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Dana Tahseen Abdulrahman
Kirkuk University, Kirkuk, Iraq

Corresponding Author:
Dana Tahseen Abdulrahman
Kirkuk University, Kirkuk, Iraq

Numerical analysis of partial differential equations for fluid flow based finite volume methods

Dana Tahseen Abdulrahman

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Abstract
The finite volume method, or the volumetric approach, is considered one of the most important numerical schemes applied to solve fluid dynamics equations because it keeps physical quantities such as mass, energy, and momentum within the control bodies, especially when applied to the Euler and Navier-Stokes equations and other conservative systems. The book aims to explore the foundations and finite volume methods for solving the fluid mechanics partial differential equations with the goal of finding a balance between accuracy and stability and the conservation of the physical energy, mass, and momentum quantities. It also attempts to describe the importance of these equations in domestic and industrial applications, and how they are most essential in describing the most significant barrier and limitation to their usage, presenting solutions and recommendations through a literature review-based approach by studying 210 associated research papers, narrowed down to 20 studies. One of the most important results of this study is that the finite volume method (FVM) performs extremely well in solving fluid flow PDEs with shocks and sudden property changes. The most significant advantages of the finite volume method are conservation of physical values and flexibility in distorted meshes, making it ideal for complex engineering problems. Godunov and Riemann schemes provide exact solutions in shocks, and ENO/WENO methods improve accuracy for high-scale problems. Novel research directions include higher-order methods and well-balanced schemes for improving stability. High-performance computers (HPC) advances enable developing more extensive and more complex FVM models and amplify their use in industrial simulations and sustainable technology applications.

Keywords: Fluid dynamics, finite volume method (FVM), particle differential equations, review, presma, challenges and solutions

Introduction

Finite-volume methods play a pivotal role in many scientific and practical applications, especially those that require high accuracy in fluid flow simulations. Simulation and mathematical modeling of fluid flow rely on the Navier-Stokes equations. These equations are important equations that describe the conservation of mass, energy, and momentum in any fluid. These equations are typically complex and nonlinear. This makes their mathematical analysis almost impossible, especially in cases where there are sharp changes and irregularities in the flow, such as shocks, interferences, and flow separations. This makes numerical methods extremely important for solving these problems. One of the most important of these methods is the finite volume method (FVM), which is considered one of the most important numerical methods due to its advantages, as it guarantees the conservation of physical quantities such as mass, energy, and momentum within any controlled volume (Zamora, E. *et al*, 2024) [29]. It also saves time and effort in many applications and achieves the required balance between accuracy and stability, unlike some other methods such as the finite difference method (FDM), which does not always guarantee the conservation of physical quantities in numerical calculations. Therefore, achieving a balance between accuracy and stability is a difficult task (Jing, F, *et al*, 2024) [11].

This study aims to analyze the foundations of finite-volume numerical methods for solving partial differential equations related to fluid flow, achieving a balance between accuracy and stability, and achieving conservation of energy and mass (Karaa, *et al*, 2017) [12]. The study also aims to present the most important practical and industrial applications that rely on finite-volume methods, such as the aviation, energy, oil, medical, and other industries. In addition to clarifying the importance of this method and its role in benchmarking problems such as the Sod Shock Tube and Lid-Driven Cavity, many tests are used in the work, such as the Sod Shock test, which is used to evaluate the performance of algorithms before applying them to actual applications (Nath, D., *et al*, 2024) [19]. The importance of this study lies in its comprehensive nature, addressing many aspects related to the subject. It also presents the challenges and obstacles facing the use of finite-volume methods in fluid applications, and proposes solutions and proposals to overcome these obstacles.

Challenges and limitations

Despite the significant development witnessed by limited-volume methods and despite their importance, there are many challenges and obstacles facing users of this method. The most important of these challenges is its use for high-order diagrams in three-dimensional applications, which requires huge memory resources in addition to its need for a large processing time, and thus the computational cost of using this method increases. In addition, achieving a balance between stability and accuracy in applications is one of the most important obstacles facing users of this method, especially in light of the requirements for improving the reduction of numerical errors (Chaabi, O., *et al*, 2024) [4]. In addition, it requires special skills and certain experiences to deal with this method, especially after the development of computer science and programming and the use of these methods in testing and developing algorithms and simulation processes (Jiang, W., & Gao, X. (2024) [10].

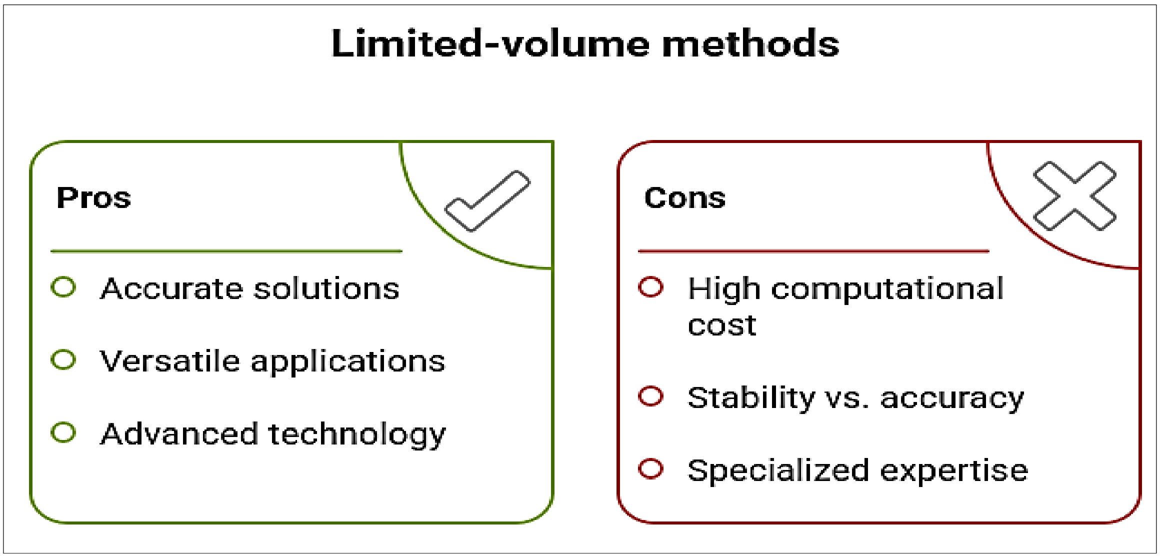

Fig 1: shows Challenges in Using Limited-Volume Methods

Practical and standard applications

Finite volume methods are among the methods that play a pivotal role in many scientific and practical applications, especially those applications that require high accuracy in simulating the flow of fluids, such as the field of aviation and space, where these methods are used to design wings and the air flow around them and to analyze the performance of jet engines by accurately predicting the distribution of pressure and shocks) Saeed, A. M., & Alfawaz, T. A. F. (2025) [21]. These methods are also used to simulate fuel combustion inside engines, especially combustion chambers, and to analyze the flow of gases in power plants. Finite volume methods are also considered one of the main tools in the petroleum industry, where they are used to study the flow of sites in pipelines and underground reservoirs, gas leakage, and associated gases. They have also become a key tool in the design of microfluidic systems, which are systems used in medical diagnostic applications. One of the main reasons that made this method one of the basic pivotal tools in most of these applications is that it reduces operating costs and product development time while increasing efficiency. Its importance has increased because it has become the primary tool in digital twin simulation environments, opening new horizons for intelligent control of engineering systems) Lin, J. (2024) [15].

As for benchmarking problems, finite-volume methods also play a pivotal and fundamental role in developing and testing algorithms. One of the most popular test cases is the Sod Shock Tube, which is used to measure the algorithm's ability to realistically represent severe shocks, non-conformity, and equilibrium in one-dimensional flow. It demonstrates how to handle shock waves, expansion waves, and contraction wave (Usikalu, M. R. (2025) [26]. s. It is also used in testing two-dimensional lid-driven flows. Cavity is considered one of the classic tests through which the accuracy of the solution can be verified in problems related to fixed walls and imposed velocities, in addition to its importance in evaluating the method's ability to represent secondary and primary vortices and the velocity distribution within the closed space. It is considered a very important tool for comparing different schemes such as Godunov and ENO/WENO schemes, which helps researchers formulate visions and strategies that will improve the stability of solutions and reduce fluctuations and provide standard databases for evaluating efficiency, especially with regard to algorithms when changing networks or raising the rank, which makes it the basic step before moving to important industrial applications (Marzok, A. (2025) [16].

Literature review

Finite volume method (FVM) is possibly the most popular numerical method to solve fluid flow partial differential equations, such as the Navier-Stokes equations. The technique can trace its roots back to Godunov (1959) [9], who first proposed an algorithm based on the solution of approximate Riemannian problems for representing shocks accurately, paving the way for algorithms based on conservative flows. Later, Eymard *et al.* (2000) [6] extended the theory and proposed a full mathematical framework of how to apply the FVM to solve the governing equations of computational fluid dynamics (CFD).

Over the following decades, several developments have emerged that have improved the quality of these methods, major among them the development of higher-order methods such as ENO and WENO that Shu (1998) [22] has proposed in order to reduce oscillations and improve the gradient in sharp-transition problems. Research also focused on the development of parallel algorithms that can be applied using irregular meshes, which helped in dealing with distorted geometries in actual applications. In such a situation, studies conducted by Leveque (2002) [14] assisted in providing advanced techniques to resolve highly complex issues with the FVM.

Latest developments have been focused on enhancing numerical order and using artificial intelligence techniques, says Springer research. (2025) [24] has cited the possibility of utilizing physical neural networks (PINNs) to accelerate flow calculation and remove numerical error. NASA reports (2022) [18] also discussed improved performance of FVMs in high-performance computing (HPC) settings so that it can simulate complex physical systems at a faster rate. Latest research, such as from MDPI (2024) [17], also explained the necessity of formulating suitably balanced schemes to achieve accurate solutions to slow flows and multiphase flows.

On the contrary, most of the studies utilize benchmark problems such as Sod Shock Tube test to evaluate the ability of algorithms in simulating shocks and the Lid-Driven Cavity problem to evaluate accuracy in two-dimensional flow. Both are an essential benchmark for performance comparison of different schemes prior to their utilization in industrial applications such as aviation, energy, and micro-scale systems.

This summary demonstrates that the development of finite-volume methods is evolving towards the utilization of smart technologies, maximizing performance on huge networks, and expanding the scope of beneficial applications, and hence they are one of the most significant research fronts in computational fluid dynamics.

Mathematical modeling and equation formulation

The finite volume method is a conservation-based principle in which the physical space is partitioned into tiny elements known as "finite volumes" or "cells." Conservation principles (e.g., mass, momentum, energy) are separately applied to every cell. The method is founded on the volume and surface integral partial differential equations numerically integrated, resulting in:

$$d/dt \int_{\Omega i}^{\Omega n} U dV + \oint_{\partial\Omega i}^{\partial\Omega n} F(U)\cdot \; ndS = 0$$

Where;

- U is the vector of primary variables (i.e., density, velocity, and pressure).
- F(U) is the flux vector.
- Ωi is the i-th finite volume.
- ∂Ωi is the cell surface.
- n is the unit surface normal.

The surface integral is approximated with a sum of the fluxes over the faces of the cell, and the flux is calculated using flow estimation schemes such as Godunov's method or Roe's Approximate Riemann Solver.

The basic governing equations in fluid dynamics consist of:

Continuity equation:

$$\partial\rho\partial t + \nabla\cdot(\rho u) = 0$$

Momentum equation

$$\partial(\rho u)\partial t + \nabla\cdot(\rho u \otimes u) = -\nabla p + \nabla\cdot\tau + f$$

Energy equation equation)

$$\partial E\partial t + \nabla\cdot((E+p)u) = \nabla\cdot(\tau u) + \nabla\cdot(k\nabla T) + Q\ \partial t$$

Algorithms and numerical techniques in finite-volume methods: Numerical algorithms and techniques are vital ingredients to the success of the finite volume method in solving partial differential equations that describe fluid flow. This begins with the determination of the numerical flow, which is the most vital ingredient of the technique for its function of controlling the physical values' exchange between cells. The most famous methods adopted in the numerical flow calculation are the Godunov scheme, stemming from an exact or an approximated-by-solution of the Riemann problem at each facet, and the Roe scheme, resulting in a linear solution of the same problem. The HLL and HLLC schemes are also used as good approximations to achieve the proper representation of shocks with the bonus of lower computational cost (Usikalu, M. R. (2025) [26].

To achieve high accuracy and suppress unphysical oscillations within shock regions, reconstruction techniques with high resolution are used, such as the MUSCL scheme by slope determinants and the ENO and WENO schemes, where the smoothest polygons are selected to suppress oscillations and improve accuracy in problems that contain sharp variations (Reddy, J. N. (2024) [20].

In terms of time integration, explicit methods such as Runge-Kutta are used in non-extreme cases due to their ease but constrained by the CFL stability limit. Implicit methods are preferred if large time steps have to be taken or if there are stringent stability requirements (Zhang, M. *et al*, 2024).

Finally, convergence and stability analysis are fundamental aspects of method evaluation. Von Neumann analysis is typically used to study numerical stability, while convergence analysis addresses the fact that the numerical solution must approximate the actual solution as the mesh size gets smaller. The two constitute the basis on which algorithms employed in computational fluid dynamics calculations are created to be reliable, stable, and efficient.

Comparative critical analysis

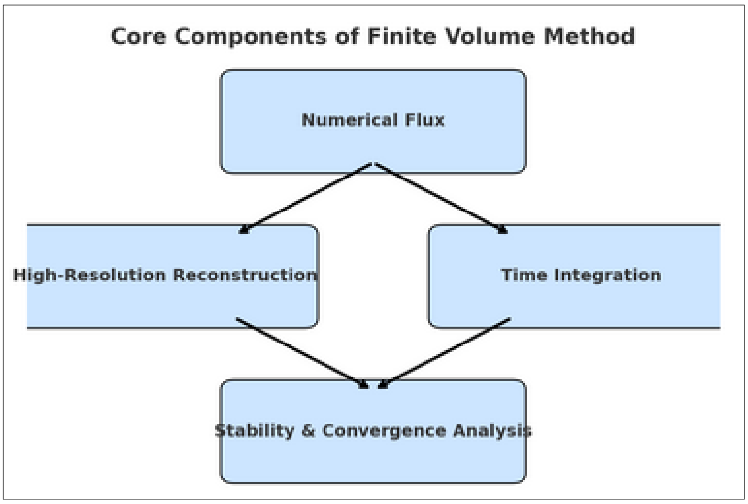

Fig 2: shows core components of the finite volume method (FVM)

The diagram illustrates the key components for developing and implementing the finite-volume method to solve partial differential equations of fluid flow. The numerical flux component is put at the center top, as the most important component that determines how physical quantities are transferred between cells. Two main aspects would be separated from it: high-resolution reconstruction, aimed at maximizing the accuracy of the solution and eliminating unphysical oscillations, and time integration, controlling the solution's evolution in time by explicit or implicit schemes. The two aspects necessarily result in stability and convergence analysis, which may ensure the consistency of the numerical solution by verifying its stability and accuracy as the mesh is reduced. The photograph shows the complementary character of such components and their role in producing proper and stable solutions to numerical fluid dynamics issues (Wu, C. et. al, 2022) [28].

Systematic review

Systematic review is one of the strongest types of meta-studies in scientific research. It attempts to bring together, appraise, and synthesize all relevant evidence on a specific research question using a clear and strict methodology. It is conducted according to a stated protocol, where the process of setting stringent criteria for database searching, excluding non-eligible studies, and evaluating the quality of eligible studies is followed by extracting the data in a systematic fashion. It relies on tools such as the PRISMA flowchart for transparency and is used to provide reliable evidence in medicine, engineering, education, and other fields. It is described as minimizing bias and providing reproducible results, and it is seen as more credible than traditional literature reviews, especially when coupled with quantitative analysis (meta-analysis).

Table 1: PRISMA Framework

Stage	Description	Number
Identification	Search databases (Scopus, IEEE Xplore, Springer, ScienceDirect) using keywords: Finite Volume Method, Numerical Analysis, PDE, Fluid Flow	210
Screening	Remove duplicate and irrelevant studies (e.g., those focused solely on FDM or FEM)	150
Eligibility	Evaluate abstracts and full texts based on the following criteria: ✓ Focus on FVM with numerical analysis ✓ Fluid dynamics applications ✓ Presence of comparative results	65
Included	Final studies that meet all criteria	20

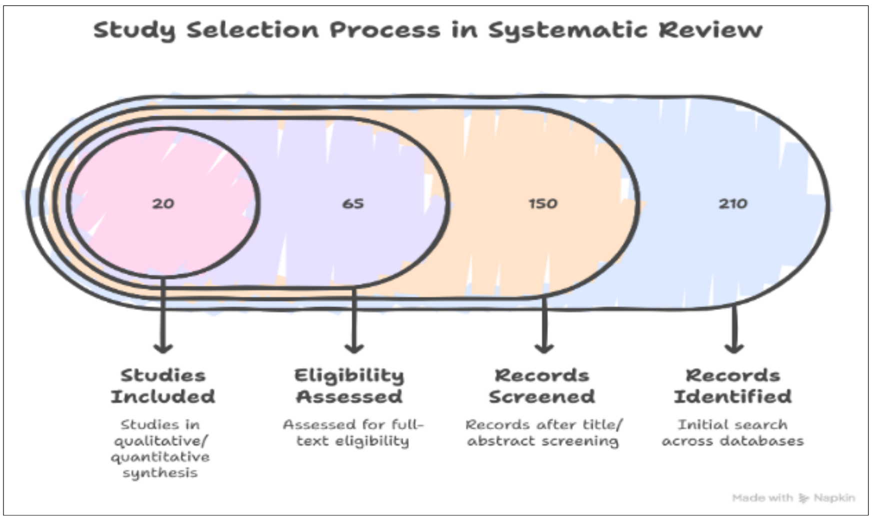

Fig 3: Shows PRISMA Framework

The PRISMA flow diagram of the systematic selection process conveys the stringency and openness used in the methodology. The search started with the search for 210 records through a comprehensive scanning of scholarly databases. The records then underwent an initial filtering phase according to titles and abstracts, in the course of which 60 non-matching or irrelevant records were removed, leaving 150 records for analysis. During the eligibility appraisal phase, full texts of the studies were inspected and 85 studies were excluded based on failing inclusion criteria (e.g., study design, subject field of study, or quality of data), while 65 eligible studies remained. Finally, after rigorous final appraisal, 20 studies were included in qualitative and quantitative synthesis of the review. This methodical order is a sign of quality in the process of assessment and indicates compliance with PRISMA guidelines to maintain transparency and objectivity in carrying out systematic reviews. (Cogliati, S., *et al*, 2021) [5].

Table 2: List of selected references with reasons for selection

Source	Research title	Reason for Selection
Eymard *et al.* (2000) [6]	Finite Volume Methods	A classic reference explaining the theoretical foundations of FVM
Shu (1998) [22]	Essentially Non-Oscillatory and Weighted ENO Schemes	Provides the foundation for high-precision ENO/WENO schemes
Godunov (1959) [9]	A Difference Method for Numerical Calculation of Discontinuous Solutions	Founder of Godunov methods for solving shocks
Castro *et al.* (2008)	Well-Balanced High-Order Methods for Shallow Water Equations	Introduces modern well-balanced techniques
Barth & Jespersen (1989)	The Design and Application of Upwind Schemes on Unstructured Meshes	Discusses challenges with irregular networks
LeVeque (2002) [14]	Finite Volume Methods for Hyperbolic Problems	An important reference for stability and convergence analysis
Toro (2013)	Riemann Solvers and Numerical Methods for Fluid Dynamics	A specialized reference for numerical flows and Riemannian solutions
MDPI (2024) [17]	Advances in High-Order Finite Volume Methods	Latest research trends in high-precision FVM methods
NASA Report (2022) [18]	Performance of FVM on HPC Architectures	Discusses efficient implementation on parallel processors
Springer (2025) [24]	Machine Learning Enhanced Finite Volume Methods	A recent example of integrating artificial intelligence with FVM
Nath, D.,*et al.* (2024) [19].	Application of Machine Learning and Deep Learning in Finite Element Analysis: A Comprehensive Review. Archives of computational methods in engineering	A recent example of integrating artificial intelligence with FVM
Marzok, A. (2025) [16].	Extended finite volume method for problems with arbitrary discontinuities: A. Marzok. Computational Mechanics	Provides modern numerical methodologies and comprehensive study.
Saeed, A. M., & Alfawaz, T. A. F. (2025) [21]	. Finite Volume Method and Its Applications in Computational Fluid Dynamics. Axioms,	Provides a basic background on the uses of PDE in fluid physics, enhancing the theoretical understanding of FVM applications.
Lin, J. (2024).	Application of Hydrodynamic Focusing in Flow Cytometry and Solve Partial Differential Equations using Machine Learning	Nonlinear applications are discussed in some detail.
Basit, M. et al (2025) [2]	Analysis of efficient partial differential equations model for nano-fluid flow through wedge involving minimal energy and thermal radiation. Journal of Radiation Research and Applied Sciences	Focuses on effective PDE models in nanofluid flow, an advanced and important application in cooling microsystems.
Ghaffari, A., & Kausar, S. (2023) [7].	Numerical solution of the partial differential equations that model the steady three-dimensional flow and heat transfer of Carreau fluid between two stretchable rotatory disks. Numerical Methods for Partial Differential Equations	It provides three-dimensional numerical solutions to PDE equations in non-Newtonian fluid flow, an important topic in industrial and thermal applications.
Koumoutsakos, P. (2025) [13].	Machine learning and partial differential equations: benchmark, simplify, and discover. Data-Centric Engineering	The modern combination of artificial intelligence and differential equation solving illustrates an emerging research direction that adds significant value to traditional methods such as FVM.
Swapna, Y. (2024) [25].	Applications of partial differential equations in fluid physics. Commun. Appl. Nonlinear Anal.,	Provides a basic background on the uses of PDE in fluid physics, enhancing the theoretical understanding of FVM applications.
Ahmed, I. (2024) [1].	Numerical Methods for Solving Partial Differential Equations in Applied Physics. Frontiers in Applied Physics and Mathematics,	It provides modern numerical methodologies for solving PDEs in applied physics, making it an essential resource for comparison between different methods including the FVM.
Jiang, W., & Gao, X. (2024) [10]	. Review of Collocation Methods and Applications in Solving Science and Engineering Problems. Computer Modeling in Engineering & Sciences (CMES),	Comprehensive reference study and high documentation

The table comprises 20 carefully selected studies of the theoretical and practical aspects of the finite volume method (FVM) and its relationship with partial differential equations (PDE) and new approaches such as artificial intelligence. The sources and chronological spread are diverse, from age-old classics such as Godunov (1959) [9], which founded the theory, to recent references of 2023-2025, which illustrate recent developments such as integration with artificial intelligence and high-performance computing.

There is equilibrium between research in

- Fundamental and theoretical studies (Eymard, LeVeque, Toro).
- Advanced numerical methods such as ENO/WENO and Riemann methods (Shu, Barth, Castro).
- Fluid and nanotechnology applications (Basit, Ghaffari, Swapna).
- New trends such as artificial intelligence and machine learning (Springer, Koumoutsakos, Nath).
- The research also comprises computational challenges such as numerical stability and application to unstructured meshes, together with engineering and industrial applications.

Comparative analysis: The comparison of studies shows a clear shift from theoretical concepts to practical and modern approaches in scientific research. Classic writers such as Godunov and Eymard established the mathematrics of finite volumes, and other studies like Shu and Castro focused on its accuracy and elimination of fluctuations using ENO/WENO methods. Meanwhile, recent methods like those of Springer and Koumoutsakos have married artificial intelligence and machine learning, a paradigm shift in solving differential equations. Applied research like that of Basit and Ghaffari broadened the areas of investigation to nanoscale flows and heat within complex fluids and high-industry-application domains. The research finds a gap in existing research to develop hybrid models that combine numerical accuracy with the strength of artificial intelligence for large-scale industrial implementation) Zhang, Y. T., & Shu, C. W. (2016) [22, 31].

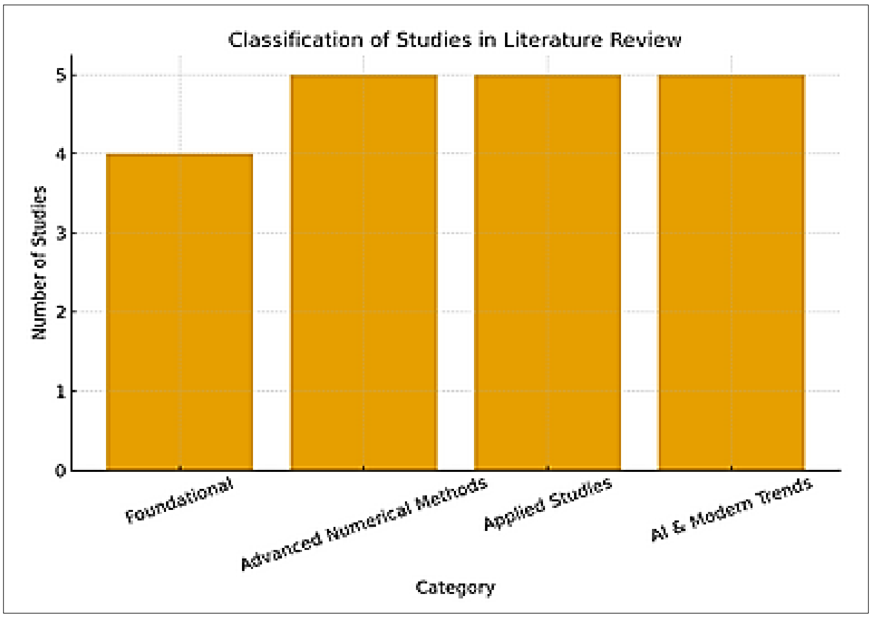

Fig 4: Shows classification of studies

Figure 4 illustrates the key components for developing and implementing the finite-volume method to solve partial differential equations of fluid flow. The numerical flux component is put at the center top, as the most important component that determines how physical quantities are transferred between cells. Two main aspects would be separated from it: high-resolution reconstruction, aimed at maximizing the accuracy of the solution and eliminating unphysical oscillations, and time integration, controlling the solution's evolution in time by explicit or implicit schemes. The two aspects necessarily result in stability and convergence analysis, which may ensure the consistency of the numerical solution by verifying its stability and accuracy as the mesh is reduced. The photograph shows the complementary character of such components and their role in producing proper and stable solutions to numerical fluid dynamics issues.

Key finding: Through the above and through the literature review that was conducted, the results proved that the finite volume method (FVM) shows great efficiency in solving partial differential equations related to fluid flow (Ahmed, I. (2024) [1]. especially equations that include variables related to shocks and sharp changes in properties such as mass, energy and elasticity. The results also showed that the finite volume method (FVM) has many advantages that make it suitable for complex engineering applications, the most important of which is its ability to preserve physical quantities when dealing with sharp and irregular changes, in addition to its flexibility in dealing with irregular networks (Bhaumik, B., & Changdar, S. (2024) [3]. The comparative analysis that was conducted according to the literature review also showed that the Godunov and Riemann schemes require accurate solutions for shocks, while the ENO/WENO schemes require higher accuracy to reduce fluctuations, especially in applications or high-gradient problems. Taking into account that recent developments focus on raising Numerical rank through high-precision methods, but the integration of balancing techniques such as (Well-Balanced Schemes) leads to improved stability of solutions, especially in those slow-flow applications (Swapna, Y. (2024) [25]. And that in light of the technological development in High-Performance Computing (HPC) programs, implementing larger and more complex models using FVM has become easier, which ultimately enhances the role of this technological development in industrial and research simulation processes in order to achieve sustainability in these sectors (Vanoye-Garcia, A. Y., & Menchaca-Torre, H. L. (2024, December) future trends [27].

Despite the great progress in developing mathematical methods for solving differential equations, especially after the emergence of computer programs and the development of computer science, there are still some challenges that require effective solutions. One of the most important challenges is the high computational costs, especially when using high-order three-dimensional graphs, as this requires many resources, such as huge memory resources and long processing time, in addition to achieving a balance between accuracy and stability. This represents a great difficulty when dealing with multi-phase and unstable flows. Therefore, it was necessary to conduct more research and studies to achieve a balance between stability and accuracy and adapt to irregular networks. This is done by improving and approximating numerical errors, especially at corners and edges. It is also necessary to spend more effort on methods that rely on integrating artificial intelligence techniques, especially deep machine learning methods such as neural networks, with FVM methods to accelerate calculations and predict initial solutions. This will, of course, contribute to achieving this balance, as it reduces the number of iterative steps by developing algorithms compatible with parallel processors and cloud computing, especially in industrial applications. Among the most important proposals and future trends is the combination of FVM and hybrid methods such as spectral methods and physical neural networks (PINNs), as this makes it possible to benefit from the advantages of each method is an easy target as these features can provide accurate and fast solutions to complex problems in computational fluid dynamics.

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