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Generalization of open sets in a topological space

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Abstract
In this paper I have summarized most of the research and the resent works on generalization of open sets in a topological space. Majority of the results and the properties on generalized open sets like semi-open set, pre-open set,α-open set,β-open set,δ-open set e.t.c. are discussed. The chatacterizations of these sets are also studied.
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1. Introduction

The role of the generalized open set in the field releted to the general topology is very significant. If we have to deeply analyze topological spaces, then it is essential to have the extended concept about open sets. Thus the thought of open set and their properties to a wider class is very crucial. This notion is also important for expanding topological concepts like continuity, compactness, separation axioms e.t.c. to a broad scale of spaces and functions.

2. Preliminaries

The pair(Z,τ) denote the topological space throughout this paper whereon no separation axiom are supposed if not explicitly specified.Assumed $M_\tau \subseteq Z$.A point $m \in Z$ is called limit point of M_τ iff every open set U_τ carrying m include a point of M_τ different from m.The subset M_τ is a closed set iff its complement is open.In this paper the symbols $IN_\tau(M_\tau)$ and $CL_\tau(M_\tau)$ are used to imply the interior and closure of M_τ respectively. Here I used abbreviations TO, OP, CL, POP, SOP, SCL, PCL for the words topological, open, closed, pre-open, semi-open, semi-closed and pre-closed respectively.

Definition 2.1: Assume $(Z,\tau),(Z^*,\tau)$ are two TO-spaces. A mapping $f_\tau : Z \to Z^*$ is continuous iff inverse image of any OP subset of Z^* is an OP subset of Z.

Definition 2.2: The function f_τ is called an OP-function if the image of any OP set is an OP set.

Definition 2.3 Two TO-spaces Z and Z^* are called homeomorphic if there be a bijection $f_\tau : Z \to Z^*$ so as f_τ and ${f_\tau}^{-1}$ are continuous.

Definition 2.4: Suppose T be a property of sets so that whenever a TO-space (Z,τ) has T then any space homeomorphic to (Z,τ) also has T.Then T is a TO-property.

Definition 2.5: A TO-space Z is named separable if Z encloses a countable dense subset.

Definition 2.6: A TO-space Z is named Hausdorff space if any pair of different points $u, v \in Z$ there exists open sets U_τ and V_τ such that $u \in U_\tau, v \in V_\tau$ and $U_\tau \cap V_\tau = \varphi$.

Definition 2.7: A subset M_τ of a TO-space Z is termed as compact if every OP cover of M_τ has a finite subcover.

3. Main results

The thought of SOP-sets in TO-spaces was presented by Levine [1] in the year 1963.

Definition 3.1: In Z,a subset F_τ is known as SOP iff there exists an OP set U_τ such that $U_\tau \subseteq F_\tau \subseteq CL_\tau(U_\tau)$.

Theorem 3.1: A subset F_τ of Z is SOP iff $F_\tau \subseteq CL_\tau(IN_\tau(F_\tau))$.
Proof. Let $F_\tau \subseteq CL_\tau(IN_\tau(F_\tau))$.Here $IN_\tau(F_\tau)$ is OP. Take $U_\tau = IN_\tau(F_\tau)$ then, $U_\tau \subseteq F_\tau$ since $IN_\tau(F_\tau) \subseteq F_\tau$.Hence $U_\tau \subseteq F_\tau \subseteq CL_\tau(IN_\tau(F_\tau)) = CL_\tau(U_\tau)$.Conversely, let F_τ be a SOP set.Then there be an OP set U_τ so that $U_\tau \subseteq F_\tau \subseteq CL_\tau(U_\tau)$.Now since $IN_\tau(F_\tau) \subseteq F_\tau$ then obiviously $U_\tau \subseteq IN_\tau(F_\tau)$.Thus $CL_\tau(U_\tau) \subseteq CL_\tau(IN_\tau(F_\tau))$. Hence $F_\tau \subseteq CL_\tau(IN_\tau(F_\tau))$.

Proposition 3.1: If F_τ be an OP set then $F_\tau \subseteq CL_\tau(IN_\tau(F_\tau))$.
Proof. Since F_τ be an OP set, $F_\tau = IN_\tau(F_\tau) \subseteq CL_\tau(IN_\tau(F_\tau))$.Thus any OP set is a SOP.The converse implication generally not true.

Example 3.1: If $Z = \{m, a, h, l, i\}$ and $\tau = \{Z, \varphi, \{m\}, \{h, l\}, \{m, h, l\}, \{a, h, l, i\}\}$.CL-subsets are $Z, \varphi, \{a, h, l, i\}, \{m, a, i\}, \{a, i\}, \{m\}$.Take $F_\tau = \{a, h, l\}$,then $IN_\tau(F_\tau) = \{h, l\}$ and $CL_\tau\big(IN_\tau(F_\tau)\big) = \{a, h, l, i\}$. Thus $F_\tau \subseteq CL_\tau(IN_\tau(F_\tau))$ and hence F_τ is a SOP set but not OP.

Proposition 3.2: Complement of a SOP set is a SCL set. .
Proof. Straightforward.

Definition 3.2: A subset F_τ of Z is SCL iff $IN_\tau(CL_\tau(F_\tau)) \subseteq F_\tau$.

Proposition 3.3: If F_τ is not an OP set and $F_\tau \subseteq CL_\tau(IN_\tau(F_\tau))$ then F_τ is also not a CL set.
Proof. As F_τ is not an OP set,$IN_\tau(F_\tau) \subset F_\tau \Rightarrow CL_\tau(IN_\tau(F_\tau)) \subset CL_\tau(F_\tau)$.Thus $F_\tau \subset CL_\tau(F_\tau)$ and hence F_τ is not a CL set.

Proposition 3.4: If F_τ be a CL set then, $IN_\tau(CL_\tau(F_\tau)) \subseteq F_\tau$.
Proof. As F_τ be a CL-set, $CL_\tau(F_\tau) = F_\tau$ and $IN_\tau(CL_\tau(F_\tau)) = IN_\tau(F_\tau) \subseteq F_\tau$.Thus every CL set is SCL.The converse implication generally not true.

Example 3.2: Continuing with the example 3.1, take $F_\tau = \{a\}$ which is neither OP nor CL, then $CL_\tau(F_\tau) = \{a, i\}$ and $IN_\tau(CL_\tau(F_\tau)) = \varphi \subseteq F_\tau$.

Proposition 3.5: A CL set F_τ is a SOP set iff the interior of F_τ be such that $CL_\tau(IN_\tau(F_\tau)) = F_\tau$.
Proof. Straightforward
The class of all SOP subsets of Z is symbolized by $SOP(Z)$.Semi-interior of $F_\tau \subseteq Z$ is symbolized by $SIN_\tau(F_\tau)$ is the union of all SOP subsets of F_τ and semi-closure of F_τ is symbolized by $SCL_\tau(F_\tau)$ is the intersection of all SCL sets enclosing the set F_τ. The set F_τ is called SOP set iff $SIN_\tau(F_\tau) = F_\tau$ and SCL iff $SCL_\tau(F_\tau) = F_\tau$.
Levine define semi-continuity in his article [1] as follows

Definition 3.3: Let Z and Z^* are two TO-spaces. Then a single valued function $f_\tau : Z \to Z^*$ is not necessary continuous is termed as semi-continuous iff for OP set U_τ is in Z^*,the inverse image ${f_\tau}^{-1}(U_\tau)$ is a SOP set in Z.
Continuity of a function assume semi-continuity but not conversely.

Definition 3.4: A mapping $f_\tau : Z \to Z^*$ is called SOP if for any OP set U_τ of Z, the set $f_\tau(U_\tau)$ is SOP in Z^*.

Definition 3.5: Two non-empty sets H_τ and K_τ in Z are called semi-separated iff
$H_\tau \cap SCL_\tau(K_\tau) = SCL_\tau(H_\tau) \cap K_\tau = \varphi$.

Definition 3.5: In Z, a set F_τ is called semi-connected if F_τ cannot be written as the union of two semi-separated sets.The space (Z, τ) is called semi-connected iff Z is semi-connected.

Definition 3.6: For the TO-space Z if every cover of Z by SOP sets has a sub-cover which is finite then Z is called semi-compact. If any countable SOP cover of Z has a sub-cover which is finite then Z is called countably semi-compact and Lindelöf if any SOP cover of Z has a countable sub-cover.

Definition 3.7: A mapping $f_\tau : Z \to Z^*$ is termed as semi-weakly continuous when for any $m \in Z$ and for any OP set U_τ in Z^* enclosing $f_\tau(m)$,there be a SOP set V_τ in Z such that $m \in V_\tau$ and $f_\tau(V_\tau) \subset SCL_\tau(U_\tau)$.

Properties on semi-open sets

- If $\{F_{\tau_i}\}_{i\in\Lambda}$ be the collection of SOP sets, then $\bigcup_{i\in\Lambda} F_{\tau_i}$ is SOP.
- Intersection of a SOP set and an OP set is SOP always.
- $IN_\tau(F_\tau) \subset SIN_\tau(F_\tau) \subset F_\tau \subset SCL_\tau(F_\tau) \subset CL_\tau(F_\tau)$.
- Intersection of two SOP sets need not be SOP.
- Not every SOP set is OP.
- If $U_\tau \subseteq F_\tau \subseteq CL_\tau(U_\tau)$ where U_τ is an OP set and F_τ is a SOP set, then the points of $F_\tau \setminus U_\tau$ are limit points of U_τ if $F_\tau \setminus U_\tau$ is non-empty.

- If V_τ is an OP-connected set and $V_\tau \subseteq F_\tau \subseteq CL_\tau(V_\tau)$, then F_τ is SOP and connected.
- Semi-connectedness is a TO-property.
- $SCL_\tau(F_\tau) = F_\tau \cup IN_\tau(CL_\tau(F_\tau))$.

The thoughts of POP sets was presented by Mashhour et. al. [2] in 1982.

Definition 3.8: In Z,a subset F_τ is POP iff $F_\tau \subseteq IN_\tau(CL_\tau(F_\tau))$.

Proposition 3.6: If F_τ is an OP set then $F_\tau \subseteq IN_\tau(CL_\tau(F_\tau))$.
Proof. Since F_τ is an OP set then,$F_\tau = IN_\tau(F_\tau)$ and so any point of F_τ is an interior point of F_τ.Let $m \in F_\tau \Rightarrow m$ is an interion point of $F_\tau \Rightarrow m$ is an interior point of $CL_\tau(F_\tau) \Rightarrow m \in IN_\tau(CL_\tau(F_\tau)) \Rightarrow F_\tau \subseteq IN_\tau(CL_\tau(F_\tau))$.Thus any OP set is a POP set but not conversely.

Proposition 3.7: Evey POP set need not be an OP set.
Proof. Continuing with the example 3.1, take $F_\tau = \{a, h\}$,then $CL_\tau(F_\tau) = \{a, h, l, i\}$ and $IN_\tau(CL_\tau(F_\tau)) = \{a, h, l, i\}$.Thus $F_\tau \subseteq IN_\tau(CL_\tau(F_\tau))$ and hence F_τ is a POP set but not an OP set.

Proposition 3.8: Complement of a POP set is a PCL set.
Proof. Straightforward.

Definition 3.9: A subset F_τ in Z is called PCL iff $CL_\tau(IN_\tau(F_\tau)) \subseteq F_\tau$.

Proposition 3.9: If F_τ is a CL set, then $CL_\tau(IN_\tau(F_\tau)) \subseteq F_\tau$.
Proof. As F_τ is a CL set, $D_\tau(F_\tau) \subseteq F_\tau$ where $D_\tau(F_\tau)$ is the derived set of F_τ.Let $m \in CL_\tau(IN_\tau(F_\tau)) \Rightarrow m \in IN_\tau(F_\tau) \cup D_\tau\big(IN_\tau(F_\tau)\big) \Rightarrow either\ m \in IN_\tau(F_\tau)\ or, m \in D_\tau(IN_\tau(F_\tau))$.Now, $D_\tau(F_\tau) \subseteq F_\tau \Rightarrow D_\tau(IN_\tau(F_\tau)) \subseteq F_\tau$.Thus $m \in D_\tau(IN_\tau(F_\tau)) \subseteq F_\tau$.Hence $CL_\tau(IN_\tau(F_\tau)) \subseteq F_\tau$.Thus every CL set is PCL but not conversely.

Proposition 3.10: If F_τ is an OP-set and $F_\tau \subseteq IN_\tau\big(CL_\tau(F_\tau)\big)$, then F_τ is also not a CL-set.
Proof. Suppose that $F_\tau \subseteq IN_\tau(CL_\tau(F_\tau))$ and F_τ is not an OP set.If F_τ is a CL set, then $CL_\tau(F_\tau) = F_\tau$ and hence $F_\tau \subseteq IN_\tau(F_\tau)$,a contradiction since $IN_\tau(F_\tau) \subset F_\tau$ as F_τ is not OP.Hence F_τ cannot be CL.

Proposition 3.11: A CL set F_τ is POP iff the closure of F_τ be such that $IN_\tau\big(CL_\tau(F_\tau)\big) = F_\tau$.
Proof. Straightforward.

The class of all POP subsets of Z is symbolized by $POP(Z)$. Pre-interior of $F_\tau \subseteq Z$ is denoted by $PIN_\tau(F_\tau)$ is the union of all POP subsets of F_τ and pre-closure of F_τ is denoted by $PCL_\tau(F_\tau)$ is the intersection of all PCL sets enclosing F_τ.Pre-derived set of F_τ is denoted by $PD_\tau(F_\tau)$.

Properties on pre-open sets

- For any $F_\tau \subset Z$, $IN_\tau(F_\tau) \subset PIN_\tau(F_\tau) \subset F_\tau \subset PCL_\tau(F_\tau) \subset CL_\tau(F_\tau)$.
- If $\{F_{\tau_i}\}_{i\in\Lambda}$ be the class of POP sets, then $\bigcup_{i\in\Lambda} F_{\tau_i}$ is POP.
- Intersection of two POP sets need not be POP.
- Intersection of a POP set and an OP set is POP.
- A subset F_τ of Z is POP iff there be an OP set U_τ in Z such that $F_\tau \subseteq U_\tau \subseteq CL_\tau(F_\tau)$.
- Eny singleton set is either POP or nowhere dense.
- For any subset F_τ of Z, $PCL_\tau(F_\tau) = F_\tau \cup PD_\tau(F_\tau)$.

Noiri's Lemma: If M_τ is SOP and N is POP, then $M_\tau \cap N_\tau$ is SOP in M_τ and POP in N_τ.

Definition 3.10: A set F_τ is called clopen iff it is both CL and OP.

Theorem 3.2: Assume M_τ and N_τ be two subsets of Z.If M_τ is PCL, then $PCL_\tau(M_\tau \cap N_\tau) \subseteq M_\tau \cap PCL_\tau(N_\tau)$.

Definition 3.11: Let $F_\tau \subseteq Z$. A point $m \in Z$ is called pre-limit point of F_τ if $\forall\ U_\tau \in POP(Z), m \in U_\tau \Rightarrow U_\tau \cap (F_\tau \backslash \{m\}) \neq \varphi$.

Theorem 3.3: For any subset F_τ of Z, F_τ is PCL iff $PD_\tau(F_\tau) \subseteq F_\tau$.

Definition 3.12: Let $F_\tau \subseteq Z$.A point $m \in Z$ is called pre-interior point of F_τ if there be a POP set U_τ such that $m \in U_\tau \subseteq F_\tau$.

Theorem 3.5: For any $F_\tau \subseteq Z$, F_τ is POP iff $PIN_\tau(F_\tau) = F_\tau$.

Mashhour *et al*. presented the thought of pre-continuity and weak continuity [2].

Definition 3.13: A function $f_\tau : Z \rightarrow Z^*$ is termed as pre-continuous if the inverse image of each OP set of Z^* is an POP set in Z

that is if $U_\tau \subset Z^*$ is an OP set then $f_\tau^{-1}(U_\tau) \subset IN_\tau(CL_\tau\left(f_\tau^{-1}(U_\tau)\right))$.

Definition 3.14: A function $f_\tau : Z \to Z^*$ is termed as pre-weakly continuous if for any point $m \in Z$ and for any OP set U_τ in Z^* enclosing $f_\tau(m)$ there be a POP set F_τ in Z such that $m \in F_\tau$ and $f_\tau(F_\tau) \subset PCL_\tau(U_\tau)$.

Theorem 3.6: A function $f_\tau : Z \to Z^*$ is pre-weakly continuous iff for any OP set U_τ in Z^*,$f_\tau^{-1}(U_\tau) \subset PIN_\tau(f_\tau^{-1}(PCL_\tau(U_\tau)))$.

Definition 3.15: A space Z is termed as pre-compact if every POP cover of Z has a subcover which is finite.

Definition 3.16: A space Z is termed as pre-connected if it cannot be express as the union of two disjoint non-empty POP sets.

Definition 3.17: A space Z is termed as pre-Hausdorff if different points in Z have disjoint pre-neighbourhood.

Definition 3.18: A subset F_τ in Z is said to be $\alpha - OP$ if $F_\tau \subseteq IN_\tau(CL_\tau(IN_\tau(F_\tau)))$.
The intersection of all $\alpha - CL$ sets enclosing F_τ is symbolized by $CL_{\tau_\alpha}(F_\tau)$.

Properties on α-open sets

- The complement of $\alpha - OP$ set is $\alpha - CL$.
- Every $\alpha - OP$ set is SOP and POP.
- Intersection of a POP set and $\alpha - OP$ set is a POP set.

Theorem 3.7
If a topology τ on Z contains only φ, Z and $\{m\}$ for fixed $m \in Z$,then every POP set is $\alpha - OP$.

Definition 3.19: A subset F_τ in Z is termed as $b - OP$ if $F_\tau \subseteq IN_\tau(CL_\tau(F_\tau))\cup CL_\tau(IN_\tau(F_\tau))$.Complement of $b - OP$ set is $b - CL$.

Definition 3.20: A subset F_τ in Z is termed $\beta - OP$ or semi-pre open (SPOP) if $F_\tau \subseteq CL_\tau(IN_\tau(CL_\tau(F_\tau)))$.Complement of $\beta - OP$ set is $\beta - CL$.

Properties on $\alpha - OP, \beta - OP, b - OP$ sets

- Every $\alpha - OP$ set is SOP, POP and $b - OP$.
- Every $\alpha - CL$ set is SCL, PCL and $b - CL$.
- Every $b - OP$ set is SPOP.
- Evey $b - CL$ set is semi-pre closed (SPCL)
- Every POP set is $b - OP$ and SPOP.
- Every PCL set is $b - CL$ and SPCL.
- Every SOP set is $b - OP$ and SPOP.
- Every SCL set is $b - CL$ and SPCL.

Theorem 3.8: A subset F_τ in Z is $\beta - OP$ if there be a POP subset U_τ of Z such that $U_\tau \subseteq F_\tau \subseteq CL_\tau(U_\tau)$.

Definition 3.21: A subset F_τ in Z is termed as regular open (ROP) if $IN_\tau(CL_\tau(F_\tau)) = F_\tau$ and regular closed (RCL) if $CL_\tau(IN_\tau(F_\tau)) = F_\tau$.

Properties

- F_τ is OP$\Rightarrow$ F_τ need not be ROP.
- F_τ is not OP $\Rightarrow$ F_τ is not ROP.
- F_τ is CL $\Rightarrow$ F_τ need not be RCL.
- F_τ is not CL $\Rightarrow$ F_τ is not RCL.

Definition 3.22 A point $m \in Z$ is called $\theta - cluster$ point of $F_\tau \subseteq Z$ if $CL_\tau(U_\tau)\cap F_\tau \neq \varphi$ for any OP neighbourhood U_τ of m.The class of all $\theta - cluster$ points of F_τ is symbolized by $CL_{\tau_\theta}(F_\tau)$.
A subset F_τ is called $\theta - CL$ if $F_\tau = CL_{\tau_\theta}(F_\tau)$.The complement of $\theta - CL$ is $\theta - OP$.

Properties on $\theta - OP$ sets

- A subset F_τ is $\theta - OP$ if $F_\tau = IN_{\tau_\theta}(F)$.
- The union of any colletion of $\theta - OP$ set is $\theta - OP$.
- The intersection of a finite colletion of $\theta - OP$ sets is $\theta - OP$.
- A subset F_τ in Z is $\theta - OP$ if for any point $m \in Z$,there be a ROP set U_τ such that $m \in U_\tau \subseteq F_\tau$.
- Every ROP set is also $\theta - OP$ but not conversely.
- Collection of all $\theta - OP$ sets in a TO-space (Z, τ) form a topology on Z.

Definition 3.22: A point $m \in Z$ is termed as $\delta - cluster$ point of $F_\tau \subseteq Z$ if $IN_\tau(CL_\tau(U_\tau)) \cap F_\tau \neq \varphi$ for every open neighbourhood U_τ of m. The set of all $\delta - cluster$ points of F is denoted by $CL_{\tau_\delta}(F_\tau)$.
A subset F_τ is called $\delta - CL$ if $F_\tau = CL_{\tau_\delta}(F_\tau)$.The complement of $\delta - CL$ is $\delta - OP$.
A point $m \in Z$ is called $\delta - interior$ point of F_τ if there be an OP set U_τ enclosing m so that $m \in U_\tau \subset IN_\tau(CL_\tau(U_\tau)) \subset F_\tau$.

Properties on $\delta - open$ sets
- A subset F_τ is $\delta - OP$ if $F_\tau = IN_{\tau_\delta}(F_\tau)$.
- Every $\delta - OP$ set can be explicated as union of ROP sets.
- $\delta - interior$ of $F_\tau \subseteq Z$ can be explicated as union of all ROP sets of Z enclosed by F_τ.
- $\delta - OP$ sets are applied to describe sepration axioms and it is used to study covering properties like compactness and $Lindel\ddot{o}f$.
- Class of all $\delta - OP$ sets in a TO-space (Z, τ) form a topology on Z.

Definition 3.22
A subset F_τ in Z is termed as
- $g - CL$ if $CL_\tau(F_\tau) \subseteq U_\tau$ whereas $F_\tau \subseteq U_\tau$ and U_τ is OP.
- $\omega - CL$ if $CL_\tau(F_\tau) \subseteq U_\tau$ whereas $F_\tau \subseteq U_\tau$ and U_τ is SOP.
- Semi generalized closed $(sg - CL)$ if $SCL_\tau(F_\tau) \subseteq U_\tau$ whereas $F_\tau \subseteq U_\tau$ and U_τ is SOP.
- Generaralized semi-closed if $SCL_\tau(F_\tau) \subseteq U_\tau$ whereas $F_\tau \subseteq U_\tau$ and U_τ is OP.
- $g^* - CL$ if $CL_\tau(F_\tau) \subseteq U_\tau$ whereas $F_\tau \subseteq U_\tau$ and U_τ is g −OP.
- Generalized pre-closed if $PCL_\tau(F_\tau) \subseteq U_\tau$ whereas $F_\tau \subseteq U_\tau$ and U_τ is OP.
- Generalized semi pre-closed if $SPCL_\tau(F_\tau) \subseteq U_\tau$ whereas $F_\tau \subseteq U_\tau$ and U_τ is OP.
- Generalized pre-regular closed if $PCL_\tau(F_\tau) \subseteq U_\tau$ whereas $F_\tau \subseteq U_\tau$ and U_τ is ROP.
- $g\alpha - CL$ if $CL_{\tau_\alpha}(F_\tau) \subseteq U_\tau$ whereas $F_\tau \subseteq U_\tau$ and U_τ is $\alpha - OP$
- $\alpha g - CL$ if $CL_{\tau_\alpha}(F_\tau) \subseteq U_\tau$ whereas $F_\tau \subseteq U_\tau$ and U_τ is OP.
- $\omega\alpha - CL$ if $CL_{\tau_\alpha}(F_\tau) \subseteq U_\tau$ whereas $F_\tau \subseteq U_\tau$ and U_τ is $\omega - OP$.
- $\alpha\omega - CL$ if $CL_{\tau_\omega}(F_\tau) \subseteq U_\tau$ whereas $F_\tau \subseteq U_\tau$ and U_τ is $\alpha - OP$.

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