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Applications of stochastic modeling for investment decision-making under market uncertainties

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Abstract
This paper develops scenario-based decision models in support of investment decision-making and analysis under market uncertainties by applying stochastic programming and dynamic stochastic programming in several application contexts, such as the optimal harvesting of forest stands, the management of electricity contract portfolios, the investments in power plants, and the valuation of real options in new product development. The applications demonstrate the feasibility of scenario-based modeling approaches, among others.

Keywords: Decision-making, stochastic programming, managerial flexibilities, financial modelling, stochastic modelling

Introduction

Profit-seeking organizations make investment decisions with financial implications under uncertainty about returns. These returns can be uncertain due to multiple market sources, such as the cost of raw materials, variable demand levels, or existing competing products. Moreover, investment decisions may involve managerial flexibilities, or real options (Trigeorgis 1996), which can allow, for example, postponing the investment to obtain more information or upgrading the facility later on to improve its efficiency. Further, organizations may have to consider the portfolio of assets that can be invested in when they seek to hedge risks. In this context, the paper analysis the investment decision-making and its financial implications under market uncertainties while acknowledging risks and real options. Models used to support of these decisions making represent a methodologically challenging and practically relevant research field combining decision analysis, stochastic modelling, and financial modelling.

Several approaches have been developed to support investment decision analysis. The conventional approach is to calculate the net present value of the investment by discounting the expected cash flows as presented in corporate finance (Brealey *et al.* 2008). However, conventional net present value calculations do not account for managerial flexibilities that allow an investor to adjust the course of the investment (e.g., Dixit and Pindyck 1994). The stochastic dynamic programming reflect the flow of information in the investment decision-making. It is based on backward induction and recursive optimization over a scenario tree that represents uncertainties. Thus, a crucial part of the problem formulation is the generation of the scenario tree. This is typically done via a lattice that is a discrete time and state approximation of the underlying stochastic processes. While the dynamic stochastic programming approach is suitable for many investment valuation and appraisal contexts, it is neither flexible enough to accommodate constraints for cash-flow positions during intermediate periods, nor can it easily account for dependencies among decisions in which future decisions depend on past decisions. Stochastic programming approaches (e.g., Birge and Louveaux 1997) do not have these limitations because they are based on mathematical programming in which constraints can be set in any time periods and can be used to link dependencies between past and future decisions. In stochastic programming approaches, uncertainties are modeled using a scenario tree as in stochastic dynamic programming thought the scenario generation can be more complex than in stochastic programming approaches (e.g., Hoyland and Wallace 2001, Growe-Kuska *et al.* 2003). An additional part of the problem formulation in stochastic programming approaches is the specification of the non-anticipatively constraints which ensure that decisions are taken without knowing in advance the future outcomes.

While stochastic programming approaches are more intensive computationally, improvements in computational capabilities have made them tractable and applicable even in large scale problems (e.g. Sen *et al*. 2006). This paper develops scenario-based decision models in support of investment decision-making and analysis under market uncertainties by applying stochastic programming and dynamic stochastic programming in several application contexts, such as the optimal harvesting of forest stands, the management of electricity contract portfolios, the investments in power plants, and the valuation of real options in new product development. The applications demonstrate the feasibility of scenario-based modeling approaches, among others when

- The management of risks is conducted in multiple time periods,
- There exist several correlated uncertainties,
- Investors are heterogeneous in their risk aversion, for example, and
- There are several actors, who interact in competitive markets.

Methodological Background

In scenario-based investment modelling, regardless of the application context, it is important to consider the following five modelling components:

1. The formulation of the objective function,
2. The generation of scenarios that represent the uncertainties,
3. The measurement and management of risks,
4. The representation of managerial flexibilities, and
5. The portfolio of investment opportunities.

As it shall be argued that these modelling components need to be considered concurrently as they are interdependent. For example, decision objectives may use for the measurement of risks, risks depend on the overall portfolio and can be measured if the uncertainties are modeled, and uncertainties influence the value and use of managerial flexibilities.

Formulation of Objective Function:

The objective function can be formulated in alternative ways, among others

1. Maximizing the expected value of the investment subject to risk constraints.
2. Minimizing risks subject to constraints on the expected return.
3. Maximizing the expected value of the investment from which is subtracted a risk term.
4. Maximizing the probability of achieving a return over a target level subject constraint 4 on the expected return.

The first approach is prevalent in practice, because many companies, particularly in the financial sector, seek to maximize the profitability of the investment subject to regulatory constraints on risk.

Modelling of Uncertainties

The generation of scenarios that represent the uncertainties can be approached in different ways. To begin with one approach is to generate scenarios based on decision analytic methods that rely on the subjective estimations of experts. Thus, scenario analysis makes it possible to analyze long-term future uncertainties that are inherently different from those that are considered relevant today. Further, scenario analysis based on experts' opinions can represent non-traditional stochastic processes and risk factors, such as political, operational, model, and liquidity risks. If, however, there are reasons to believe that historical data may characterize future developments of uncertainties, then it is appealing to consider methods that are based on data, for example, by deriving the parameters for scenario generation to match the moments or other statistical properties of data (e.g., Casey and Sen 2005, Pennanen 2005, Høyland and Wallace 2001, Smith 1993, Gülpinar *et al*. 2004). These approaches include the following scenario generation methods: (i) simulating scenarios from their distributions, (ii) selecting scenarios by solving optimization problems, which satisfy the stated conditions, and (iii) using a hybrid of these two approaches. Such approaches can approximate the stochastic process of a single uncertainty using, for example, the recombining binomial tree model of Cox *et al*. (1979). Their model provides an arbitrage-free pricing environment by deriving risk-neutral scenario probabilities under which the scenario outcomes are discounted using the risk free rate. Similar approaches can also model multiple correlated stochastic processes of several uncertainties that can exhibit mean reversion and volatility clustering. These approaches have several advantages that is they can match the market observed prices of the financial contracts, they can provide an arbitrage free pricing environment, and their parameters can be estimated based on historical time series data. The granularity of the generated scenarios may not be at the level of the required accuracy, particularly in the case of managing extreme risks. One approach is to apply the importance five sampling method. The principle in importance sampling is to generate scenarios that relate to a certain percentile of the probability distributions in order to capture more accurately extreme outcomes. Methods for reducing the number of scenarios have been developed for problems that would be otherwise intractable. These methods rely on algorithms that seek to reduce the number of scenarios so that the remaining scenarios approximate the original problem in terms of chosen probabilistic measures such as mean or higher moments. The appropriateness of the scenario generation method depends on the application context. If, for example, the problem deals with financial portfolio optimization or financial contracts, then a requirement for the generated scenarios is that they provide an arbitrage free pricing environment (e.g., Klaassen 2002).

Measurement of Risk and Characterization of Risk Aversion: In the classic mean-variance model (Markowitz 1952); risk aversion can be modeled by setting constraints for the standard deviation or variance. However, because variance and the standard deviation penalize upside potential as well, other risk measures have been suggested. These include measures such as (i) the lower semi-absolute deviation, which measures the expected shortfall of the terminal cash position relative to the expectation and (ii) the expected downside risk, which measures the downside deviation relative to a pre-specified target level. Alternatively, risk aversion can be based on the expected utility theory, where the returns are mapped to a utility level using utility functions that are strictly increasing and concave for risk-averse investors (e.g., Delquie 2008). In

decision analysis, risk aversion using utility functions is often combined with decision trees and dynamic stochastic programming (e.g., Keeney and von Winterfeldt (1991), Smith and Nau (1995), Smith and Mccardle 1998). Yet, extreme risks are often of the greatest concern to decision makers. To model the aversion of the extreme risks, constraints can be set for the value-at-risk (VAR) risk measure, 6 which quantifies the maximum amount of money that may be lost over a certain period of time, with a certain level of confidence. While VAR is the de facto standard of the financial industry (e.g., Risk Metrics Group 2009), it has been criticized (e.g., Embrechts *et al*. (1999), Alexander and Baptista 2002, Szegö 2002) because it is not a coherent measure, i.e., it fails to fulfil the subadditivity requirement, with the result that diversification may increase VAR. Due to this deficiency, an alternative risk measure Conditional-VAR has been proposed, which measures the expected loss with a confidence level ß ∈ [0, 1], conditional on the occurrence of the tail event 1 − ß. CVAR is a coherent and a convex risk measure and hence suitable for linear optimization problems. In practice, companies seem to use risk constraints that are set within their financial planning models for investment in terms of cash flows, such as cash-flow-at-risk, which is a cash-flow based version of VAR.

Representation of Real Options

Real options offer managerial flexibility, whose value can be significant enough so that it needs to be explicitly included in the investment valuation. The five most commonly cited managerial flexibilities are decisions to (i) abandon, (ii) defer, (iii) expand, (iv) contract and (v) switch the operating mode of investments (Trigeorgis 1996). An investment opportunity can constitute even a set or a sequence of real options as presented by Grenadier and Weiss (1997). The value of managerial flexibilities is fundamentally driven by uncertainties. However, as Huchzermeier and Loch (2001) and Santiago and Vakili (2005) demonstrate, an increase in an uncertainty does not necessarily increase the value of managerial flexibilities. Furthermore, Smit and Trigeorgis (2004) highlight the importance of considering competition when managerial flexibilities in R&D projects are evaluated. They suggest, among others, that the value of a managerial flexibility to delay a product's launch may be eliminated by competition. The valuation of an investment with managerial flexibilities requires a holistic approach in which the project and its embedded managerial flexibilities are valued together. Holistic approach is essential because the value of the managerial flexibilities is not necessarily additive.

Portfolio of Investments

Portfolio of investments is not, in general, the sum of the risks of the individual investments because investments can be correlated, such that they hedge each other's risks thereby reducing the risk of the portfolio (Markowitz 1952). Furthermore, the available resources may prevent investments in all desirable opportunities resulting in an optimal portfolio selection problem. Such resource-constrained portfolio problems can be particularly challenging when the investment decisions are of the "no go /go" type resulting in a knapsack optimization problem.

Approach Method and Key Managerial Questions to the Method: To validate the feasibility of the scenario-based modelling approach and its benefits, it is pertinent to apply scenario-based modelling in a wide range of investment decision contexts. These decision contexts should be representative enough to draw general conclusions regarding the approach. In particular, the following four perspectives are considered when selecting the application contexts. First, the selected application contexts need to reflect how risks can be managed at multiple levels, e.g., several time periods and confidence levels concurrently. Risk management in multiple concurrent time periods is needed, for example in the financial sector because risks need to be curtailed consistently below a pre-specified level due to regulatory reasons. Further, multiple level risk management may be needed if a firm is close to financial distress or if the planning horizon is long as in the case of a nuclear power plant whose investment is evaluated over its entire operating time. Second, the selected application contexts should include investment decision-making with multiple correlated uncertainties of the variable. Investment decisions often have several correlated uncertainties, for example if the investment deals with a facility that provides services or goods, whose demand and price are uncertain. One approach to accommodate this is to use revenue as a numeraire to represent the impact of both uncertainties. This approach can be computationally less intensive. However, when the value of the investment depends on how the operations are managed, this approach may not be suitable, because it does not provide information about the demand and price levels, which may be needed in the management of operations. Hence, the explicit representation of multiple correlated uncertainties can be helpful even if it is computationally more intensive, because the number of scenarios increases exponentially in the number of uncertainties and time periods. Besides evaluating the computational tractability of the scenario-based approach when including explicitly multiple correlated uncertainties, it is also important to assess the need to model the correlations themselves. Third, the application contexts should be selected to reflect the feasibility of the scenario based modeling approach in incorporating the heterogeneity of investors. Investors can be heterogeneous, among others, in terms of the level of risk aversion, the financial conditions, and the existing asset portfolio. The representation of investor heterogeneity is beneficial, for example when the impacts of different Government policies for different types of investors are analyzed and how these policies may influence the evolution of the industry as a whole. Fourth, it is relevant to consider application contexts in which competition is present, as is the case in industries that develop new products. Besides the challenge to model and represent the different levels of the competition in scenario-based approach, it is also of interest to consider the impacts of the competitive environment on investment decision making.

Summary of Main Findings

The below table that is Table 1.1 summarizes the key results of the paper which provides brief answers for the research questions.

Table 1.1: Main contributions

Chapter	Key research questions	Methodological contributions	Essential findings
1	How can a forest owner manage risks of the forest stand portfolio efficiently? What are the implications of applying several risk constraints concurrently?	Introduces a multi-level risk management in the forest portfolio optimization.	The reduction of extreme risks is initially efficient, in terms of reducing significant amount of risk with small decrease in the expected terminal wealth, but as more risk is reduced the less efficient it becomes. The introduction of risk constraints at several time periods allows forest owners to curtail risks according to their preferences.
2	What are the main drivers of the risks faced by electricity retailers with different risk preferences under price and demand uncertainties?	Develops a framework for dynamic portfolio analysis that accounts for correlated uncertainties.	Risk-averse electricity retailers are most susceptible to the drivers of forward risk premiums, while competitive electricity retailers to the price related uncertainties
3	How does climate policy risk influence investment behavior and market structure in the electricity sector?	Extends the analysis of investment decisions to account for heterogeneous firms.	Carbon policy uncertainty leads to more concentrated and less competitive markets.
4	How does competition affect the value of real options and their interactions in new product development?	Includes the competitive environment in investment analysis.	The value of real options may not increase monotonically with increasing competition. The competition affects whether options are complements or substitutes

References

1. Adkins Paxson RD. “The effect of depreciation allowances on the stochastic replacement decision”. Real Options Conference, Rio de Janeiro, 2008.
2. Alvarez LHR, Koskela E. The forest rotation problem with stochastic harvest and amenity value, Natural Resources Modeling 2007;20(4):477-509.
3. Artzner P, Delbaen F, Eber JM, Heath D. Coherent measures of risk. Mathematical Finance 1999;9(3):203–228.
4. Bell ML, Hobbs BF, Elliott EM, Ellis H, Robinson Z. An evaluation of multi criteria methods in integrated assessment of climate policy. Journal of Multi-Criteria Decision Analysis 2001;10(5):229-256.
5. Benati S. The optimal portfolio problem with coherent risk measure constraints. European Journal of Operational Research 2003;150(3):572-584.
6. www.google.com.