A^{(u)}x)_n| \le \frac{1}{U_n} \sum_{k=0}^{n} u_{n-k}|x_k| \le \frac{M}{U_n} \sum_{k=0}^{n} u_{n-k} = M,$$

so $\|A^{(u)}x\|_\infty \le \|x\|_\infty$. Also $A^{(u)}1 = 1$, so the norm is exactly 1.

(ii) If $x \in c$ with $\lim x_k = L$, then

$$(A^{(u)}x)_n - L = \frac{1}{U_n} \sum_{k=0}^{n} u_{n-k}(x_k - L).$$

Fix $\varepsilon > 0$ and choose N with $|x_k - L| < \varepsilon$ for $k \geq N$. Split the sum into $k < N$ and $k \geq N$. The tail part is bounded by ε, while the finite part is bounded by C/U_n for some constant C, hence goes to 0 since $U_n \to \infty$. Thus $(A^{(u)}x)_n \to L$. The case $x \in c_0$ is the special case $L = 0$.

4. Maddox-type paranormed spaces and matrix domains

4.1: Maddox-type variable exponent spaces

Let $p = (p_k)_{k\geq 0}$ be a sequence of positive reals with

$$0 < H_1 := \inf_{k\geq 0} p_k \le \sup_{k\geq 0} p_k =: H_2 < \infty,$$

and fix $M \geq \max\{1, H_2\}$.

Definition 4.2: (Maddox-type spaces)**.** Define:

(i) $c_0(p) := \{x \in \omega : |x_k|^{p_k} \to 0\}, \qquad q_{c_0(p)}(x) := \sup_{k\geq 0} |x_k|^{p_k/M}.$

(ii) $c(p) := \{x \in \omega : \exists \ell \in \mathbb{C},\ |x_k - \ell|^{p_k} \to 0\}, \quad q_{c(p)}(x) := \inf_{\ell\in\mathbb{C}} \sup_{k\geq 0} |x_k - \ell|^{p_k/M}.$

(iii) $\ell_\infty(p) := \left\{x \in \omega : \sup_{k\geq 0} |x_k|^{p_k/M} < \infty\right\}, \qquad q_{\ell_\infty(p)}(x) := \sup_{k\geq 0} |x_k|^{p_k/M}.$

(iv) $\ell(p) := \left\{x \in \omega : \sum_{k=0}^{\infty} |x_k|^{p_k} < \infty\right\}, \qquad q_{\ell(p)}(x) := \left(\sum_{k=0}^{\infty} |x_k|^{p_k}\right)^{1/M}.$

Remark 4.2. If $p_k \equiv p$ is constant, these spaces reduce (up to equivalent paranorms) to classical spaces: $c_0(p) = c_0$, $c(p) = c$, $\ell_\infty(p) = \ell_\infty$, $\ell(p) = \ell_p$.
It is known that these are complete paranormed FK-spaces with continuous coordinate functionals; see [1, 2, 3, 5, 7].

Matrix domains
Definition 4.3: (Matrix domain)**.** Let $(X,q_X) \subseteq \omega$ be a paranormed FK-space and let A be a triangle. Define
$X_A := \{x \in \omega : Ax \in X\}, \quad q_A(x) := q_X(Ax).$
Remark 4.4. Even if X is of absolute type, X_A is typically of non-absolute type when A has nontrivial off-diagonal entries, since membership depends on linear combinations in Ax and may involve cancellations.

Definition 4.4: For $\lambda \in \{c_0,c,\ell_\infty,\ell\}$ and $X = \lambda(p)$, set
$\lambda(A,p) := X_A = \{x \in \omega : Ax \in \lambda(p)\}$, $q_{\lambda(A,p)}(x) := q_{\lambda(p)}(Ax)$.
If $A = A^{(u)}$ is a Nörlund matrix, we also write $\lambda^{(u)}(p) := \lambda(A^{(u)},p)$.

5. Topological structure of triangular matrix domains
Theorem 5.1: (FK-structure and completeness)**.** *Let* $(X,q_X) \subseteq \omega$ *be a complete paranormed FK-space and let A be a triangle. Then* (X_A,q_A) *is a complete paranormed FK-space.*
Proof. Linearity of X_A and the paranorm properties of $q_A(x) = q_X(Ax)$ follow from linearity of A and the paranorm axioms of q_X.
Let $(x^{(m)})$ be Cauchy in (X_A,q_A). Then $(Ax^{(m)})$ is Cauchy in X since
$qX(Ax(m) - Ax(r)) = qA(x(m) - x(r)) \to 0.$
By completeness of X, there exists $y \in X$ with $Ax^{(m)} \to y$ in X.
Since A is a triangle, the equation $Az = y$ has a unique solution $z \in \omega$ obtained by back substitution, and $z \in X_A$ because $Az = y \in X$. Finally,
$q_A(x^{(m)} - z) = q_X(Ax^{(m)} - Az) = q_X(Ax^{(m)} - y) \to 0,$
so $x^{(m)} \to z$ in X_A. Hence X_A is complete.
To see continuity of coordinate functionals on X_A, note that A^{-1} is a triangle $B = (b_{nk})$, and for $x \in X_A$ we have $x = BAx$. Thus for each k,

$$\text{xk} = \sum_{I=0}^{k} bkj(Ax)j,$$

a finite linear combination of continuous coordinate functionals on X composed with the continuous map $x \to Ax$. Hence each $x \to x_k$ is continuous on X_A. Therefore X_A is an FK-space.

Proposition 5.2: (Isometric isomorphism)**.** *Let* (X,q_X) *and A be as in Theorem 5.1. Then*

$T_A : (X_A,q_A) \to (X,q_X)$, $T_A(x) = Ax$,

is a linear bijective isometry. In particular, X_A *and X are linearly homeomorphic.*
Proof. By definition, T_A is linear and well-defined, and $q_X(T_Ax) = q_X(Ax) = q_A(x)$, so it is an isometry into X. Since A is invertible on ω with inverse triangle $B = A^{-1}$, for each $y \in X$ we have $x := By \in \omega$ and $Ax = y$, hence $x \in X_A$ and $T_A(x) = y$. Thus T_A is surjective. Injectivity follows from invertibility of A. Therefore T_A is a bijective isometry.

Corollary 5.3: *Let* $\lambda \in \{c_0,c,\ell_\infty,\ell\}$*, let p satisfy* $0 < H_1 \leq p_k \leq H_2 < \infty$*, and let A be any triangle (in particular a N¨orlund matrix* $A^{(u)}$*). Then* $\lambda(A,p)$ *is linearly isomorphic (indeed, isometric via* $x \to 7\ Ax$*) to* $\lambda(p)$.
Proof. Apply Proposition 5.2 with $X = \lambda(p)$.

Lemma 5.4: (Inclusion via relative boundedness)**.** *Let* (X,q_X) *be a complete paranormed space and let A,B be triangles. Assume that* $C := BA^{-1}$ *defines a bounded linear operator on X (i.e. there is* $K > 0$ *such that* $q_X(Cz) \leq Kq_X(z)$ *for all* $z \in X$*). Then* $X_A \subseteq X_B$ *and*
$q_B(x) \leq K\, q_A(x) \quad (x \in X_A).$
Proof. If $x \in X_A$, set $z := Ax \in X$. Then $Bx = BA^{-1}z = Cz \in X$, so $x \in X_B$. Moreover,
$q_B(x) = q_X(Bx) = q_X(Cz) \leq Kq_X(z) = Kq_X(Ax) = Kq_A(x).$

6. Schauder bases in matrix domains
Theorem 6.1: (Basis transfer)**.** *Let* $(X,q_X) \subseteq \omega$ *be a paranormed FK-space that admits a Schauder basis* $(u^{(n)})_{n\geq 0}$ *with continuous*

coordinate functionals. Let A be a triangle and X_A its matrix domain. Then the sequence

$$e_A^{(n)} := A^{-1}u^{(n)}, \qquad n \geq 0,$$

is a Schauder basis for X_A, and its coordinate functionals are continuous.
Proof. Let $T_A : X_A \to X$ be the isometric isomorphism $T_A(x) = Ax$. For any $x \in X_A$, put $y := Ax \in X$. Since $(u^{(n)})$ is a Schauder basis of X, there are unique scalars (α_n) such that

$$y = \sum_{n=0}^{\infty} \alpha_n u^{(n)} \text{ in } X.$$

Applying A^{-1} gives

$$x = A^{-1}y = \sum_{n=0}^{\infty} \alpha_n A^{-1}u^{(n)} = \sum_{n=0}^{\infty} \alpha_n e_A^{(n)}.$$

If $S_N = \sum_{n=0}^{N} \alpha_n e_A^{(n)}$, then $AS_N = \sum_{n=0}^{N} \alpha_n u^{(n)} \to y$ in X, hence

$$q_A(x - S_N) = q_X(Ax - AS_N) = q_X\left(y - \sum_{n=0}^{N} \alpha_n u^{(n)}\right) \to 0.$$

Uniqueness of coefficients follows by applying A and using uniqueness in X.
For coordinate functionals, if φ_n are the continuous coefficient maps on X associated with $(u^{(n)})$, define $\psi_n(x) := \varphi_n(Ax)$. Then ψ_n is continuous on X_A and returns the coefficient of $e_A^{(n)}$ in the expansion of x.

Corollary 6.2: (Column basis for Maddox-type spaces)**.** *Let $X \in \{c_0(p), c(p), \ell_\infty(p)\}$, and let A be a triangle with inverse $B = (b_{nk})$. Then the columns of B form a Schauder basis of X_A:*

$e_B^{(n)} := (b_{kn})_{k\geq 0}, \quad n \geq 0.$

Proof. For $X \in \{c_0(p), c(p), \ell_\infty(p)\}$ the canonical unit vectors $(e^{(n)})$ form a Schauder basis. Apply Theorem 6.1 with $u^{(n)} = e^{(n)}$ and note that $A^{-1}e^{(n)}$ is exactly the nth column of B.

7. Kothe-Toeplitz duals of matrix domains

Let A be a triangle and $B = A^{-1} = (b_{nk})$.
For $y \in \omega$, define the sequence $yB \in \omega$ by

$$(yB)_n := \sum_{k=0}^{\infty} y_k b_{kn} = \sum_{k=n}^{\infty} y_k b_{kn}, \qquad n \geq 0, \quad (1)$$

where the sum is finite for each fixed n since $b_{kn} = 0$ when $k < n$.

Lemma 7.1: (Pairing transfer)**.** *Let $X \subseteq \omega$ be any sequence space, let A be a triangle, and let X_A be the matrix domain. Then for every $x \in X_A$ and $y \in \omega$,*

$\langle x,y\rangle = \langle Ax,yB\rangle,$

whenever either side is absolutely convergent (hence the rearrangements below are valid). Proof. Let $x \in X_A$ and set $z := Ax \in X$. Since $x = Bz$ and B is lower triangular,
$x_k = \sum_{n=0}^{k} b_{kn} z_n.$

Hence

$$\langle x, y\rangle = \sum_{k=0}^{\infty} x_k y_k = \sum_{k=0}^{\infty}\sum_{n=0}^{k} b_{kn} z_n y_k = \sum_{n=0}^{\infty}\left(\sum_{k=n}^{\infty} y_k b_{kn}\right) z_n = \sum_{n=0}^{\infty} z_n (yB)_n = \langle z, yB\rangle = \langle Ax, yB\rangle,$$

where we used the definition of yB in (1).

Theorem 7.2: (Duals via the inverse matrix)**.** *Let $X \subseteq \omega$ be a sequence space and let A be a triangle with inverse $B = A^{-1}$. Then:*

$(X_A)^\alpha = \{y \in \omega : yB \in X^\alpha\}$,
$(X_A)^\beta = \{y \in \omega : yB \in X^\beta\}$, $(X_A)^\gamma = \{y \in \omega : yB \in X^\gamma\}$.

Proof. We show the α-case; the others are analogous.
Assume $y \in (X_A)^\alpha$. Let $z \in X$ be arbitrary and write $x := Bz \in X_A$ (since $Ax = z$). Then by Lemma 7.1,

$$\sum_{n=0}^{\infty} |z_n(yB)_n| = \sum_{n=0}^{\infty} |(Ax)_n(yB)_n| = \sum_{k=0}^{\infty} |x_k y_k| < \infty,$$

so $yB \in X^\alpha$.

Conversely, suppose $yB \in X^\alpha$ and take any $x \in X_A$ with $z := Ax \in X$. Again by Lemma 7.1,

$$\sum_{k=0}^{\infty} |x_k y_k| = \sum_{n=0}^{\infty} |z_n(yB)_n| < \infty,$$

so $y \in (X_A)^\alpha$. This proves the first identity. The β-case replaces absolute convergence by convergence; the γ-case uses boundedness of partial sums and the same transfer identity.

8. Specialized matrix domains from Fibonacci, Pell, Motzkin, Catalan weights
Fix $\lambda \in \{c_0, c, \ell_\infty, \ell\}$ and an exponent sequence p as in Definition 4.1. For an admissible weight sequence u, define $\lambda^{(u)}(p) := \lambda(A^{(u)}, p) = \{x \in \omega : A^{(u)}x \in \lambda(p)\}$.

Then all general results above apply with $A = A^{(u)}$.

Example 8.1: (Fibonacci-generated domains)**.** Let $u_n = F_{n+1}$ (Example 3.2(a)) and $A^{(F)} := A^{(u)}$. Then

$$(A^{(F)}x)_n = \frac{1}{\sum_{k=1}^{n+1} F_k} \sum_{k=0}^{n} F_{n-k+1} x_k$$

$$x \in c_0^{(F)}(p) := c_0(A^{(F)}, p) \text{ iff } |(A^{(F)}x)_n|^{p_n} \to 0.$$

And

Example 8.2: (Motzkin-generated domains)**.** Let $u_n = M_n$ (Example 3.2(c)) and $A^{(M)} := A^{(u)}$. Then

$$(A^{(M)}x)_n = \frac{1}{\sum_{k=0}^{n} M_k} \sum_{k=0}^{n} M_{n-k} x_k$$

$$x \in \ell^{(M)}(p) := \ell(A^{(M)}, p) \text{ iff } \sum_{n=0}^{\infty} |(A^{(M)}x)_n|^{p_n} < \infty.$$

and
Remark 8.3. By Proposition 5.2, each $\lambda^{(u)}(p)$ is (isometrically) isomorphic to $\lambda(p)$ via $x \mapsto A^{(u)}x$. Hence properties such as separability, existence of Schauder bases (when available), and dual formulas transfer from $\lambda(p)$ to $\lambda^{(u)}(p)$, while the concrete membership test is encoded by the specific weights u (Fibonacci, Pell, Motzkin, Catalan, etc.).

9. Conclusion
We provided a general framework for matrix domains X_A of paranormed FK-spaces under triangles, emphasizing Norlund matrices generated by admissible combinatorial sequences. The key structural features completeness, FK-property, basis transfer, and dual descriptions are stable under passage to such matrix domains, and the inverse triangle A^{-1} provides the natural mechanism for transporting duality. These results serve as a convenient platform for further extensions, e.g. Orlicz-paranormed matrix domains and classes of matrix mappings between different domains.

References
1. Maddox IJ. Spaces of strongly summable sequences. Quarterly Journal of Mathematics (Oxford Series). 1967;18(1):345-355. doi:10.1093/qmath/18.1.345.
2. Maddox IJ. Paranormed sequence spaces generated by infinite matrices. Proceedings of the Cambridge Philosophical Society. 1968;64(2):335-340. doi:10.1017/S0305004100042894.
3. Başar F, Altay B. Matrix mappings on the space bs(p) and its α-, β- and γ-duals. Aligarh Bulletin of Mathematics. 2002;21(1):79-91.
4. Altay B, Başar F. On the paranormed Riesz sequence spaces of non-absolute type. Southeast Asian Bulletin of Mathematics.

2003;26(5):701-715.
5. Altay B, Başar F. Some paranormed sequence spaces of non-absolute type derived by weighted mean. Journal of Mathematical Analysis and Applications. 2006;319(2):494-508. doi:10.1016/j.jmaa.2005.06.055.
6. Choudhary B, Mishra SK. On Köthe-Toeplitz duals of certain sequence spaces and their matrix transformations. Indian Journal of Pure and Applied Mathematics. 1993;24(5):291-301.
7. Başar F, Yeşilkayagil M. A survey for paranormed sequence spaces generated by infinite matrices. TWMS Journal of Pure and Applied Mathematics. 2019;10(1):3-38.