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A note on: Stop loss and take profit using up-crossings

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Abstract
There are many trading strategies in foreign exchange market. These strategies have some pre-determined parameters such as stop loss and take profit thresholds. Determination of these parameters makes a tradeoff between risk and returns of trader. In this paper, using the up-crossing concept of stochastic process field, these parameters are determined. First, for return based up-crossings, mixture, limiting and exact distributions and related stop-loss and take profit strategies are derived. Then, another formulation using Ross method and optimal stopping are presented to study the price based up-crossings. Simulations are given and conclusions show the economic importance of this research.

Keywords: Exact distribution, foreign exchange, limiting and mixture distributions, optimal stopping, risk and return, stop loss, take profit; trading, up-crossing

1. Introduction
Foreign exchange (also known as Forex or FX) refers to the global, over-the-counter market (OTC) where traders, investors, institutions and banks, exchange speculate on, buy and sell world currencies. Trading is conducted over the ‘interbank market’, an online channel through which currencies are traded 24 hours a day, five days a week. FX is one of the largest trading markets, with a global daily turnover estimated to exceed US$5 trillion. All transactions made on the FX market involve the simultaneous purchasing and selling of two currencies. These are called ‘currency pairs’, and include a base currency and a quote currency. The display below shows the FX pair EUR/USD (Euro/US Dollar), one of the most common currency pairs used on the FX market.

There are many technical trading strategies in the literature that their performances have long been examined. For example, Brock *et al.* (1992) [4] compared the performances of buy-and-hold, moving average and trading range break strategies in the Dow Jones index. Profitability of technical trading rules in the stock markets of Malaysia, Thailand and Taiwan is studied by Bessembinder and Chan (1995) [2]. There has also been growing interest in nonlinear trading rules (see for example, Andrada-Felix *et al.*, 2003; Nam *et al.*, 2005) [1, 8]. Chong *et al.* (2015) [8] conduct similar analysis for the Chinese markets and find that most rules fail to produce significant returns, except for the SETAR (200) and MA (50) models during the pre-SOE reform period. Chong and Lam (2018) [6] investigated the synergy of combining SETAR (200) and MA (50) rules. They understand that SETAR (200) and MA (50) outperform other rules in the U.S. market. Important parameters of every trade are stop loss and take profit thresholds. Determination of these parameters makes a tradeoff between risk and returns of trader.

This paper uses the up-crossing tools of returns of price of financial asset to derive the stop loss and take profit parameters of trading rules. To this end, let s_t be the price of a financial asset such as stock at time $t \in [0,T]$. Suppose that s_0 is the initial value of price and it is observed in $0 = t_0 < t_1 < \cdots < t_n = T$ where $n < \infty$, and $\delta = t_i - t_{i-1}, i = 1,2,\dots,n$. Let $r_i = \frac{s_{t_i} - s_{t_{i-1}}}{s_{t_{i-1}}}$ be the i-th return. Define the hitting time (up-crossing) $\tau = \inf\{t, r_t > U\}$ for some pre-determined threshold U. Suppose that r_i's are independent and identically distributed random variables with common distribution function F. For example, when the price dynamic satisfies the Ito stochastic differential equation $ds = \mu s dt + \sigma s dB$ where the drift parameter is a real unknown number, volatility (diffusion) parameter is positive and B is standard Brownian motion, then, F is normal distribution $N(\mu\delta, \sigma^2\delta)$.

This paper is organized as follows. In the next section, the exact, limiting and mixture distributions for τ are given and based on these information stop loss and take profit limits are proposed. Price based up-crossings are studied via Ross method and optimal stopping methods in section 3. Simulation results and conclusions are given in section 4, 5.

2. Mixture, limiting and exact distributions. Here, exact, approximation, and limiting mixture distributions are fitted to τ.

(a) Mixture distribution: There are two cases: $\tau < \infty$ at which r_t passes the threshold U and the second case that $r_t, t = 1,2,\dots,n$ remains below the level U, that is $\tau = \infty$. The probability of this event is

$$P(\tau = \infty) = P(max_{1\le i\le n} r_i < U) = F^n(U).$$

Let $q = F(U)$. Then, $P(\tau = \infty) = q^n$ is the mixing parameter. In the case of $N(\mu\delta, \sigma^2\delta)$, then

$$P(\tau = \infty) = \Phi^n\left(\frac{U-\mu\delta}{\sqrt{\delta}\sigma}\right).$$

Indeed, If $\tau < \infty$, it is easy to see that

$$P(\tau > k|\tau < \infty) = \frac{P(max_{1\le i\le k} r_i \le U, max_{k+1\le i\le n} r_i > U)}{P(max_{1\le i\le n} r_i > U)} = \frac{q^k - q^n}{1 - q^n}.$$

Then,

$$P(\tau = k|\tau < \infty) = \frac{(1-q)q^{k-1}}{1-q^n}$$

That is, given $\tau < \infty$, then τ has a truncated geometric distribution on {1,2,…,n} with probability q. Thus, the distribution of τ is a mixture between truncated geometric distribution over {1,2,…,n} and event $\tau = \infty$ with mixing factor q^n. This is the exact distribution of τ. Notice that

$$E(\tau^k|\tau < \infty) = \sum_{i=1}^{n} i^k \frac{(1-q)q^{i-1}}{1-q^n}$$

which is computable by software such as Excel, although, closed forms for them may be obtained.

(b) Approximate technique: There is also an approximation to mixture distribution fitted to $\frac{\tau}{T}$. Indeed, with probability of q^n, series r_t do not pass threshold U (i.e., $\tau = \infty$) and if it passes (i.e., $\tau < \infty$), our simulation results show that the conditional distribution of $\frac{\tau}{T}$ given $\tau < \infty$ is well approximated by a beta distribution. To this end, let

$$E\left(\frac{\tau}{T}\middle|\tau < \infty\right) = m \text{ and } var\left(\frac{\tau}{T}\middle|\tau < \infty\right) = v$$

To fit beta distribution $b(\alpha, \beta)$ to $\frac{\tau}{T}$, it is enough to consider method of moment (MM) estimates of them given by $\hat{\alpha}_{MM} = c \times m$, $\hat{\beta}_{MM} = c \times (1 - m)$, where $c = \frac{m(1-m)}{v} - 1$. The values of m, v can be derived using the moment of geometric distribution.

(c) Limiting cases: As follows, the limiting distribution of τ, when $n \to \infty$ is studied. Actually, q depends to n. Let $-n \times \ln(q_n) \to \gamma > 0$ when $n \to \infty$. Then, for $x \in (0,1)$, we have $P(\tau > nx|\tau < \infty) \to \frac{e^{-\gamma x}-1}{e^{-\gamma}-1}$ which is a truncated exponential distribution on (0,1). Next, suppose that $\varphi = \inf\{t; r_t < L\}$. Again, it is easy to see that given $\varphi < \infty$ then φ has a truncated geometric distribution with mixing probability $1 - q'$ $(q' = F(L))$ on $\{1,2,\dots,n\}$. Also, φ/T has beta approximated distribution and as soon as $-n \times \ln(1 - q'_n) \to \gamma' > 0$, then the limiting distribution of $P(\varphi \le nx \,|\, \tau < \infty)$ has a truncated distribution on (0,1).

As follows, stop loss and take profit limits are derived using the above results. To this end, let $\hat{u}$ and $\hat{l}$ be the desired upper and lower bound for future movements of return, respectively. Then, let stop loss and take profit parameters be $(1-q^n)\hat{u}$ and $(1-(1-q')^n)\,\hat{l}$. The following proposition summarizes the above discussion.

Proposition 1. Let $\tau=\inf\{t, r_t>U\}$ and $\varphi=\inf\{t; r_t<L\}$. Then, (*1*), (*2*) are correct.

(1) Both τ and φ have mixture distributions on $\{1,2,\dots,n\}$. Given $\tau,\varphi<\infty$, then both have exact geometric, approximated beta and limiting exponential distributions.

(2) The stop loss and take profit strategies are given by $(1-q^n)\hat{u}$ and $(1-(1-q')^n)\,\hat{l}$, respectively, where $\hat{u}$ and $\hat{l}$ are the trader desired upper and lower bound for future movements of return**.**

3. Price based up-crossings. Hereafter, another formulation for the distribution of up-crossings are derived. In above sections, up-crossings are defined based on returns of financial asset. Here, price based up-crossings are studied. To this end, notice that $s_k>L$ implies that $s_k=s_0\prod_{i=1}^k(1+r_i)>L$. Hence, $\ln(s_k)=\ln(s_0)\sum_{i=1}^k\ln(1+r_i)>\ln(L)$. Since $\ln(1+r_i)\approx r_i\geq -1$ for small r_i's, then $s_k>L$ is equivalent to $\sum_{i=1}^k r_i>\ln(\frac{L}{s_0})$. Let $N(y)=\min\{k;\sum_{i=1}^k r_i>y\}$. As follows, two methods, namely Ross method and optimal stopping methods are proposed to promote this section.

(a) Ross method: Following Ross (2010) [9], page 342, example 5j and page 378, problem 7.62, notice that $p_k(y)=P(N(y)\geq k+1)=P(max_{1\leq i\leq k}\sum_{j=1}^i r_j\leq y)$. Assuming that r_i's are independent, and identically distributed, it is seen that

$$p_k(y)=\int_{-1}^{\infty}P(0,r_2,r_2+r_3,\dots,r_2+\dots+r_k\leq y-r)dF_{r_1}(r)$$

$$=\int_{-1}^{y}P(r_1,r_1+r_2,\dots,r_1+\dots+r_{k-1}\leq y-r)dF_{r_1}(r)$$

$$=E_{r_1}\big(p_k(y-r_1)1(r_1\leq y)\big).$$

This equation uses this fact that $(r_2,r_2+r_3,\dots,r_2+\dots+r_k)$ has the same distribution with $(r_1,r_1+r_3,\dots,r_1+\dots+r_k)$ and their maximum have the same distribution using the continuous mapping theorem for maximum function (see, Billingsley, 1999) [3]. In above equation, F_{r_1} is the distribution function of the first return, r_1, also, E_{r_1} means taking expectation with respect to r_1 and $1(r_1\leq y)$ is one if $r_1\leq y$ and zero otherwise. Also, notice that the above equation is achievable using a Monte Carlo simulation where as the initial value $p_1(y)=F_{r_1}(y)$. Numerical methods such as Spline procedure together the Monte Carlo simulation may be used to estimate the functional form of $p_k(y), k>1$. Also, assuming

$$p_k\to^{k\to\infty}p_\infty,$$

Then

$$p_\infty(y)=E_{r_1}\big(p_\infty(y-r_1)1(r_1\leq y)\big)$$

Which is an integral equation. To approximate p_∞, suppose that $p_\infty(y)\approx a+by+cy^2$, then using method of Hansen (2000) [7], it is seen that

$$a+by+cy^2=aP(r_1\leq y)+bE\big((y-r_1)1(r_1\leq y)\big)+cE\big((y-r_1)^2 1(r_1\leq y)\big).$$

Using the numerical methods and Monte Carlo simulations, the functional form of

$P(r_1\leq y), E\big((y-r_1)1(r_1\leq y)\big)$ and $E\big((y-r_1)^2 1(r_1\leq y)\big)$

are derived and then coefficients a,b,c are derived. The following proposition summarizes the above discussion.

Proposition 2: Let $p_k(y)=P(N(y)\geq k+1)$ and $N(y)=\min\{k;\sum_{i=1}^k r_i>y\}$. Then, assuming r_i's are independent, and identically distributed with common distribution function F_{r_1}, we have

1. $p_k(y)=E_{r_1}\big(p_k(y-r_1)1(r_1\leq y)\big), p_1(y)=F_{r_1}(y).$

2. Assuming $p_k \to^{k\to\infty} p_\infty$, then $p_\infty(y) = E_{r_1}\big(p_\infty(y - r_1)1(r_1 \le y)\big)$.

(b) Optimal stopping: Here, the optimal stopping technique based on dynamic programming solution is used to find the optimal stopping time at which $P(s_\tau > L)$ is maximized. Let $G_t = 1(s_t > L)$. Then, the stopping time τ is needed to find such that $max_\tau E(G_\tau)$ is attained. Tijms (2012) [11] applied this technique in stochastic games such as dice games and proposed dynamic programming solutions. In mathematics, the theory of optimal stopping is concerned with the problem of choosing a time to take a particular action, in order to maximize an expected reward or minimize an expected cost. Optimal stopping problems can be found in areas of statistics, economics, and mathematical finance (related to the pricing of American options). A key example of an optimal stopping problem is the secretary problem. Optimal stopping problems can often be written in the form of a Bellman equation, and are therefore often solved using dynamic programming (see, Shiryaev and Zhitlukhin, 2013, page 7) [10].

Suppose that the price is observed in n discrete times. The dynamic programming solution to above mentioned problem is given by

$$\pi_k = \max\big(G_k, E(\pi_{k+1}|G_k)\big), \pi_n = G_n.$$

It is easy to see that $1 - \pi_{n-1} = (1 - \delta_n)(1 - G_{n-1})$. Generally,

$$\pi_{n-k} = 1 - (1 - G_{n-k})\prod_{j=0}^{k-1}(1 - \delta_{n-j}),$$

$$\delta_{n-j} = P(s_{n-j} > L | s_{n-j+1} \le L).$$

Following Shiryaev and Zhitlukhin (2013) [10], the optimal stopping time is the first point at which $\pi_{n-k} = G_{n-k}$. Indeed, at first k where $\prod_{j=0}^{k-1}(1 - \delta_{n-j}) = 1$ or equivalently,

$$\sum_{j=0}^{k-1}\log(1 - \delta_{n-j}) = 0\,.$$

Let $\beta_j = -\log(1 - \delta_{n-j})$, then the first time point at which $\sum_{j=0}^{k-1}\beta_j \approx 0$. If for some k^* the value of β_{k^*} becomes large, drop that points and all pioneer points and start search from that point, backwardly.

To calculate δ_{n-j} notice that

$$\delta_{n-j} = P\big(s_{n-j} > L \big| s_{n-j+1} < L\big) = P\big(s_{n-j} > L \big| s_{n-j+1} = L - \varepsilon\big)$$

$$= P\left(r_{n-j} > \frac{\varepsilon}{L - \varepsilon}\right) = 1 - F_{n-j}\left(\frac{\varepsilon}{L - \varepsilon}\right),$$

for some pre-determined ε and F_{n-j} is the distribution function of $(n - j)$-th return. The following proposition summarizes the above discussion.

Proposition 3: The first point at which $\sum_{j=0}^{k-1}\beta_j \approx 0$, then it is an up-crossing time point.

4. Simulations: Here, the daily closing price of stock of *Intel. Co* for 21 December 2018 to 19 December 2019 (251 observations). The time series plot of returns r_t are plotted as follows. It is seen that there is no a non-stability in mean or variance. The auto-correlation (ACF) and partial auto-correlation (PACF) functions of returns are given denoting that this series is a white noise processes. The following table gives the summary statistics of return process. Clearly, return data has a light symmetric tail which is different of normal distribution. The first attempt is to fit a distribution F to this data set. It is easy to see that $r_t' = sign(r_t) \times |r_t|^{\frac{1}{1.5}}$ has a normal distribution with mean 0.005 and 0.058, respectively. Notice that $|r_t'| = |r_t|^{\frac{1}{1.5}}$ and $P(max|r_t| \le U) = P(max|r_t'| \le U^{1.5}) = 1 - \alpha$. Using a Monte Carlo simulation, with 1000 repetitions, it is seen that $U = 0.375$, for $\alpha = 0.05$. Table 2 gives the values of U for various values of $1 - \alpha$'s. Table 3 gives the take profit limits which are derivable by multiplying the first and second row of Table 2.

Next, it is interested to find the stop-loss limits. Again, notice that

$$P(min|r_t| > L) = P(min|r_t'| > L^{1.5}) = 1 - \alpha.$$

For almost all $\alpha \ge 0.6$, then $L = 0.9996$. Table 4 gives the values related stop-loss strategies..

Table 1: Summary statistics of return process

Stats.	Mean	Median	Max	Min	1st Qu.	3rd Qu.	Skew	Kurt	Stdev.
Value	0.0013	0.0012	0.0810	-0.0898	-0.0069	0.0111	-0.2556	7.9948	0.0175

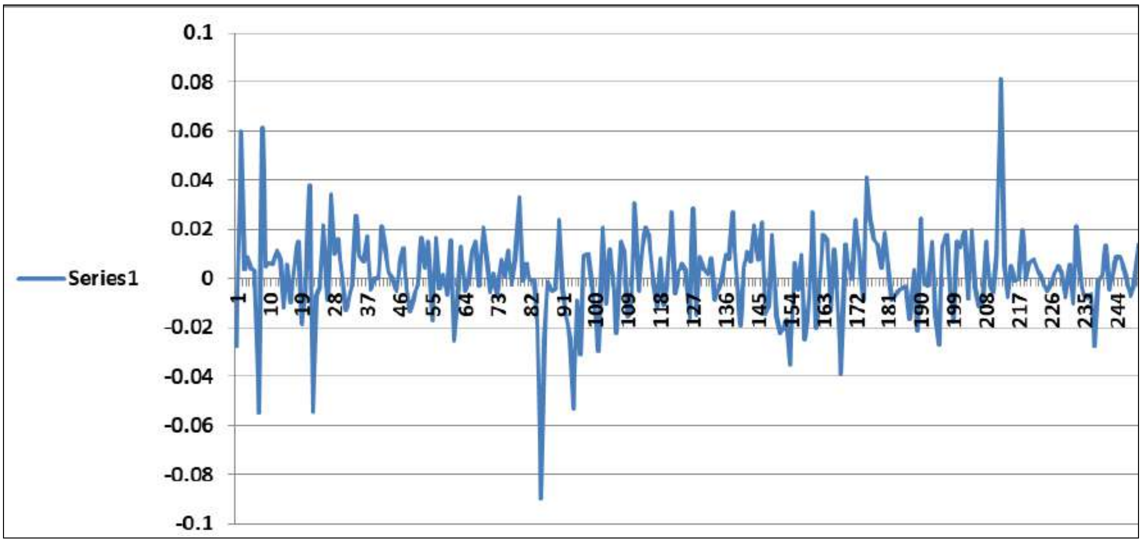

Fig 1: Time series plot of return of Intel Co.

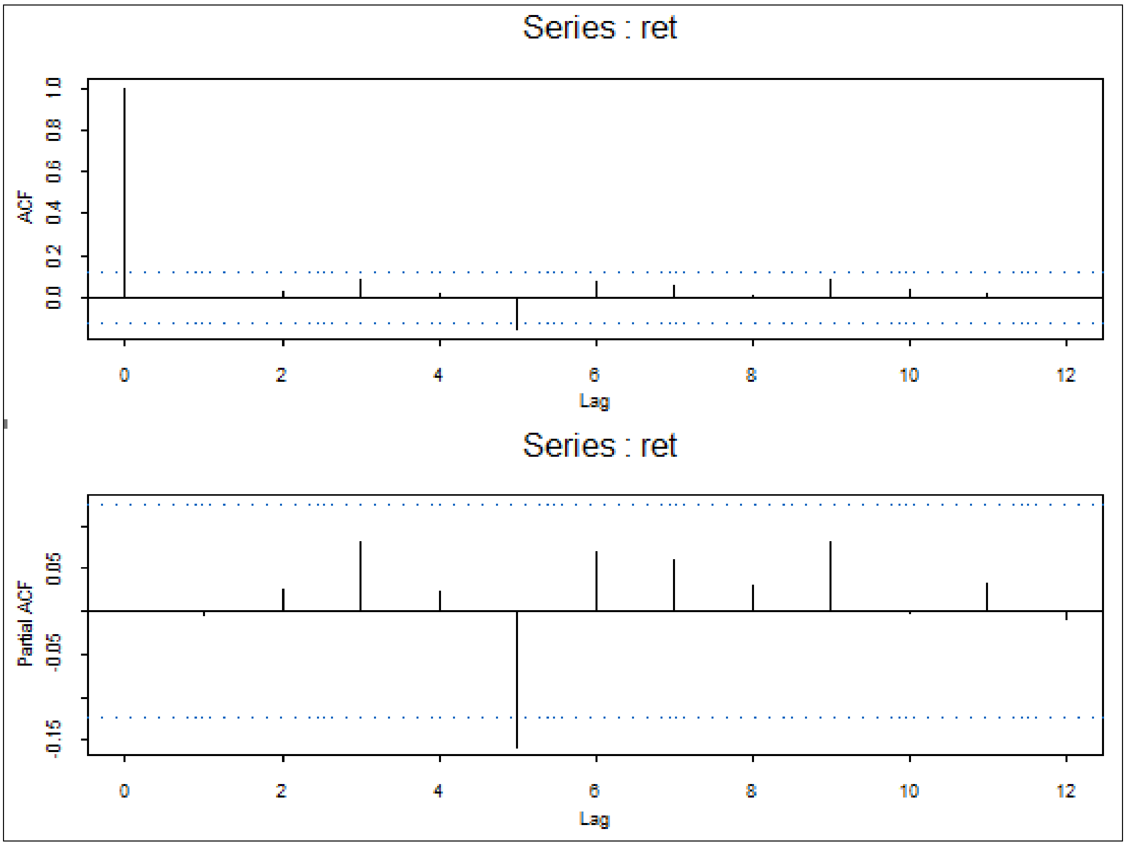

Fig 2: ACF and PACF of return of Intel Co.

Table 2: Values of U for various values of $1-\alpha$'s

$1-\alpha$	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95
U	0.313	0.316	0.319	0.324	0.329	0.337	0.344	0.375

Table 3: Values of take-profit limits for various $1-\alpha$'s

$1-\alpha$	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95
Take-profit	0.1878	0.2054	0.2233	0.243	0.2632	0.28645	0.3096	0.35625

Table 4: Values of stop-loss limits for various $1-\alpha$'s

$1-\alpha$	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95
Stop-loss	0.59976	0.64974	0.69972	0.7497	0.79968	0.84966	0.89964	0.94962

5. Conclusions. This note presents a applied statistical approach for presenting stop-loss and take profit parameters which are frequently used in FOREX market. These parameters are determined using the mixture distribution of up-crossing limits of returns of price of financial asset such as stock. These distributions are mixture since there are possibilities that return process does not pass the upper and lower thresholds. It is assumed that returns are statistically independent, although, the same results may be applied in the case of dependent returns.

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