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Optimizing inventory costs in warehousing with linear time-dependent demand and preservation technology for degrading products

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Abstract
This research paper emphasis to resolves the problem of reducing costs of inventory of any type of warehouses for individual commodities whose demand is linearly growing time to time. Due to their deteriorating nature, owner get high losses if these types of items were not used early. To avoid these losses, Preservation technologies are suggested as an inexpensive intervention. A mathematical model related to the inventory is developed along with the demand of time-dependent and efforts in preservation. I have explored a numerical problem with the various conditions and situations for different items. It is confirmed that by various preservation methods into inventory planning considerably minimizing costs related to products deteriorations.

Keywords: Inventory optimization, degrading products, time-dependent demand, preservation technology, warehouse inventory model, cost minimization

1. Introduction

Inventory management is a challenging task that needs careful planning, oversight, and control. Businesses can quickly satisfy consumer demand, save expenses, maximize cash flow, and enhance overall operational efficiency with effective inventory management. Businesses can attain appropriate inventory levels, reduce stock outs and surplus inventory, and improve customer satisfaction by using strategies including demand forecasting, order management, inventory tracking, and inventory control models [3]. Continual evaluation, adjustment, and cooperation with suppliers and other supply chain players are necessary for effective inventory management.

Three major reasons to maintaining an inventory can be stated as:-

- **Time:** A certain amount of inventory stock is required which can be used during “lead time” which is the time delay in supply chain from supplier to the customer.
- **Unpredictability:** A certain stock of inventories is kept meeting during the changes in demand, supply & product flexibility.
- **Economies of scale:** Inventory is required due to economies of scale which results from purchasing, relocating & keeping the stock in quantity.

Demand forecasting is an important part of inventory management since it enables companies to foresee future demand for their goods [2]. Businesses may accurately estimate future demand patterns by examining past data, market trends, and customer behavior. They can change their inventory levels based on this information to make sure they have the proper products accessible when they need them. By predicting demand, firms may avoid stock outs, which can result in lost revenue and disgruntled consumers when demand exceeds supply [7]. Additionally, it aids in avoiding overstocking, where excess inventory uses up capital and results in needless holding expenses.

2. Warehouse inventory models for degrading products

Deteriorating products are products whose quantity or quality depletes over time. Some general inventory models for these kinds of products are:

- Basic EOQ model with deterioration
- Time-dependent demand models
- Models with preservation technology
- Partial backlogging models

In the paper, a linear time-dependent demand function is adopted, and deterioration is mitigated by the use of preservation technology.

In contemporary logistics and supply chain systems, inventory management of products that deteriorate or degrade over time is a serious challenge. In contrast to traditional inventory models that assume products are in usable form forever, deteriorating inventory models acknowledge that products like food, drugs, flowers, and chemicals depreciate or diminish due to natural factors like spoilage, evaporation, or chemical disintegration. Such models play a pivotal role in industries where quality control and minimizing waste are very important [9].

Researchers have made a number of warehouse inventory models that consider product deterioration in the last few decades. One basic model belonging to this class is the Economic Order Quantity (EOQ) model but with an added constant rate of deterioration. The deterioration of inventory items is often represented as a fraction of the existing stock diminishing over time, typically quantified through a decay rate parameter (θ). While this approach facilitates basic analysis, it relies on the assumption of constant demand an assumption that does not consistently reflect real-world scenarios [11].

To address this limitation, advanced models have been developed that incorporate demand patterns changing over time. These are frequently modeled using linear, exponential, or power-law functions. Among them, linear time-dependent demand is especially useful for capturing demand trends that shift uniformly whether increasing or decreasing due to influences such as market expansion, seasonal variation, or promotional activities. The inclusion of preservation technologies and techniques into various inventory models for desirables products has gained attention as a planning to mitigate losses from detrition. Solutions like refrigeration, vacuum sealing, and chemical preservatives are employed to help for protection from various losses. These solutions and models aim to provide a balance between the expenses associated with preservation and the benefits derived from extended product usability [1].

In present warehouse conditions, the selection of appropriate inventory model relies on various factors such as types of products, pattern of demand, warehouse capacity, and possibilities of technology. Incorporating of degradation problems, demands, and preservation measures provides a more integrated and realistic process for inventory decision and cost reduction.

3. Model Assumptions

- Demand rate is a linear function of time: D(t)= a+bt
- Shortages are not allowed.
- Deterioration is constant over time (rate = θ).
- Preservation technology reduces deterioration cost by factor γ at a cost Cp.
- Inventory is replenished instantaneously.
- Time horizon T is finite.

4. Notations

Symbol	Description
D(t)	Time-dependent demand = a+bt
Q(t)	Inventory level at time t
θ	Deterioration rate
T	Cycle length
h	Holding cost per unit per time
Co	Ordering cost per cycle
Cp	Preservation technology cost per unit time
Γ	Effectiveness factor of preservation
TC	Total cost per cycle

5. Inventory Model Formulation

The inventory depletion equation with deterioration and demand is:

$$\frac{dQ(t)}{dt} = -D(t) - \theta Q(t)$$

With $D(t) = a + bt$, and integrating over $[0, T]$, we derive:

$$Q(t) = \left(\frac{aT + \frac{bT^2}{2}}{1 - e^{-\theta T}} \right) e^{-\theta t}$$

Total Cost (TC):

$$TC = \frac{C_0}{T} + \frac{h}{t} \int_0^T Q(t)dt + \frac{\theta}{T} \int_0^T Q(t)dt - \gamma Cp$$

6. Numerical Problem

Given:

$$a = 100, b = 10, \theta = 0.05, h = ₹2, Co = ₹500,$$

$$Cp = ₹100, \gamma = 0.7, T = 5$$

Step 1: Total Demand over time

$$D_T = \int_0^5 (100 + 10t)dt = 100t + 5t = 500 + 125 = 625\ units$$

Step 2: Average Inventory Level (approximated numerically)

Using midpoint approximation, deterioration-adjusted inventory ≈ 312 units

Step 3: Total Cost Calculation

- Ordering cost = ₹500 / 5 = ₹100
- Holding cost = ₹2 × 312 / 5 = ₹124.8
- Deterioration cost = ₹0.05 × 312 / 5 = ₹3.12
- Preservation benefit = ₹100 × 0.7 = ₹70

$$TC = 100 + 124.8 + 3.12 - 70 = ₹157.92$$

7. Graphical Representation

Inventory level over time with and without preservation

- Without preservation, faster decay is visible.
- Preservation maintains a higher average inventory level.

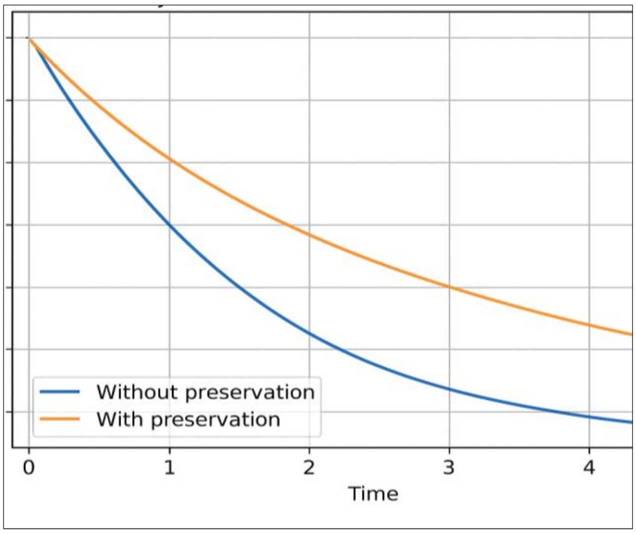

Fig 1: Inventory level over time with and without preservation

8. Results and Discussion

The numerical example illustrates that preservation technology, when optimally integrated, can result in huge cost reduction. The cost of deterioration is largely compensated for by the effort of preservation. Higher stock turnover from increased demand over time results in lower cumulative deterioration yet further enhancing cost-effectiveness. The analysis reveals that preservation cost is warranted and makes inventory sustainable. Graphs confirm that time-dependent demand increases inventory turnover, which is helpful when properly managed with technology.

9. Conclusion

This paper proposes and analyzes a warehouse inventory model designed for degrading products under linear time-dependent demand. The integration of preservation technology proves effective in minimizing total inventory-related costs. The study concludes that adopting such models supports better operational decisions and economic benefits in supply chains dealing with perishable or degradable goods.

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