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Shatha Awad Al-Fatlawi
Department of Statistics, College of Administration and Economics, University of Al Qadisiyah, Al Diwaniyah, Iraq

Sanaa J Tuama
Department of Statistics, College of Administration and Economics, University of Al Qadisiyah, Al Diwaniyah, Iraq

Corresponding Author:
Shatha Awad Al-Fatlawi
Department of Statistics, College of Administration and Economics, University of Al Qadisiyah, Al Diwaniyah, Iraq

Choosing best model selection in elastic-net quantile regression model

Shatha Awad Al-Fatlawi and Sanaa J Tuama

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Abstract
The effectiveness of model selection techniques is significantly influenced by shrinkage parameters. The more precise the shrinkage parameterization process is, the greater the likelihood of obtaining efficient, generalizable models. One of the methods for selecting important variables is the elastic- net method, which is considered a very efficient method for selecting variables and then selecting models. By utilization the Elastic Net method with the quantile regression model, we will obtain an effective statistical model in selecting models and estimating the coefficients of these models. Two methods will be used in this paper to choose the shrinkage parameter, and the simulation approach and the real data technique were used to determine which of these two methods is better. Based on the results, it was determined that the Bayesian method is the best method for choosing the optimal model after determining the shrinkage parameters in the Elast-net technique.

Keywords: Shrinkage parameter, quantile regression model, elast-net technique, cross-validation method, Bayesian method

Introduction
The good statistical model formulation is a priority for researchers that achieves best estimators. One of these statistical models is the regression model, which focuses on estimating the relationship between the response variable and a set of explanatory variables. However, when the explanatory variables are very large maybe effecting on predictive accuracy,to overcame this problem model selection has been used. Model selection is the process of choosing the good model from a set of available models for a given explanatory variables. The goal of model selection is to choose the model that will work well with rest explanatory variables (Mohammed and Raheem, 2020) [11]. Efroymson, was the first to propose the idea of model selection in 1960 through the stepwise technique. This method summarizes by selecting a small group of explanatory variables that have explanatory weight in the studied regression model. But the stepwise technique has many drawback, therefore the (Mallows, 1974), proposed an approach to selecting the best model is called a method Mallows C_K. Also (Akaike,1973) [1] proposed a criterion for selecting the best model from a set of models, and the name of this criterion was Akaike information criterion, abbreviated as AIC. In 1978 Schwarz introduce another criterion is Bayesian information criterion (BIC) for choose best model. But all these criterions have drawback especially when the numbers of explanatory variables is large, because the model will become more complex. In a short period of time, researchers proposed a set of regularization methods that are characterized by a good and flexible approach in selecting good models. Tibshirani,1996 [14] was proposed in the subject of variable selection, and this method is called the least absolute selection and shrinkage operator (lasso) method. This technique provides a new approach to variable selection, and thus produces regression models with very high explanatory power. In 2005 introduce (Zou and Hastie) new methods mixing with Ridge method and lasso method is called elastic-net method which have a good property. all above methods regularized with classical regression model, in some times assumptions not achieving there for the classical model cannot provide us a good estimation to overcame these problems quantile regression model has been used (Al-Guraibawi, Raheem, and Mohammed, 2025) [2]. The quantile regression model is proposed by (Koenker and Bassett 1978) [4] it has many features compared with other regression models. The quantile regression model mixing with regularization methods in many situations. Koenker 1981 [5] proposed model selection of quantile regression model by using AIC and BIC

criterions. Ravikumar *et al*, 2007 [12] proposed a new approach mixing between lasso method and quantile regression model which is strong method for choosing formative explanatory variables. Zou and Zhang 2009 [21] proposed a new approach mixing between Elastic-net method and quantile regression model which is strong method for choosing formative explanatory variables. In this paper, we will two method for computing the value of shrinkage parameters of Elastic-net and choosing best model selection. The our paper is organized as following: First section elastic-net method second section elastic-net quantile regression method. Third section computing the value of shrinkage parameters of Elastic-net fourth section simulation approach, fifth section real dataset sixth section conclusions and recommendations.

2. elastic-net method

The elastic-net is a good approach for choosing the variables and shrinkage other parameters. Also, this approach is consider a good statistical tool analysis the models until when high correlation between the explanatory variables and when the sample size less than number of explanatory variables. The elastic-net approach is mixture between ridge function and lasso function, we can write the mathematical formula as following:

$$\hat{\beta}_{elastic-net} = \sum(y_i - \bar{y})^2 + \lambda_1 Ridge\ function + \lambda_2 lasso\ function \ (1)$$

Where;

$\sum(y_i - \bar{y})^2$ is the loss function, λ_1, λ_2 is the shrinkage function $\lambda_1, \lambda_2 \geq 0, Ridge\ function = \|\beta\|_2^2, lasso\ function = \|\beta\|$. The elastic-net method is a good tool to improving accurate of forecasting to lasso method when exciting high correlation between explanatory variables. But it hasn't oracle properties.

3. Elastic-net quantile regression model

Quantile regression was proposed by (Koenker and Bassett (1978)) [4] as an extension to classical regression model in conditional different quantiles of a dependent variables can dealing with low tail distribution. Quantile regression model is capable of providing complete information about different quantiles of the stochastic relationships between dependent and explanatory variables. Recently, Quantile regression model has received much interest in theoretical and application studies. Where, Quantile regression model is applied in different fields of knowledge such as: body mass index (Yu *et al*., 2013) [17] growth chart (Wei *et al*., (2006)) [16], ecological studies (Cade and Noon, (2003)) [3] agricultural economics (Kostov and Davidova, (2013)) [10], Microarray study (Wang and He, (2007)) [15] and so on. The Quantile regression models have a good property compared with other regression models. Where, Quantile regression model belongs to a robust regression models' family (Koenker and Geling, (2001)) [8]. Quantile regression model does not require any supposition about the random residuals distribution. It is providing greatest statistical an efficiency than classical regression models when the random error is non-normal. Also, Q Reg model is robust against the economic problems. All these features made the Q Reg model of an informative model in application fields. The following mathematical formula belongs to Q Reg model.

$$y_i = x_i^T \beta_\tau + \varepsilon_i, \tau \in (0,1) \ (2)$$

For any θth quantile, $(0 < \tau < 1)$, the θth quantile regression can be denoted as $Q_{y_i|x_i}(\tau) = x'_i \beta_\tau$, where y_i is the response variable (dependent variable), x_i^T is a k-dimensional vector of covariates (independent variables), β_τ is a coefficients vector of Q Reg model. To estimate the coefficients of Elastic-net quantile regression model (zhang and lu (2018)) [18] proposed the following equation.

$$\min_{\beta_\theta} \sum_{i=1}^{n} \rho_\theta(y_i - x_i^T \beta_\theta) + \lambda_1 Ridge\ function + \lambda_2 lasso\ function \ (3)$$

where $\rho_\tau(u)$ is the checkfunction defined by $\rho_\tau(u) = u\{\tau - \mathrm{I}(u \leq 0)\}, and$ where $I(u < 0)$ is the indicator function and λ_1, λ_2 is the shrinkage function $\lambda_1, \lambda_2 \geq 0, Ridge\ function = \|\beta\|_2^2, lasso\ function = \|\beta\|$, the equation (3) is not differentiable at (0), there is no solution for equation (3) (Koenker, (2005)) [9] shows the minimization of (3) can be achieved by a linear programming method which is proposed (Koenker and D'Orey, 1987)) [7]. Via using the "rqpen package"

4. Shrinkage parameters of Elastic-net

in equation there are many methods for estimation the shrinkage parameters (λ_1, λ_2). Therefor using of an efficient method for estimating shrinkage parameters is an important thing in producing optimal model selection. In this study we focus on three method for estimation shrinkage parameter as following:

4.1 Cross-Validation Method

This method depends on dividing the data into a training set and a test set. Then, the model is estimated using the training set with a variety of values for the shrinkage parameter. Finally, the value for the shrinkage parameter that achieves the best performance on the test set is chosen.

The algorithm of Cross-Validation Method shown as following

- The data is divided into a training set and a test set to evaluate the model's performance on data that it has not been trained on.
- The model is estimated using the training set with a variety of shrinkage parameter values.
- The shrinkage parameter value that achieves the best performance on the test set is chosen using appropriate evaluation metrics, such as mean squared error or root mean squared error.

4.2 Bayesian method

This method is important for choosing optimal shrinkage parameters as the following steps:

- find prior distribution for shrinkage parameter such as uniform distribution or Gaussin distribution or other distributions as case study
- Find posterior distribution for shrinkage parameters via training data
- Choosing of shrinkage parameter that achieving maximum probability after compute of probability for each value of shrinkage parameters

5. Simulation Approach: To compare of performance to model selection with Elastic-net quantile regression model at two approach for compute shrinkage parameters. It was employed with three levels that are $\theta, 0.25, 0.50, and\ 0.95$. For each simulation study The methods studied are assessed depend on Akaike information criterion (AIC), that is computed as following:$AIC = 2K - 2\ln(L)$,where L is likelihood function And Bayesian information criterion (BIC), that is computed as following: $BIC = \ln(n)\,(k - 2\ln(L)($,where L is likelihood function, k is the nuber of independent varibles. In this simlution approach three teypes of random error have been used, $\varepsilon \sim N(0,1), \varepsilon \sim N(2,2)\ and\ \varepsilon \sim Laplace\ (0,1)$. In this simulation,we will used two simulation examples:

First simulation example

We suppose that the true vector's parameters in this simulation scenario are as follows: (0,0,1,0,0,0,0). This vector defines the actual model of the initial simulation technique as follows:

$$y = x_{3i} + \varepsilon_i \quad i = 1,2, \dots \dots n$$

The multivariate normal distribution will be used to generated the seven independent variables, which will have the following variance and covariance definitions and an arithmetic mean of 0: $(\Sigma_x)_{ij} = (0.5)^{|i+j|}$. That is, the explanatory variables are distributed, where k represents the number of explanatory variables. $X \sim N_k(0, \Sigma_x)$.

Table 1: show the values of Akaike information criterion (AIC) and Bayesian information criterion (BIC) with s $\varepsilon \sim N(0,1)$.

Sample size	Cross-Validation Method						Bayesian method					
	$\theta_1 = 0.25$		$\theta_1 = 0.50$		$\theta_1 = 0.95$		$\theta_1 = 0.25$		$\theta_1 = 0.50$		$\theta_1 = 0.95$	
	AIC	BIC	AIC	BIC	AIC	BIC	AIC	BIC	AIC	BIC	AIC	BIC
n=25	27.223	36.974	25.503	35.254	29.505	39.256	21.223	36.974	25.503	35.254	29.505	39.256
n=50	42.777	58.073	49.847	65.143	22.893	38.190	32.761	41.414	27.674	36.573	27.056	30.963
n=100	80.709	101.55	99.700	120.54	34.86	55.709	52.781	56.563	32.643	39.894	31.453	37.571
n=150	111.63	135.72	142.94	167.03	43.393	67.478	60.767	68.464	35.673	48.451	35.005	49.763
n=200	133.05	159.44	57.286	83.672	51.923	78.309	62.672	71.511	44.353	52.621	46.464	56.603
n=250	154.81	182.98	206.50	234.67	60.587	88.759	82.005	93.511	52.672	61.473	52.735	67.643

The aforementioned findings indicate that the Akaike information criterion (AIC) and Bayesian information criterion (BIC) values calculated for the elastic-net shrinkage parameters using the Bayesian method are much lower than Akaike information criterion (AIC) and Bayesian information criterion (BIC) values calculated for the elastic net shrinkage parameters using the Cross-Validation Method. This result demonstrates that the models selected using the Bayesian method are more efficient than the models selected using the Cross-Validation method, for all sample sizes and quantile levels.

Table 2: show the values of Akaike information criterion (AIC) and Bayesian information criterion (BIC) with $\varepsilon \sim N(2,2)$.

Sample size	Cross-Validation Method						Bayesian method					
	$\theta_1 = 0.25$		$\theta_1 = 0.50$		$\theta_1 = 0.95$		$\theta_1 = 0.25$		$\theta_1 = 0.50$		$\theta_1 = 0.95$	
	AIC	BIC	AIC	BIC	AIC	BIC	AIC	BIC	AIC	BIC	AIC	BIC
n=25	28.472	32.371	28.792	40.273	31.206	42.289	21.672	30.361	23.089	34.451	39.351	35.610
n=50	32.723	43.192	27.281	36.372	33.183	47.411	22.743	35.561	25.451	31.341	24.562	34.492
n=100	45.435	51.429	37.827	43.239	41.083	52.121	34.241	48.351	28.452	37.967	38.672	51.821
n=150	49.082	63.295	55.673	71.193	61.138	78.295	38.823	48.451	51.361	62.461	48.545	57.415
n=200	86.242	98.182	75.682	86.206	84.285	96.652	64.219	71.182	69.325	76.261	64.581	72.131
n=250	146.261	1559.926	161.087	172.183	100.328	116.327	84.818	94.263	94.432	112.451	82.525	91.528

The aforementioned findings indicate that the Akaike information criterion (AIC) and Bayesian information criterion (BIC) values calculated for the elastic-net shrinkage parameters using the Bayesian method are much lower than Akaike information criterion (AIC) and Bayesian information criterion (BIC) values calculated for the elastic net shrinkage parameters using the Cross-Validation Method. This result demonstrates that the models selected using the Bayesian method are more efficient than the models selected using the Cross-Validation method, for all sample sizes and quantile levels.

Table 3: show the values of Akaike information criterion (AIC) and Bayesian information criterion (BIC) with $\varepsilon \sim Laplace\ (0,1)$.

Sample size	Cross-Validation Method						Bayesian method					
	$\theta_1 = 0.25$		$\theta_1 = 0.50$		$\theta_1 = 0.95$		$\theta_1 = 0.25$		$\theta_1 = 0.50$		$\theta_1 = 0.95$	
	AIC	BIC	AIC	BIC	AIC	BIC	AIC	BIC	AIC	BIC	AIC	BIC
n=25	19.674	28.342	29.526	37.892	49.452	54.078	13.724	23.724	19.172	28.026	34.677	46.145
n=50	22.484	28.415	31.462	47.482	68.672	78.513	16.362	24.362	24.382	44.134	46.134	58.183
n=100	28.485	37.253	44.452	56.472	37.562	51.856	21.782	32.782	37.531	45.851	33.715	46.833
n=150	24.561	34.573	37.542	46.572	58.481	66.452	19.245	28.245	34.321	38.231	41.204	54.361
n=200	26.471	36.453	39.919	49.411	84.363	92.003	20.561	34.561	31.215	41.791	54.772	63.741
n=250	57.562	64.482	65.253	73.362	94.456	119.471	34.452	41.452	43.251	51.185	76.341	83.193

As can be seen from the above results, the Bayesian method's Akaike information criterion (AIC) and Bayesian information criterion (BIC) values for the elastic-net shrinkage parameters are significantly lower than the Cross-Validation Method's AIC and BIC values for the same parameters. According to this results, the models chosen through the Bayesian approach are more effective than those chosen through the Cross-Validation method for all sample sizes and quantile levels.

Second Simulation Example

We suppose that the true vector's parameters in this simulation scenario are as follows: (0.85, 0.85, 0.85, 0.85, 0.85, 0.85, 0.85). This vector defines the actual model of the initial simulation technique as follows:

$$y = 0.85x_{1i} + 0.85x_{2i} + 0.85x_{3i} + 0.85x_{4i} + 0.85x_{5i} + 0.85x_{6i} + 0.85x_{7i} + 0.85x_{8i} + \varepsilon_i \quad i = 1,2, \dots \dots n$$

The multivariate normal distribution will be used to generated the seven independent variables, which will have the following variance and covariance definitions and an arithmetic mean of 0: $(\Sigma_x)_{ij} = (0.5)^{|i+j|}$. That is, the explanatory variables are distributed, where k represents the number of explanatory variables. $X \sim N_k(0, \Sigma_x)$.

Table 4: show the values of Akaike information criterion (AIC) and Bayesian information criterion (BIC) with s $\varepsilon \sim N(0,1)$.

Sample size	Cross-Validation Method						Bayesian method					
	$\theta_1 = 0.25$		$\theta_1 = 0.50$		$\theta_1 = 0.95$		$\theta_1 = 0.25$		$\theta_1 = 0.50$		$\theta_1 = 0.95$	
	AIC	BIC	AIC	BIC	AIC	BIC	AIC	BIC	AIC	BIC	AIC	BIC
n=25	38.761	49.451	53.674	65.672	75.682	88.082	23.673	39.723	43.285	58.952	61.672	73.219
n=50	45.810	57.837	50.874	61.573	78.734	91242	27.192	38.435	34.328	61.643	61.743	76.261
n=100	32.674	37.801	42.563	57.735	65.894	81.792	28.682	34.295	31.289	43.783	47.351	56.281
n=150	95.621	108.643	127.472	139.511	147.451	159.281	75.082	88.182	89.411	94.549	86.451	104.206
n=200	74.963	85.473	81.682	96.763	82.735	100.827	57.239	64.083	59.121	70.271	75.325	91.083
n=250	54.082	69.571	62.734	81.603	96.511	111.673	39.673	46.138	52.295	79.371	82.432	105.087

As can be seen from the above results, the Bayesian method's Akaike information criterion (AIC) and Bayesian information criterion (BIC) values for the elastic-net shrinkage parameters are significantly lower than the Cross-Validation Method's AIC and BIC values for the same parameters. According to this result, the models chosen through the Bayesian approach are more effective than those chosen through the Cross-Validation method for all sample sizes and quantile levels.

Table 5: show the values of Akaike information criterion (AIC) and Bayesian information criterion (BIC) with $\varepsilon \sim N(2,2)$.

Sample size	Cross-Validation Method						Bayesian method					
	$\theta_1 = 0.25$		$\theta_1 = 0.50$		$\theta_1 = 0.95$		$\theta_1 = 0.25$		$\theta_1 = 0.50$		$\theta_1 = 0.95$	
	AIC	BIC	AIC	BIC	AIC	BIC	AIC	BIC	AIC	BIC	AIC	BIC
n=25	19.451	27.351	25.672	34.945	38.674	46.342	14.342	24.253	21.172	32.562	29.172	36.892
n=50	26.341	34.562	25.545	36.472	39.484	46.526	18.526	28.573	21.382	31.481	31.382	43.472
n=100	18.967	31.872	29.581	37.152	43.485	59.462	15.462	22.892	23.851	33.003	37.531	55.411
n=150	27.461	34.245	31.525	45.417	51.561	73.472	20.472	31.482	29.231	42.724	49.321	65.562
n=200	19.261	23.461	34.610	46.821	101.47	121.57	17.572	29.078	24.772	41.387	84.215	97.363
n=250	21.451	34.169	46.492	66.415	61.562	78.415	18.415	27.513	38.341	43.452	64.251	76.078

As can be seen from the above results, the Bayesian method's Akaike information criterion (AIC) and Bayesian information criterion (BIC) values for the elastic-net shrinkage parameters are significantly lower than the Cross-Validation Method's AIC and BIC values for the same parameters. According to this result, the models chosen through the Bayesian approach are more effective than those chosen through the Cross-Validation method for all sample sizes and quantile levels.

Table 6: show the values of Akaike information criterion (AIC) and Bayesian information criterion (BIC) with $\varepsilon \sim Laplace\ (0,1)$.

Sample size	Cross-Validation Method						Bayesian method					
	$\theta_1 = 0.25$		$\theta_1 = 0.50$		$\theta_1 = 0.95$		$\theta_1 = 0.25$		$\theta_1 = 0.50$		$\theta_1 = 0.95$	
	AIC	BIC	AIC	BIC	AIC	BIC	AIC	BIC	AIC	BIC	AIC	BIC
n=25	41.856	56.562	36.172	43.026	61.772	80.351	27.241	43.240	32.301	37.134	37.341	45.432
n=50	31.452	42.481	44.382	52.185	74.341	92.241	27.341	34.782	37.425	48.241	56.132	82.241
n=100	24.841	36.363	32.531	54.677	85.145	127.34	19.635	25.544	26.137	34.066	45.451	53.342
n=150	22.362	34.456	30.321	46.134	96.183	119.83	17.028	26.231	24.434	39.108	85.852	97.522
n=200	24.458	33.782	29.215	42.715	91.833	152.25	1.426	1.573	0.094	0.321	1.934	1.532
n=250	29.387	40.245	37.251	51.204	89.361	147.45	22.563	35.925	37.152	63.017	64.572	122.45

As can be seen from the above results, the Bayesian method's Akaike information criterion (AIC) and Bayesian information criterion (BIC) values for the elastic-net shrinkage parameters are significantly lower than the Cross-Validation Method's AIC and BIC values for the same parameters. According to this result, the models chosen through the Bayesian approach are more effective than those chosen through the Cross-Validation method for all sample sizes and quantile levels.

Real data: To compered between two methods (Bayesian method's) (Cross-Validation Method's) for computing the elastic-net shrinkage parameters with quantile regression model, we will use the thrombocytopenia data which are collected from Afaj hospital. The sample size of real dataset is 153 observations, three quantile levels used the identical simulation process. In our paper, we will used one response variable is called thrombocytopenia and ten independent variables as following:

x_1: (Erythrocyte Sedimentation Rate) (ESR), x_2: (Silurian cholesterol) (S.cholesterol), x_3: (Low-density lipoprotein)(LDL), x_4: (high-density lipoprotein) (HDL), x_5: (Aplastic anemia) (A.AN), x_6: Lack of vitamin B12, x_7: The age of patient, x_8: The age of patient, x_9: The weight of patientx_{10}: (idiopathic thrombocytopenic purpura(ITP). In below the brief information about thrombocytopenia is A disorder known as thrombocytopenia occurs when the blood's platelet count falls below the normal range, or less than 150,000 platelets per microlitre of blood. A low platelet count can cause major bleeding issues because platelets are necessary for blood clotting and halting bleeding.

Table 7: show the values of Akaike information criterion (AIC) and Bayesian information criterion (BIC) with real data

Sample size	**Cross-Validation Method**						**Bayesian method**					
	$\theta_1 = 0.25$		$\theta_1 = 0.50$		$\theta_1 = 0.95$		$\theta_1 = 0.25$		$\theta_1 = 0.50$		$\theta_1 = 0.95$	
	AIC	BIC	AIC	BIC	AIC	BIC	AIC	BIC	AIC	BIC	AIC	BIC
n=153	91.524	102.64	75.234	86.241	84.806	93.422	67.776	72.52	64.811	71.764	65.912	75.428

The aforementioned findings indicate that the Akaike information criterion (AIC) and Bayesian information criterion (BIC) values calculated for the elastic-net shrinkage parameters using the Bayesian method are much lower than Akaike information criterion (AIC) and Bayesian information criterion (BIC) values calculated for the elastic net shrinkage parameters using the Cross-Validation Method. This result demonstrates that the models selected using the Bayesian method are more efficient than the models selected using the Cross-Validation method, for all quantile levels. Since the Bayesian method has demonstrated its effectiveness and superiority in selecting the shrinkage parameters in the elastic-net technique using the quantile regression model, we will exclusively use the Bayesian method to select models in real data, as illustrated below.

Table 8: Parameter estimates for three quantile regression levels.

		Bayesian method		
Variables	**Symbol variables**	$\theta_1 = 0.25$	$\theta_1 = 0.50$	$\theta_1 = 0.95$
E. S. R (Erythrocyte Sedimentation Rate)	x_1	0.000	0.000	0.000
S. cholesterol (Silurian cholesterol)	x_2	0.242	0.429	0.573
LDL (Low-density lipoprotein)	x_3	0.002	0.101	0.000
HDL (high-density lipoprotein)	x_4	0.000	0.000	0.000
A.AN (Aplastic anemia)	x_5	0.000	0.000	0.000
Lack of vitamin B12	x_6	-0.472	-2.581	-1.215
The age of patient	x_7	0.152	0.392	0.851
HIV (Human Immunodeficiency Virus)	x_8	-0.454	-0.945	-0.791
The weight of patient	x_9	0.000	0.000	0.000
ITP (idiopathic thrombocytopenic purpura	x_{10}	0.823	0.528	0.204

At quantile level 0.25

In this case the best model is selected in quantile regression model at quantile level 0.25 with AIC (67.776) and BIC (72.52) is

$$y = 0.242x_{2i} + 0.002x_{3i} - 242x_{6i} + 0.152x_{7i} - 454x_{8i} + 0.823x_{10i} + \varepsilon_i$$

In above model, there are six independent variables have affecting in response variable but four independent variables not have effecting in response variable, we can exclude them from our model.

At quantile level 0.50

In this case the best model is selected in quantile regression model at quantile level 0.50 with AIC (64.811) and BIC (71.764) is

$$y = 0.429x_{2i} + 0.101x_{3i} - 2.581x_{6i} + 0.392x_{7i} - 945x_{8i} + 0.528x_{10i} + \varepsilon_i$$

In above model, there are six independent variables have affecting in response variable but four independent variable not have effecting in response variable, we can exclude them from our model.

At quantile level 0.95

In this case the best model is selected in quantile regression model at quantile level 0.50 with AIC (65.912) and BIC (75.428) is

$$y = 0.573x_{2i} - 1.215x_{6i} + 0.851x_{7i} - 791x_{8i} + 0.204x_{10i} + \varepsilon_i$$

In above model, there are five independent variables have affecting in response variable but five independent variable not have effecting in response variable, we can exclude them from our model.

Conclusions and Recommendations

Conclusions: The shrinkage parameters have the ability to achieve the selection of efficient models that achieve good statistical properties. Because the Bayesian method handles the shrinkage parameters as variables whose estimates can be updated constantly, we find that it is the optimal approach for choosing quantile regression models when employing the elastic net method. In this paper, we note that the shrinkage parameters are directly affected by the sample size during all simulation experiments.

Recommendations
We recommend using the Bayesian method in selecting shrinkage parameters. Methods for selecting models that result in selecting a model that is more efficient in representing the phenomenon under study. Extend this study to include other variable selection methods such as Lasso and adaptive Lasso. Also, we recommend using more quantile levels this is to monitor the behavior of variable selection with different quantile levels.

References

1. Akaike H. Maximum likelihood identification of normal autoregressive moving average models. Biometrika. 1973;60(2):255-265.
2. Al-Guraibawi M, Raheem SH, Mohammed BK. A new modified robust Mahalanobis distance based on MRCD to diagnose high leverage points. Pakistan J Stat. 2025;41(1):1-12.
3. Cade BS, Noon BR. A gentle introduction to quantile regression for ecologists. Front Ecol Environ. 2003;1(8):412-420.
4. Koenker R, Bassett GJ. Regression quantiles. Econometrica. 1978;46(1):33-50.
5. Koenker R. Asymptotic properties of M-estimators of conditional quantile. Econometrica. 1981;49(3):817-845.
6. Koenker R. A note on studentizing a test for heteroscedasticity. J Econom. 1981;17(1):107-112.
7. Koenker R, D'Orey V. Algorithm AS 229: Computing regression quantiles. J R Stat Soc Ser C Appl Stat. 1987;36(3):383-393.
8. Koenker R, Geling O. Reappraising medfly longevity: a quantile regression survival analysis. J Am Stat Assoc. 2001;96(454):458-468.
9. Koenker R. *Quantile Regression*. Cambridge: Cambridge University Press; 2005. 366 p.
10. Kostov P, Davidova S. A quantile regression analysis of the effect of farmers' attitudes and perceptions on market participation. J Agric Econ. 2013;64(1):112-132.
11. Mohammed MA, Raheem SH. Determine of the most important factors that affect the incidence of heart disease using logistic regression model (applied study in Erbil Hospital). Econ Sci. 2020;15(56):175-184.
12. Ravikumar P, Wainwright MJ, Lafferty JD, Wasserman L. Lasso for quantile regression. J Am Stat Assoc. 2007;102(479):1119-1130.
13. Schwarz KW. Turbulence in superfluid helium: steady homogeneous counterflow. Phys Rev B. 1978;18(1):245-262.
14. Tibshirani R. Regression shrinkage and selection via the lasso. J R Stat Soc Ser B Methodol. 1996;58(1):267-288.
15. Wang H, He X. Detecting differential expressions in GeneChip microarray studies: a quantile approach. J Am Stat Assoc. 2007;102(477):104-112.
16. Wei Y, Pere A, Koenker R, He X. Quantile regression methods for growth charts. Stat Med. 2006;25(8):1369-1382.
17. Yu S, Zhang G, Li J, Zhao Z, Kang X. Effect of endogenous hydrolytic enzymes pretreatment on the anaerobic digestion of sludge. Bioresour Technol. 2013;146:758-761.
18. Zhang Z, Lu J. Elastic-net quantile regression: a flexible and efficient approach. J Am Stat Assoc. 2018;113(523):1209-1222.
19. Zhou J, Sun HC, Wang Z, Cong WM, Wang JH, Zeng MS, *et al*. Guidelines for diagnosis and treatment of primary liver cancer in China (2017 edition). Liver Cancer. 2018;7(3):235-260.
20. Zou H, Hastie T. Regularization and variable selection via the elastic net. J R Stat Soc Ser B Stat Methodol. 2005;67(2):301-320.
21. Zou H, Zhang HH. On the adaptive elastic-net with a diverging number of parameters. Ann Stat. 2009;37(4):1733-1751.