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Integrated inventory models under time-dependent demand, deterioration, lost sales, and trade credit in a supply chain environment

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Abstract
Inventory management continues to mainly evolve as the global supply chains face increasing complexity from the market volatility, inflationary trends, customer behaviour, as well as the supply network form of dependencies. This paper presents an integrated technique to inventory modelling that carries deterministic and probabilistic call for, volume flexibility, product deterioration, lost income, and permissible put off in payments. Three principal stock modelling frameworks are explored, specifically deterministic-probabilistic models with quantity flexibility and lost sales, ramp-type seasonal call for fashions with deterioration, and integrated seller-purchaser fashions with trade credit underneath inflation. Mathematical formulations are derived for every model, followed via optimization conditions and solution algorithms. Numerical illustrations support the sensible utility of the proposed models. This examines gives considerable theoretical and sensible insights for inventory and delivers chain optimization.

Keywords: Inventory management, time-dependent demand, product deterioration, lost sales, trade credit, supply chain optimization, volume flexibility, deterministic and probabilistic models, ramp-type demand, inflation-adjusted inventory models

Introduction
Operations Research (OR) has mainly had become a proper cornerstone of the modern decision-making in supply chain as well as the inventory management. Rooted in analytical modelling, OR affords a systematic basis for fixing complex logistical, economic, and operational troubles. One of its maximum distinguished programs lies inside the improvement of stock models aimed toward balancing price-performance with carrier overall performance.
Traditional Economic Order Quantity (EOQ) and Economic Production Quantity (EPQ) fashions expect regular demand and static value parameters. However, real-international supply chains function underneath volatile conditions involving time-various call for, deteriorating inventory, misplaced sales, and alternate credits (Bardhan *et al.*, 2021) [1]. With the upward thrust of worldwide alternate, dynamic customer expectancies, and useful resource constraints, modern-day inventory structures must comprise bendy production, probabilistic call for, and strategic charge rules. This paper synthesizes and extends latest improvements in inventory modelling by means of integrating a couple of dimensions of uncertainty and fee into a cohesive choice-making framework.

2. Deterministic and Probabilistic Inventory Models with Volume Flexibility and Lost Sales
2.1 Conceptual Framework
Classical inventory fashions, inclusive of the Economic Order Quantity (EOQ) and Economic Production Quantity (EPQ) models, form the muse of traditional stock control. These models are mostly deterministic, assuming that parameters like manufacturing price, demand, and charges remain steady through the years. While those assumptions enable mathematical tractability and theoretical clarity, they limit the fashions' ability to symbolize real-global situations. In realistic scenarios, in particular in dynamic manufacturing environments, these assumptions often do not preserve (Das *et al.*, 2021) [2]. Production costs may also differ due to device ability, hard work availability, and order precedence. Similarly, demand is not often

consistent; it varies over the years in reaction to marketplace developments, seasonal modifications, promotional activities, and different stochastic elements. The want for flexibility in both production and call for modelling has pushed the evolution of inventory models in the direction of extra practical and adaptive frameworks. In unique, Flexible Manufacturing Systems (FMS) permit businesses to adjust manufacturing costs in step with situational needs. These systems are designed to deal with a variety of product configurations, batch sizes, and demand fluctuations. As a end result, the unit production fee will become a feature of the production fee. When the manufacturing price will increase, positive overhead charges which include hard work and energy get allotted over a larger wide variety of devices, thereby lowering the unit cost (Jana *et al.*, 2021). However, excessively excessive manufacturing quotes might also cause device wear, nice defects, or expanded energy consumption, which could, in turn, enhance the unit fee once more. Therefore, determining a choicest manufacturing fee is a important part of stock choice-making under FMS.

Simultaneously, in many actual-global settings, call for is inherently probabilistic. It may also observe nicely-described statistical distributions, inclusive of exponential, Poisson, or normal, relying on the character of the product and the patron base. Incorporating stochastic call for into inventory models adds any other layer of realism, reflecting the uncertainty that managers need to navigate of their making plans approaches. The integration of bendy manufacturing rates and stochastic demand transforms the traditional inventory hassle into a more complete optimization task, one which bills for operational agility and marketplace volatility.

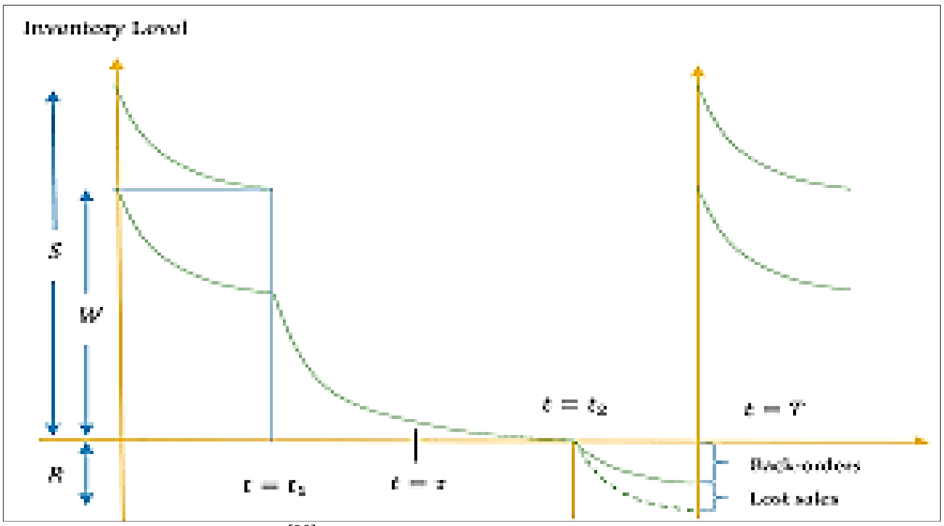
This paper addresses this undertaking by means of growing an integrated inventory version that mixes deterministic and probabilistic settings. The version includes exponential demand styles, representing both increasing and memory less behaviours which might be not unusual in technological goods and fashion objects (Macías *et al.*, 2021) [4]. Additionally, exponential deterioration is blanketed to represent the herbal degradation of stock gadgets through the years. Many products, mainly perishable goods like meals, prescription drugs, and high-tech gadgets, lose cost or application as time progresses. Modelling this deterioration is vital for accurate stock valuation and replenishment planning.

A specific factor of the proposed model is its incorporation of both partial backlogging and lost income. In conventional models, stockouts are both absolutely backlogged and absolutely lost. However, in practice, some customers can be inclined to wait for the product to be restocked, even as others may additionally cancel their orders or flip to competitors. The model displays this practical conduct via introducing a backlogging price that captures the proportion of clients inclined to attend. The ultimate demand is handled as lost income, which incurs extra penalty prices and affects lengthy-time period customer retention.

The common cost structure of the gadget is derived from multiple interrelated additives. These encompass the cost of manufacturing, which varies with production fee; maintaining fees, which account for storage and capital tied up in stock; scarcity costs, associated with unfulfilled demand that is in the end backlogged; deterioration expenses, reflecting the loss of value in stock over the years; and lost sale costs, representing the monetary impact of completely unfulfilled call for (Mishra *et al.*, 2021) [5]. The inclusion of these cost elements guarantees that the model displays the total monetary implications of inventory choices in complex operational environments.

By integrating these factors—volume flexibility, stochastic call for, partial backlogging, deterioration, and lost income—the version provides a robust framework for managing inventory in contemporary deliver chains. The proposed technique complements choice-making abilities, allowing firms to strike a balance among carrier stages, operational efficiency, and price minimization. This is especially important in industries where fast product turnover, aggressive stress, and purchaser pleasure are crucial to success.

In end, the conceptual framework laid out in this phase lays the muse for a holistic stock model that aligns with modern operational challenges. It bridges the distance between theoretical elegance and realistic relevance with the aid of incorporating more than one real-global variable that have an impact on stock performance (Nand *et al.*, 2021) [6]. The following segment will formalize the deterministic thing of the model through mathematical system, offering deeper insights into how stock evolves over time in a controlled but dynamic manufacturing environment



(Source: Momena *et al.*, 2021) [20]

Fig 1: A Two-Storage Inventory Model with Trade Credit Policy and Time-Varying Holding Cost

2.2 Deterministic Model Formulation: In the deterministic setting, production starts from the zero inventories as well as builds up to a stock level at time t1. Inventory is depleted with the aid of both demand and deterioration until stockouts at t2. Between t2 and t3, shortages gather. Production resumes at t3 and keeps until the backlog is cleared at T. The machine then enters a brand-new replenishment cycle.

Differential equations define the stock dynamics across these periods. Boundary situations are applied to derive closed-form answers for inventory ranges. Cost features are then built for every form of price component and optimized the usage of mathematical strategies.

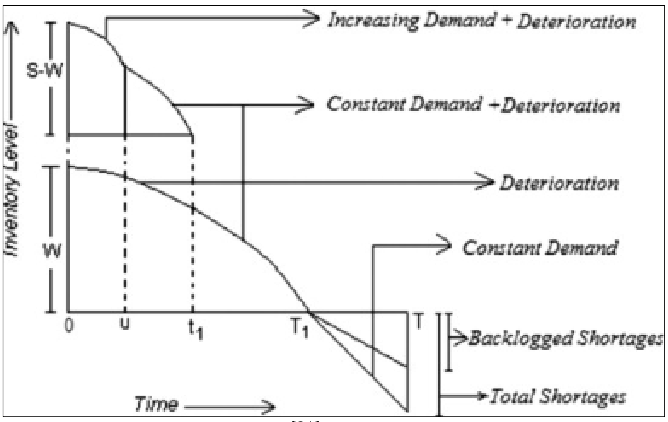
2.3 Probabilistic Model Formulation

In the probabilistic variant, demand follows a proper negative exponential distribution. Two cases are analysed: systems without the actual shortages as well as the systems with shortage. In the first case, production meets expected call for, and cost minimization makes a speciality of preserving and deterioration prices (Nath *et al.*, 2021) [7]. In the second one case, shortages and misplaced sales are brought, ensuing in a greater complicated value structure. Expectation over the opportunity density feature is used to derive common charges, and optimization is executed via calculus-based totally techniques.

3. Ramp-Type Demand Models with Variable Deterioration for Seasonal Products

3.1 Modelling Time-Dependent Demand: In many actual-world stock structures, especially the ones related to seasonal or elegant merchandise, call for does no longer observes a steady pattern. Instead, it fluctuates in a manner that displays patron behaviour, marketplace trends, and the life cycle of the product itself. These merchandise often exhibit what's called a ramp-type call for sample, that's characterised by three awesome and consecutive levels. The first phase is a growth length, for the duration of which patron interest inside the product will increase, often pushed by means of promotional campaigns, novelty, or rising seasonal wishes (Pervin *et al.*, 2021) [8]. The call for at some point of this section grows progressively, every so often exponentially, because the product profits market traction. Following this preliminary boom, the second one section is a constant-kingdom or saturation period, wherein call for stabilizes at a fantastically high and steady stage. This degree often represents the peak of product recognition, wherein purchaser hobby is sustained and stock turnover is predictable. The very last segment of the call for cycle is the decline or decay length, at some point of which call for begins to taper off. This lower is probably because of seasonal transitions, obsolescence, the provision of more modern alternatives, or surely a herbal drop in client interest. The call for on this phase regularly techniques 0, often asymptotically, over time.

Accurately modelling such dynamic call for conduct is vital for inventory decision-making, especially when aiming to decrease total system costs over a finite planning horizon. Traditional models that anticipate regular or linearly increasing demand are insufficient in these scenarios, as they fail to seize the complexities of actual marketplace conduct. In evaluation, the ramp-kind demand feature allows for a greater practical and time-touchy technique. To represent this conduct mathematically, the proposed version makes use of a piecewise exponential demand characteristic. This feature captures the 3 phases with the aid of employing special mathematical expressions over specific time intervals. During the increase section, call for increases as a feature of time, regularly represented by way of an exponential increase curve (Sharma *et al.*, 2021) [9]. In the constant-nation section, demand stays steady, modelled via a fixed price. In the decline segment, the call for function decays exponentially, drawing near zero over time. These piecewise capabilities are built using Heaviside functions or other switching mechanisms to transition easily from one segment to the subsequent. Another essential factor of the version is the consideration of inventory deterioration. Many seasonal or elegant merchandise are perishable or difficulty to obsolescence, which means their fee or usability decreases with time (Sivashankari, *et al.*, 2021) [12]. Therefore, the version assumes a variable deterioration rate that reflects the getting older or spoilage of inventory over time. This deterioration isn't always uniform; it is able to boost up at some point of intervals of low demand or prolonged garage. By incorporating deterioration into the inventory dynamics, the version becomes drastically extra relevant to industries along with clothing, meals, generation, and pharmaceuticals. Furthermore, the model assumes that shortages are not allowed, which displays strict carrier-stage coverage frequently visible in aggressive markets wherein stockouts can result in permanent purchaser loss. To mirror the increasing burden of inventory garage over time, retaining prices are modelled as linearly increasing functions. This assumption acknowledges the growing economic and logistical stress related to extended inventory protecting, including space constraints, security costs, and insurance. Together, those factors create a strong and comprehensive version for managing stock underneath time-established demand. It contains realistic demand variability, product deterioration, and the financial implications of conserving inventory over a finite making plans horizon.



(Source: Agrawal *et al.*, 2013) [21]

Fig 2: Ramp type demand model

3.2 Replenishment Strategies under Finite Horizon

Given the ramp-type demand sample and the restrictions imposed by using deterioration and keeping fees, replenishment strategies ought to be designed carefully to optimize average performance (Sundararajan *et al.*, 2021) [13]. The proposed version divides the finite planning horizon into six awesome instances primarily based on the relationship between the replenishment cycle and the three phases of demand. Each case represents a exclusive state of affairs in which a replenishment cycle either begins or ends in the course of the increase, constant-kingdom, or decay phase.

In each of the six instances, the inventory behaviour through the years is ruled by using differential equations that describe the fee of stock depletion beneath the blended consequences of demand and deterioration. These equations are solved analytically or numerically to determine the immediate stock levels throughout the cycle. Based on those stock profiles, the model calculates the related cost functions, which encompass ordering prices, keeping expenses, and deterioration charges (Verma *et al.*, 2021) [14]. The overall system cost over the making plans horizon is acquired by summing those individual price additives across all replenishment cycles. The version in addition establishes the convexity of the price function in each case. Convexity guarantees that there exists a unique international minimum for the value function, that's critical for identifying the finest replenishment cycle lengths. To determines the ultimate answer; the model makes use of an iterative optimization algorithm, inclusive of the Newton-Raphson approach or different one-dimensional search strategies. These strategies are well-perfect for fixing nonlinear equations derived from the first-order situations for optimality. The algorithm evaluates one-of-a-kind cycle lengths, computes the corresponding machine prices, and iteratively adjusts the parameters until the minimal price configuration is completed. This established approach permits the model to adapt dynamically to changing demand profiles in the planning horizon (Mohan *et al.*, 2021) [15]. It guarantees that inventory is replenished just in time to fulfil call for even as minimizing overstocking and the associated charges. The model also lets in for flexibility inside the variety and period of replenishment cycles, making sure alignment with operational capacities and demand styles. By combining distinctive call for modelling with rigorous optimization, this replenishment method presents a powerful tool for inventory planners. It helps better useful resource allocation, reduces waste because of obsolescence or spoilage, and in the long run complements carrier ranges in environments characterized by risky or seasonal demand.

3.3 Algorithm for Cost Minimization

A set of rules is provided to decide the ideal replenishment cycle lengths throughout the seasonal time horizon. It sequentially evaluates each of the six cases, updating cycle lengths and computing cumulative device expenses. The technique guarantees that the total time blanketed via all replenishment cycles aligns with the finite season length. Numerical validation confirms the practical relevance and accuracy of the approach.

4. Integrated Inventory Models under Permissible Delay in Payments and Inflation

4.1 Supply Chain and Trade Credit Context

Modern inventory systems often mainly involve the multiple stakeholders in a particular supply chain, including vendors as well as the buyers (Mahlangu *et al.*, 2021) [16]. These events may also perform under cooperative agreements that contain delayed fee options or alternate credit. Under such phrases, buyers might also defer charge to providers for a particular period without incurring interest, or they will pay interest after the grace period expires.

The integration of alternate credit score regulations into inventory fashions offers financial flexibility and impacts replenishment choices. The presence of inflation and possibility prices further complicates choice-making, necessitating a comprehensive modelling method.

4.2 Mathematical Formulation of the Integrated Model

The integrated vendor-buyer inventory model which mainly assumes deterministic demand with the actual product deterioration. The supplier offers a two-phase credit score period: no hobby is charged if charge is made in the first duration (M), and a decrease or better interest is charged relying on whether or not payment is made among M and N or after N. The client earns hobby on income revenue during the deferral length.

The client's and seller's inventory ranges are described via differential equations, incorporating deterioration and variable prices (Kumar *et al.*, 2021) [17]. Boundary conditions permit for the derivation of average inventory levels, even as value components include protecting, deterioration, and setup, scarcity, lost income, and interest fees or income.

4.3 Payment Scenarios and Cost Structures

Three fundamental price eventualities are analysed. In the primary case, price is made before or at M, resulting in no hobby cost and tremendous interest income. In the second one case, price is made between M and N, and the customer pays hobby at a moderate rate. Two sub-cases arise relying on whether or not revenue at M is enough to cowl the whole price. In the third case, price is made after N, triggering a better interest fee. Three sub-instances are considered based on revenue sufficiency and payment shape.

For each state of affairs, total cost in keeping with unit time is formulated and minimized with recognize to cycle lengths (Khan *et al.*, 2021) [18]. Analytical answers and optimality conditions are derived for all sub-cases.

5. Managerial Implications and Strategic Insights

5.1 Optimizing Inventory Performance through Strategic Modelling

The models developed in this particular study offer some for the substantial value to decision-makers in supply chain as well as the operations management. Parameters encompass initial call for, cost coefficients, deterioration charge, and interest quotes.

The ramp-kind demand version is applied to a four-length season with multiple replenishment cycles. Optimal cycle lengths are derived, and total system prices are computed. Adjustments are made to make certain cycle lengths match the finite horizon exactly, and the outcomes display enormous value financial savings and stepped forward inventory overall performance.

In the incorporated vendor-client version, various interest rates and credit score intervals are tested to analyze their effect on overall value (Rahman *et al.*, 2021) [19]. The consequences display that premier price timing and replenishment scheduling can significantly lessen both operational and economic charges.

5.2 Volume Flexibility and Trade Credit in Strategic Supply Chain Management

The models developed on this observe offer substantial fee to selection-makers in supply chain and operations management. Incorporating volume flexibility allows producers to dynamically adapt production levels primarily based on call for. Ramp-type call for modelling enhances forecasting for seasonal and perishable goods.

Trade credit score policies offer monetary leverage to buyers, but should be balanced against capacity hobby liabilities (Khan *et al.*, 2021) [18]. By integrating price timing into stock choices, firms can align financial and operational strategies to optimize running capital.

From a strategic perspective, cooperation between providers and shoppers in figuring out replenishment schedules, charge terms, and production making plans can cause win-win consequences. Supply chain companions need to use these incorporated models to negotiate exchange agreements and plan joint inventory policies.

6. Conclusion and Future Directions

This research contributes to the actual field of the inventory management by integrating multiple form of the real-world factors—time-dependent demand,, deterioration, lost income, alternate credit, and inflation—right into a unified modelling framework. The analytical formulations and numerical answers offered offer robust selection-assist gear for companies working in uncertain and complex environments.

Future research should extend this painting to multi-echelon supply chains regarding multiple carriers and shops. Incorporating sustainability metrics, dynamic pricing, and device getting to know-primarily based demand forecasting would enhance version applicability. Additionally, simulation-primarily based validation across distinct industries can improve generalizability and help statistics-driven inventory coverage layout.

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