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Comparative analysis of 2-out-of-2 g system with single cold standby and arrival time of the server

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Abstract
This paper analyzed a G system of 2-out of-2 with a single cold standby identical unit. A single server is offered to provide the services after some arrival time to rectify the faults. The unit is replaced by new one if the repair is not possible by the server in a given pre-specific time (maximum allowed time). The time for failure and when unit is tested for replacement, repair time and also for arrival are taken as exponential and rayleigh distribution respectively. The concept of base state with fuzziness and using it while applying RPGT for the profit analysis of the system. To allocate the behavior of some key parameters of the system to check the efficacy of the system model under particular situations is shown graphically.

Keywords: 2-Out-of-2: G System, fuzziness measure, maximum allowed time, regenerative point graphical technique

1. Introduction

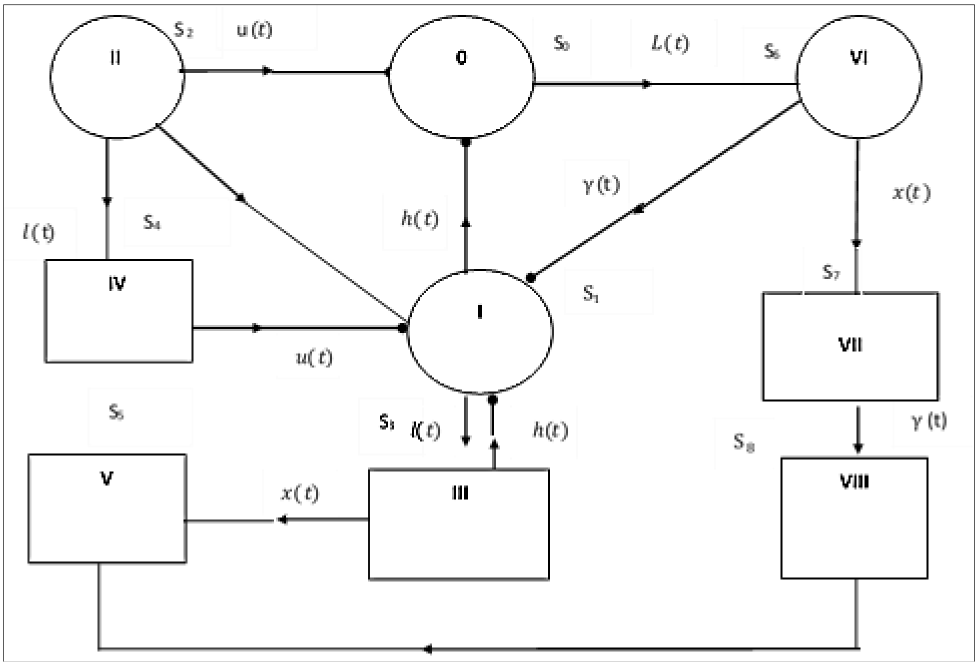
In the field of reliability engineering, numerous studies have explored different reliability aspects and stochastic behaviors of systems. Engineers and system designers have achieved notable improvements in system performance through the application of redundancy techniques and efficient repair strategies. Among these, the cold standby redundancy approach has been recognized as one of the most effective methods, as it enhances system reliability without altering the reliability of individual components. Furthermore, most system models developed by researchers are based on the assumption that a repair server is immediately available to address any faults that occur within the system analyzed system models with redundancy and immediate visit of the server. However, this assumption appears to be unrealistic in practical scenarios, as the server may not always be able to reach the system immediately possibly due to being occupied with previously assigned tasks. In such cases, a certain delay in the server's arrival at the system is expected. Barak and Malik (2014) [11] have proposed reliability models of a standby system with arrival time of the server and maximum repair time. Further, there may be the situation that a server cannot repair the failed unit in a given maximum repair time. In such a situation either the server may be replaced by an expert or the unit may be replaced by new one in order to avoid unnecessary expenses on repair and also to reduce the downtime. Gnedenko, B. V., & Ushakov, I. A. (2022) [2] carried out Sensitivity Analysis of a k-out-of-n redundant system. It can also be noted here that in most of the studies referred to the subject reliability the exponential distribution has been considered for different random variables associates with failure and repair times.

This paper explores a notable characteristic of 2-out-of-2;G systems composed of max repair time and arrival time using exponential and Rayleigh distribution, Each unit in the system operates in one of two states: operational or completely failed. The system is considered to be in an operational (up) state as long as at least two units are functioning. A single server is assigned to perform repair activities and requires some time to reach the system after a failure occurs. If the server is unable to repair the failed unit within a predefined duration (referred to as the maximum allowed time), the unit is replaced with a new one after a certain replacement period. The random variables corresponding to failure time, arrival time, repair time, and replacement time are statistically independent. After repair, a unit functions as good as new, with its lifetime restored to that of the original. Analytical expressions for key reliability and economic indicators such as transition probabilities, mean sojourn times, mean time to system failure (MTSF), system availability, server busy period due to repair and replacement, Expected number of unit replacements, and profit function are obtained in the steady state using the semi-Markov process and regenerative point graphical technique. The performance of these measures is examined for different parameter values and cost conditions under specific cases of the exponential and Rayleigh distribution. A comparative analysis of several important reliability measures between both distributions has also been presented.

2. Notations

Table 1: Notations and Descriptions

o/Cs	The unit is in operative/ cold standby mode
ε	Constant failure rate.
h(t)/H(t)	Pdf/cdf of Repair time.
l(t)/L(t)	pdf/cdf of Failure time
γ(t)/Υ(t)	Pdf/cdf of Arrival time.
u(t)/U(t)	Pdf/cdf of Replacement time.
$x(t)/X(t)$	The rate for which unit undergoes for replacement.
$\dot{q_{ij}}(t)/\dot{Q_{ij}}(t)$	Pdf/cdf of direct transition time from a regenerative state regenerative state (0,t].
$\dot{q_{ij.k}}(t)$ $/\dot{Q_{ij.k}}(t)$	pdf/cdf of first passage time for a regenerative state S_i to regenerative state S_j or to failed state S_j visiting state S_k once in (0,t]
δ_i	The mean sojourn time spent in state $S_i \in E$ before transition to any other state
δ_i'	The total unconditional time spent in state before transition to any other regenerative state given that the system entered regenerative state i at time t=0
f_i	Fuzziness measure of the i-state
n_i	Expected time spend while doing a job,given that the system entered regenerative state i at time t=0
FU_r/FUR	Unit is failed under repair/ under repair continuously from previous state.
FW_r/FWR	Unit is failed and waiting for repair/ waiting for repair continuously from previous state
$Furp/FURP$	Unit is failed and under Replacement/ under Replacement continuously from previous state
$'$	Derivative of function
Pdf/cdf	probability density function/cumulative density function



The possible transition states of the system models are shown in figure 1.10.
Stage 0 =(o,o):cs); Stage I=(o,o): Fur) ; Stage II= (o,o):FUrp);
Stage III= (Wo,FWr,: FUr); Stage IV= (Wo,FWr, :FURP);
State V= (Wo, FUrp,:FWR);StageVI=(o,o):FWr);
StageVII=(Wo, FWr, :FWR); StageVIII:(WO, FWR,:FUr)

Fig 1.10: State Transition Diagram

$x(t)$

3. Transition Probabilities

Simple probabilistic considerations yield the following expressions for the non-zero elements

$$p_{ij} = Q_{ij}(\infty) = \int_0^\infty q_{ij}(t)dt\ as$$

$q_{ij}(t)$	$p_{ij} = q_{ij}^{*}(0)$
$p_{06} = \int_0^\infty l(t)dt$	$l(t) = 2\varepsilon t^b e^{-2\varepsilon \frac{t^{b+1}}{b+1}}; b = 0,1$
$p_{10} = \int_0^\infty h(t)\overline{L(t)}\overline{X(t)}dt$	$x(t) = \emptyset e^{-\emptyset \frac{t^{b+1}}{b+1}}; b = 0,1$
$p_{12} = \int_0^\infty x(t)\overline{L(t)}\overline{H(t)}dt$	$u(t) = \beta e^{-\beta \frac{t^{b+1}}{b+1}}; b = 0,1$
$p_{13} = \int_0^\infty l(t)\overline{X(t)}\overline{H(t)}dt$	$h(t) = \omega e^{-\omega \frac{t^{b+1}}{b+1}}; b = 0,1$
$p_{20} = \int_0^\infty u(t)\overline{L(t)}dt$	$\gamma(t) = \rho e^{-\rho \frac{t^{b+1}}{b+1}}; b = 0,1.$
$p_{24} = \int_0^\infty l(t)\overline{X(t)}dt$	$p_{61} = \int_0^\infty \gamma(t)\overline{L(t)}dt$
$p_{31} = \int_0^\infty h(t)\overline{X(t)}dt$	$p_{67} = \int_0^\infty l(t)\overline{\Gamma(t)}dt$
$p_{35} = \int_0^\infty x(t)\overline{H(t)}dt$	$p_{78} = \int_0^\infty \gamma(t)dt$
$p_{41} = p_{51} = \int_0^\infty u(t)dt$	$p_{81} = \int_0^\infty h(t)\overline{X(t)}dt$
	$p_{85} = \int_0^\infty x(t)\overline{H(t)}dt$

It is clear that Summations of all the terms of p_{ij}in each box is equal to 1.

4. Mean Sojourn Times

The mean sojourn times (δ_i) in the state S_i are

$$\delta_0 = \int_0^\infty P(T > t)dt = \sum_{\substack{i=0 \\ j=6}} m_{ij} \;\; \delta_1 = \sum_{\substack{i=1 \\ j=0,2,3}} m_{ij},$$

$$\delta_2 = \sum_{\substack{i=2 \\ j=0,4}} m_{ij} \;\; \delta_6 = \sum_{\substack{i=6 \\ j=1,7}} m_{ij}$$

$$\delta_1' = \sum_{\substack{i=1 \\ j.k=0,2.1.3,3.5}} m_{ij.k} \;\; \delta_2' = \sum_{\substack{i=2 \\ j.k=0,1.4}} m_{ij.k} \;\; \delta_6' = \sum_{\substack{i=6 \\ j.k=1,1.78,1.785}} m_{ij.k}$$

5. MTSF (Mean Time to System Failure)

The regenerative un-failed states to which the system can transit before entering any failed state are i=0, 1,2and 6.
The mean time to system failure (MTSF) is given by

$$MTSF = \frac{(0-0)\delta_0+(0-6-1)\delta_1+(0-6-1-2)\delta_2+(0-6)\delta_6}{1-(0-6-1-0)-(0-6-1-2-0)} = \frac{\delta_0+p_{61}(\delta_1+p_{12}\delta_2)+\delta_6}{1-p_{61}(p_{10}+p_{20}p_{12})}$$

6. Availability (Steady state)

The regenerative state at which system is available are i=0,1,2,6 and j=0,1,2,6 .Base State:=1

$$A_0 = \frac{(1-0)f_0\delta_0 + (1-1)f_1\delta_1 + (1-2)f_2\delta_2 + (1-6)f_6\delta_6}{(1-0)\delta_0 + (1-2-0)\delta_0 + (1-1)\delta_1' + (1-2)\delta_2' - \{(1-0) + (1-2-0)\}\delta_6'}$$

$$= \frac{\{(1-0)+(1-2-0)\}\delta_0 + (1-1)\delta_1 + (1-2)\delta_2 + \{(1-0-6) + (1-2-0-6\}\delta_6}{(1-0)\delta_0 + (1-2-0)\delta_0 + (1-1)\delta_1' + (1-2)\delta_2' + \{(1-0-6) + (1-2-0-6)\}\delta_6'}$$

$={}^{N_1}/_{D_1}$ *where* $N_1 = (p_{10} + p_{12}p_{20})\delta_0 + \delta_1 + p_{12}\delta_2 + (p_{10} + p_{12}p_{20})\delta_6$; $D_1 = (p_{10} + p_{12}p_{20})\delta_0 + \delta_1' + p_{12}\delta_2' + (p_{10} + p_{12}p_{20})\delta_6'$

7. Busy Period (due to repair/replacement)

The regenerative state where the server is busy while doing repair /replacement are i=1, 2,

$$\mathrm{B}_0 = \mathrm{N}^1 \div D_1$$

$$\mathrm{N}^1 = (1-0-6-1)\eta_1^*(0) + (1-2)\eta_2^*(0) = p_{10}\, p_{61} W_1^*(0) + p_{12} W_2^*(0)$$

D_1 is specified earlier.

8. Expected Number of Visits of the Server

The regenerative state where the server visits (afresh) for the repair/replacement are i=1, 6

$$V_0 = N^2 \div D_1$$

$$N^2 = (1-0-6) + (1-3-1) + (1-0-6-7-8-5-1) + (1-2-0-6-7-8-1) + (1-2-0-6-1) + (1-3-5-1)$$

$$= p_{10} + p_{11,3} + p_{61.78}(p_{10} + p_{12}p_{20}) + p_{12} + p_{11.35}$$

D_1 is specified earlier.

9. Profit Analysis

Profit of Figure 1.10 is obtained as

$$\mathrm{P}_0 = \mathrm{C}_0\mathrm{A}_0 - \mathrm{C}_1\mathrm{B}_0 - \mathrm{C}_2\mathrm{V}_0$$

Where

C_0=Revenue per unit up-time of the system.

C_1=Cost per unit time for which server is busy due to replacement/repair.

C_2= Cost per unit time visit of the serviceman.

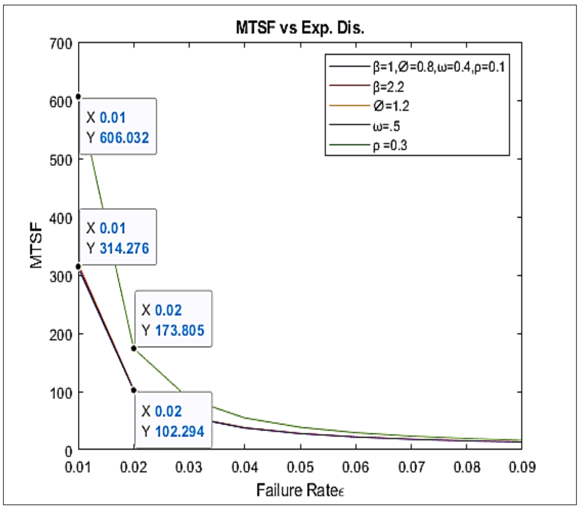


Fig 1.11: MTSF vs. exponential distribution

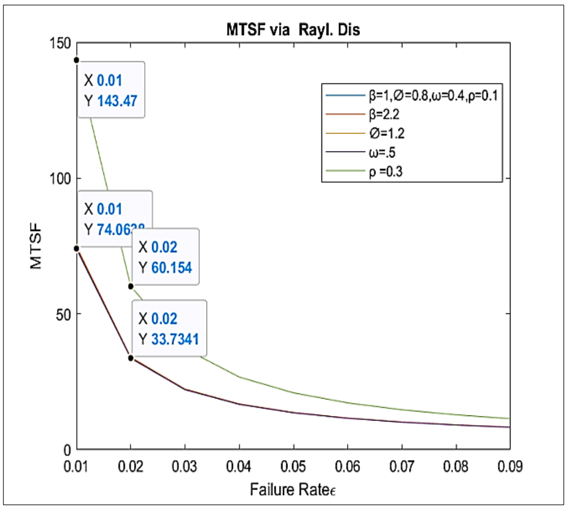
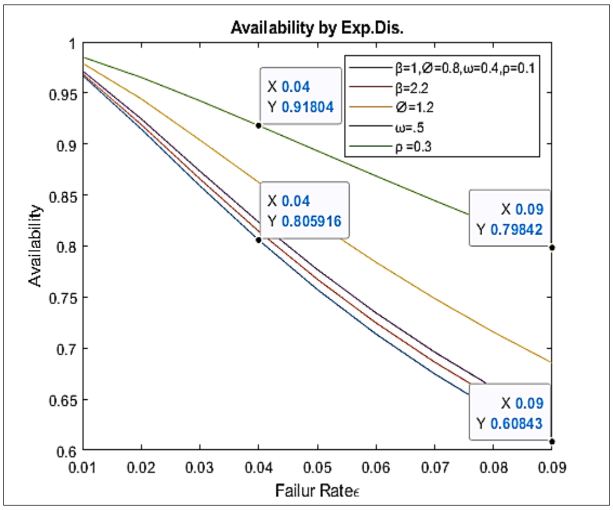


Fig 1.12: MTSF via Rayleigh distribution



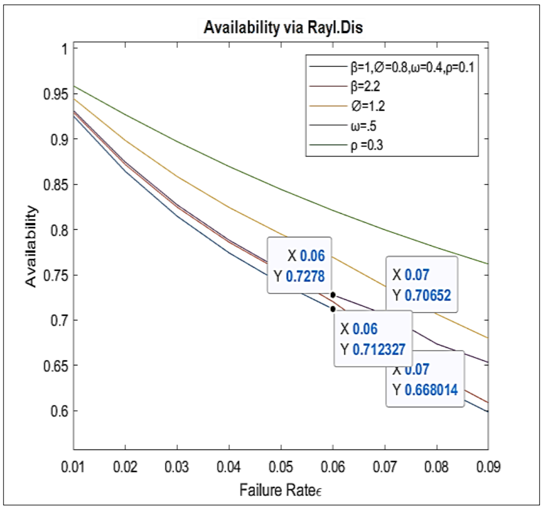
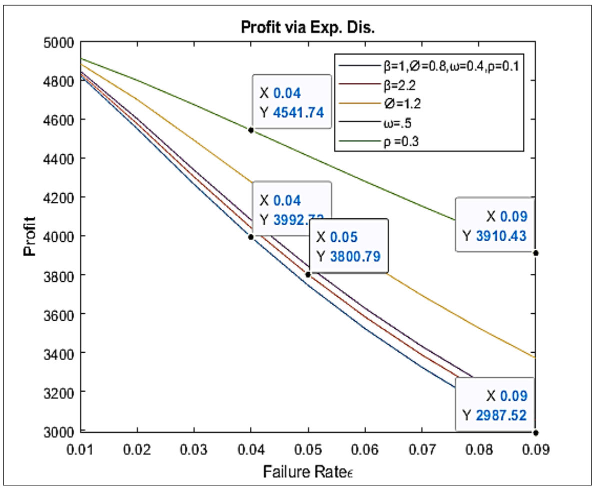


Fig 1.13: Availability by exponential distribution **Fig 1.14:** Availability via Rayleigh distribution



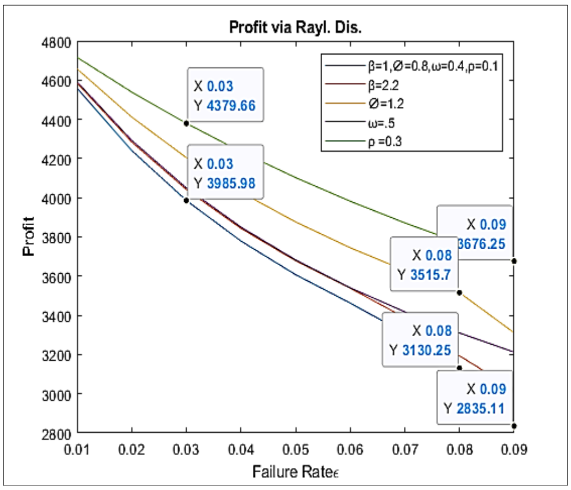


Fig 1.15: Profit via exponential distribution **Fig 1.16:** Profit via Rayleigh distribution

10. Conclusion

The graphical analysis of various parameters with respect to exponential and Rayleigh distributions, covering random variables such as failure time, repair time, replacement time, maximum repair time, and server arrival time is presented in Figures 1.11 to 1.16. Under the given system conditions, it is noteworthy that all reliability measures the Mean Time to System Failure (MTSF), availability and overall profit of the system follows indirect relation with Failure rate. While in comparative distribution, all three measures are significantly higher when the failure, repair, and replacement times of the units follow an exponential distribution rather than Rayleigh distribution. The specific points show in graph show maximum gap or drastic change in values.

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