Journal of Mathematical Problems, Equations and Statistics

E-ISSN: 2709-9407
P-ISSN: 2709-9393
JMPES 2025; 6(1): 345-349

www.mathematicaljournal.com
Received: 25-04-2025
Accepted: 30-05-2025

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Profit and average period of the repairman analysis of four unit system using regenerative point graphical technique

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Abstract
This paper discusses Availability Analysis of Four Unit classification by Preventive Maintenance in addition to Degradation. In One Unit, and a single repairman who is also in 24x7 available by the RPGT. In this study, we have considered a four units in which there is a main unit in which components are so arranged that it can work in reduced and further reduced state after degradation also other units A, B & C are occupied either at full ability or in failed state-II is considered also the repair rates are perfect. In the unit or subsystem 'D', it can apply two types of maintenance as one is perfect maintenance. Various path probabilities mean sojourn time is discuss by drawing tables for greater than ever failure/repair rates and graphs.

Keywords: Average period of the repairman, profit analysis, RPGT

1. Introduction

This article discusses a system with four units, one of which has flawless repair using the RPGT and other of which has preventive maintenance applied before full failure and degeneration in one unit post-failure. In this study, we have considered a four units in which there is a main unit in which components are so arranged that it can work in reduced and further reduced state after degradation also other units A, B & C are occupied either at full ability or in failed state-II is considered also the repair rates are perfect. In the unit or subsystem 'D', it can apply two types of maintenance as one is perfect maintenance. Multiple pieces are assembled to form different systems. Many process industries, like the cloth and utensil industries, among many others, use four unit systems. The system fails as a whole if one unit malfunctions. In light of the significance of a single unit within the larger system, we have examined a system consisting of four subunits in this chapter, with the primary single unit undergoing preventive maintenance and the first unit experiencing degradation upon complete failure. A transition diagram of the organism is bent by accounting for different scenarios and path probabilities. From this diagram, a table for primary, secondary, and tertiary cycles is created. In [2018], Kumar *et al.* [1, 2] investigated the behavior of a bucks plant. Kumar [2019] [4, 5, 6] used RPGT to perform a warmth analysis on a freezing standby architecture consisting of two indistinguishable units by server letdown and prioritized for PM. The current paper is divided into two parts, one of which be out cold deputy mode and the other is in use. The sole distinction linking online as well as cold deputy equipment is between the good and utterly failed states. In their 2019 occupation on organism model and analysis, Rajbala *et al.* [2019] [4] looked at a case revise of an EAEP built-up capability. Kumar *et al.* [2017] [7] studied the behavior of the urea fertilizer sector. Kumari [2021] [8] investigated limited cases via PSO. Rajbala [2022] [9] looked at the RAP issue in the cylinder built-up factory using a heuristic technique. Kumar *et al.* (2017) [7] looked into the profit function and mathematical formulation of an edible oil refinery facility. Kumar [2019] [4] looked at statistical formulation and behavior study in a paper mill. The sensitivity analysis of a 3:4::G outstanding system plant was examined by Kumar [2018] [1, 2].

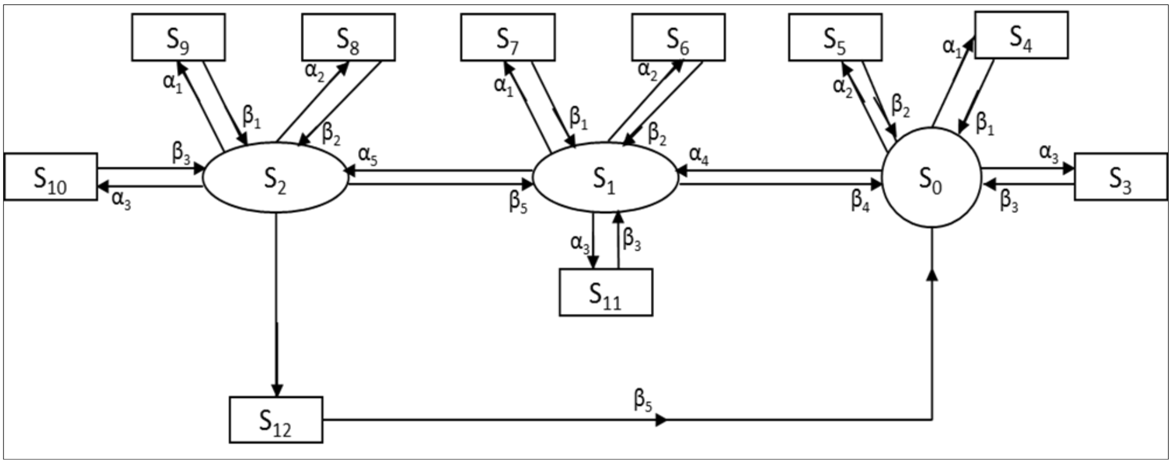
2. Assumptions and Notations

- Repairs are perfect.
- Repair unit works like a innovative one.
- $D_1 \rightarrow$ First Degraded State
- $D_2 \rightarrow$ Second Degraded State

- Increasing Failure Rates
- $\alpha_1 \rightarrow$ A to a, $\alpha_2 \rightarrow$ B to b, $\alpha_3 \rightarrow$ C to c, $\alpha_4 \rightarrow$ D to D_1, $\alpha_5 \rightarrow$ D_1 to D_2, $\alpha_6 \rightarrow$ D_2 to d
- Increasing Repair Rates
- $\beta_1 \rightarrow$ a to A, $\beta_2 \rightarrow$ b to B, $\beta_3 \rightarrow$ c to C, $\beta_4 \rightarrow$ D_1 to D, $\beta_5 \rightarrow$ D_2 to D_1, $\beta_6 \rightarrow$ d to D

3. Model Description

Considering the various possibilities and following the assumptions of the coordination is drawn as under in Figure 1



S_0 = ABCD, S_1 = ABCD$_1$, S_2 = ABCD$_2$, S_3 = ABcD, S_4 = aBCD, S_5 = AbCD, S_6 = AbCD$_1$, S_7 = aBCD$_1$, S_8 = AbCD$_2$, S_9 = aBCD$_2$, S_{10} = ABcD$_2$, S_{11} = ABcD$_1$, S_{12} = ABcd

Fig 1: Transition Diagram

4. Transition Probability and the Mean sojourn times.

Table 1: Transition Probabilities

$q_{i,j}(t)$	$P_{ij} = q^*_{i,j}(t)$
$q_{0,1}= \alpha_4 e^{-(\alpha_1+\alpha_2+\alpha_3+\alpha_4)t}$	$P_{0,1}= \alpha_4/(\alpha_1+\alpha_2+\alpha_3+\alpha_4)$
$q_{0,3}= \alpha_3 e^{-(\alpha_1+\alpha_2+\alpha_3+\alpha_4)t}$	$P_{0,3}= \alpha_3/(\alpha_1+\alpha_2+\alpha_3+\alpha_4)$
$q_{0,4}= \alpha_1 e^{-(\alpha_1+\alpha_2+\alpha_3+\alpha_4)t}$	$P_{0,4}= \alpha_1/(\alpha_1+\alpha_2+\alpha_3+\alpha_4)$
$q_{0,5}= \alpha_2 e^{-(\alpha_1+\alpha_2+\alpha_3+\alpha_4)t}$	$P_{0,5}= \alpha_2/(\alpha_1+\alpha_2+\alpha_3+\alpha_4)$
$q_{1,0}= \beta_4 e^{-(\alpha_1+\alpha_2+\alpha_3+\beta_4+\alpha_5)t}$	$P_{1,0}= \beta_4/(\alpha_1+\alpha_2+\alpha_3+\beta_4+\alpha_5)$
$q_{1,2}= \alpha_5 e^{-(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\beta_4)t}$	$P_{1,2}= \alpha_5/(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\beta_4)$
$q_{1,6}= \alpha_2 e^{-(\alpha_1+\alpha_2+\alpha_3+\beta_4+\alpha_5)t}$	$P_{1,6}= \alpha_2/(\alpha_1+\alpha_2+\alpha_3+\beta_4+\alpha_5)$
$q_{1,7}= \alpha_1 e^{-(\alpha_1+\alpha_2+\alpha_3+\beta_4+\alpha_5)t}$	$P_{1,7}= \alpha_1/(\alpha_1+\alpha_2+\alpha_3+\beta_4+\alpha_5)$
$q_{1,11}= \alpha_3 e^{-(\alpha_1+\alpha_2+\alpha_3+\beta_4+\alpha_5)t}$	$P_{1,11}= \alpha_3/(\alpha_1+\alpha_2+\alpha_3+\beta_4+\alpha_5)$
$q_{2,1}= \beta_5 e^{-(\alpha_1+\alpha_2+\alpha_3+\beta_5+\alpha_6)t}$	$P_{2,1}= \beta_5/(\alpha_1+\alpha_2+\alpha_3+\beta_5+\alpha_6)$
$q_{2,8}= \alpha_2 e^{-(\alpha_1+\alpha_2+\alpha_3+\beta_5+\alpha_6)t}$	$P_{2,8}= \alpha_2/(\alpha_1+\alpha_2+\alpha_3+\beta_5+\alpha_6)$
$q_{2,9}= \alpha_1 e^{-(\alpha_1+\alpha_2+\alpha_3+\beta_5+\alpha_6)t}$	$P_{2,9}= \alpha_1/(\alpha_1+\alpha_2+\alpha_3+\beta_5+\alpha_6)$
$q_{2,10}= \alpha_3 e^{-(\alpha_1+\alpha_2+\alpha_3+\beta_5+\alpha_6)t}$	$P_{2,10}= \alpha_3/(\alpha_1+\alpha_2+\alpha_3+\beta_5+\alpha_6)$
$q_{2,12}= \alpha_6 e^{-(\alpha_1+\alpha_2+\alpha_3+\beta_5+\alpha_6)t}$	$P_{2,12}= \alpha_6/(\alpha_1+\alpha_2+\alpha_3+\beta_5+\alpha_6)$
$q_{3,0}= \beta_3 e^{-\beta_3 t}$	$P_{3,0} = \beta_3/ \beta_3 = 1$
$q_{4,0}= \beta_1 e^{-\beta_1 t}$	$P_{4,0} = \beta_1/ \beta_1 = 1$
$q_{5,0}= \beta_2 e^{-\beta_2 t}$	$P_{5,0} = \beta_2/ \beta_2 = 1$
$q_{6,1}= \beta_2 e^{-\beta_2 t}$	$P_{6,1} = \beta_2/ \beta_2 = 1$
$q_{7,1}= \beta_1 e^{-\beta_1 t}$	$P_{7,1} = \beta_1/ \beta_1 = 1$
$q_{8,2}= \beta_2 e^{-\beta_2 t}$	$P_{8,2} = \beta_2/ \beta_2 = 1$
$q_{9,2}= \beta_1 e^{-\beta_1 t}$	$P_{9,2} = \beta_1/ \beta_1 = 1$
$q_{10,2}= \beta_3 e^{-\beta_3 t}$	$P_{10,2} = \beta_3/ \beta_3 = 1$
$q_{11,1}= \beta_3 e^{-\beta_3 t}$	$P_{11,1} = \beta_3/ \beta_3 = 1$
$q_{12,0}= \beta_6 e^{-\beta_6 t}$	$P_{12,0} = \beta_6/ \beta_6 = 1$

Table 2: Mean Sojourn Times

$R_i(t)$	$\mu_i=R_i^*(0)$
$R_0^{(t)} = e^{-(\alpha_1+\alpha_2+\alpha_3+\alpha_4)t}$	$\mu_0 = 1/(\alpha_1+\alpha_2+\alpha_3+\alpha_4)$
$R_1^{(t)} = e^{-(\alpha_1+\alpha_2+\alpha_3+\beta_4+\alpha_5)t}$	$\mu_1 = 1/(\alpha_1+\alpha_2+\alpha_3+\beta_4+\alpha_5)$
$R_2^{(t)} = e^{-(\alpha_1+\alpha_2+\alpha_3+\alpha_6+\beta_5)t}$	$\mu_2 = 1/(\alpha_1+\alpha_2+\alpha_3+\alpha_6+\beta_5)$
$R_3^{(t)} = e^{-\beta_3 t}$	$\mu_3 = 1/\beta_3$
$R_4^{(t)} = e^{-\beta_1 t}$	$\mu_4 = 1/\beta_1$
$R_5^{(t)} = e^{-\beta_2 t}$	$\mu_5 = 1/\beta_2$
$R_6^{(t)} = e^{-\beta_2 t}$	$\mu_6 = 1/\beta_2$
$R_7^{(t)} = e^{-\beta_1 t}$	$\mu_7 = 1/\beta_1$
$R_8^{(t)} = e^{-\beta_2 t}$	$\mu_8 = 1/\beta_2$
$R_9^{(t)} = e^{-\beta_1 t}$	$\mu_9 = 1/\beta_1$
$R_{10}^{(t)} = e^{-\beta_3 t}$	$\mu_{10} = 1/\beta_3$
$R_{11}^{(t)} = e^{-\beta_3 t}$	$\mu_{11} = 1/\beta_3$
$R_{12}^{(t)} = e^{-\beta_6 t}$	$\mu_{12} = 1/\beta_6$

5. Path Probability

Now path probability of the system from the base position of system are given as

$V_{0,0} = 1$

$V_{0,1} = p_{0,1} = [\alpha_4/(\alpha_1+\alpha_2+\alpha_3+\alpha_4)]$

$V_{0,2} = p_{0,1}\, p_{1,2} = [\alpha_4\, \alpha_5/(\alpha_1+\alpha_2+\alpha_3+\alpha_4)(\alpha_1+\alpha_2+\alpha_3+\alpha_5+\beta_4)]$

$V_{0,3}=$ …….continuous

6. Evaluation of Parameters of the System

ATSF (T_0): Reformative un-failed circumstances to which organization container transit (first state '0'), in the past inflowing whichever vain state are: 'i' = 0 to 4 attractive 'ξ ' = '0'.

$$\text{ATSF } (T_0) = \left[\sum_{i,sr}\left\{\frac{\left\{pr\left(\xi\xrightarrow{sr(sff)}i\right)\right\}\mu i}{\Pi_{m_1\neq\xi}\left\{1\text{-}V_{\overline{m_1 m_1}}\right\}}\right\}\right] \div \left[1\text{-}\sum_{sr}\left\{\frac{\left\{pr\left(\xi\xrightarrow{sr(sff)}\xi\right)\right\}}{\Pi_{m_2\neq\xi}\left\{1\text{-}V_{\overline{m_2 m_2}}\right\}}\right\}\right]$$

$T_0 = (V_{0,0}\, \mu_0 + V_{0,1}\, \mu_1 + V_{0,2}\, \mu_2)/\ [\{1\text{-}(0,1,0)\}\{1\text{-}(1,2,1)\}]$

$= (p_{0,0}\mu_0 + p_{0,1}\mu_1 + p_{0,2}\mu_2)\div(1\text{-}p_{0,1}p_{1,0})\ (1\text{-}p_{1,2}p_{2,1})$

Availability of System: Reformative circumstances at which organization is to hand are 'j' = 0 - 4 and reformative status are 'i' = 0 to 10 attractive 'ξ' = '0' whole fraction of interlude on behalf of which union is easily reached is prearranged in

$$A_0 = \left[\sum_{j,sr}\left\{\frac{\{pr(\xi^{sr\to}j)\}fj,\mu j}{\Pi_{m_1\neq\xi}\left\{1\text{-}V_{\overline{m_1 m_1}}\right\}}\right\}\right] \div \left[\sum_{i,sr}\left\{\frac{\{pr(\xi^{sr\to}i)\}\mu_i^1}{\Pi_{m_2\neq\xi}\left\{1\text{-}V_{\overline{m_2 m_2}}\right\}}\right\}\right]$$

$$= \left[\sum_j V_{\xi,j}\,, f_j, \mu_j\right] \div \left[\sum_i V_{\xi,i}\,, f_j, \mu_i^1\right]$$

Average Period of Repairman: The reformative states where head waiter 'j' = 1 - 12 and reformative position are 'i' = 0 to 10, pleasing ξ = '0', entirety fraction of interlude in support of which wine waiter ruins in prepared as repair of faulty units.

$$B_0 = \left[\sum_{j,sr}\left\{\frac{\{pr(\xi^{sr\to}j)\},nj}{\Pi_{m_1\neq\xi}\left\{1\text{-}V_{\overline{m_1 m_1}}\right\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\{pr(\xi^{sr\to}i)\}\mu_i^1}{\Pi_{m_2\neq\xi}\left\{1\text{-}V_{\overline{m_2 m_2}}\right\}}\right\}\right]$$

$$B_0 = \left[\sum_j V_{\xi,j}\,, n_j\right] \div \left[\sum_i V_{\xi,i}\,, \mu_i^1\right]$$

$B_0 = N \div D$

$B_0 = [(\beta_4+\alpha_6+\beta_3)/(\beta_1+\beta_2+3\beta_4+\beta_6+\alpha_2^2+2\alpha_1)]+[(2\alpha_1+\alpha_3\,\beta_4+\beta_6)/(\beta_1+2\alpha_3+2\alpha_4+4\alpha_3^3)]$

Expected Fractional Number by repair man: States where the repairman do vacation's a unmarked are S_0, S_1 and S_2 taking ξ'= 0', number of repair man's visit

$$V_0 = \left[\sum_{j,sr}\left\{\frac{\{pr(\xi^{sr\to}j)\}}{\Pi_{k_1\neq\xi}\left\{1\text{-}V_{\overline{k_1 k_1}}\right\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\{pr(\xi^{sr\to}i)\}\mu_i^1}{\Pi_{k_2\neq\xi}\left\{1\text{-}V_{\overline{k_2 k_2}}\right\}}\right\}\right]$$

6.1 Particular Cases

Profit Analysis of the system

As main objective of the industry persons are to optimize the profit function, which mainly depends upon the smooth working of system/ industry. Now the profit function mainly depends upon the parameters like availability of system proper maintenance of units and easily availability of server. After reducing the server's repair time and maximize the availability of working tie of sub unit, we can optimize the profit. Considering impact of preventive maintenance and different failure and repair rates on all the parameters mentioned above, finally we have to find the performance of the system under study. This can be done by using the Performance function as

$$P_0 = C_1T_0 + C_2A_0 - C_3B_0 + \Upsilon$$

Where $T_0 \rightarrow$ average time for system failure, $A_0 \rightarrow$ availability of system, $B_0 \rightarrow$ busy time for which server remains busy, $\Upsilon \rightarrow$ miscellaneous factor which depends upon circumstances in which machine work.
Using the value of above parameter, we get

$$= C1\ ([(\alpha_4\ \alpha_5+\beta_4)/(2\alpha_1+3\alpha_2+\beta_4+\beta_5+ 6\alpha_4^2+2\alpha_3)]/[\ (\alpha_1+\alpha_2+\alpha_3+\alpha_4)^2/(\beta_5+\alpha_6+\alpha_2+\alpha_3+\alpha_1)^3]) + C2\ ([(\alpha_1+\alpha_2 + \alpha_3 + \alpha_4)]+[\alpha_4\ \alpha_5/(\alpha_1 + \alpha_3 + \alpha_5+\beta_4)]/\ [(4\alpha_6+\ \alpha_4)(\alpha_1+\alpha_5)]/[(\alpha_1+\ \alpha_3 + \alpha_5 +\beta_4)(\alpha_1 + \alpha_2 + \alpha_3 +\alpha_6+\beta_5)]) + C3\ ([(\beta_4+\alpha_6+\beta_3)/(\beta_1+\beta_2+3\beta_4+\beta_6+\alpha_2^2+2\alpha_1)]+\ [(2\alpha_1+\alpha_3\ \beta_4+\beta_6)/(\beta_1+2\alpha_3+2\alpha_4+\ 4\alpha_3^3)]) + \Upsilon.$$

Table 3: Performance/Output Function

Output Function (P_0)	$\alpha_4 = 0.05$	$\alpha_5 = 0.075$	$\alpha_6 = 0.10$
$\beta_4 = 0.90$	67.2568	64.2630	57.2190
$\beta_5 = 0.925$	116.7520	112.1615	102.3125
$\beta_6 = 0.95$	138.1260	131.2160	124.2400

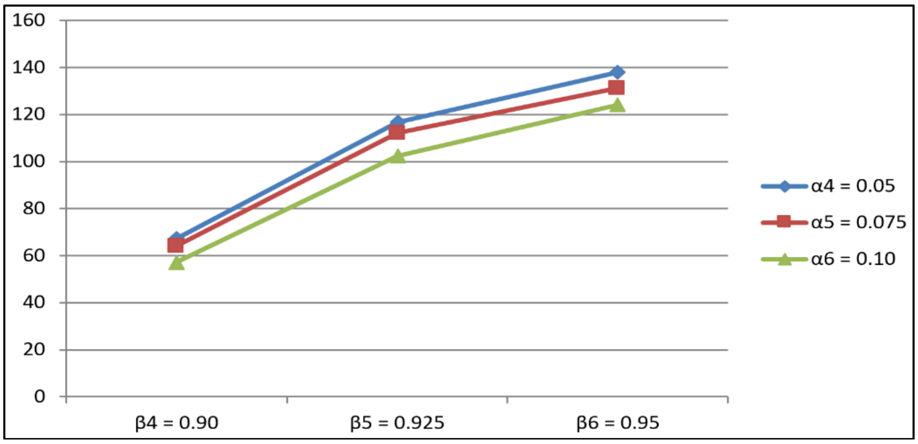


Fig 2: Profit Function Graph

Table 4: Average Period of the Repairman (B_0)

B_0	β_1	β_2	β_3	β_4	β_5
0.60	3.36	3.38	3.40	3.42	3.44
0.65	3.34	3.36	3.38	3.40	3.42
0.70	3.32	3.34	3.36	3.38	3.40
0.75	3.30	3.32	3.34	3.36	3.38
0.80	3.28	3.30	3.32	3.34	3.36

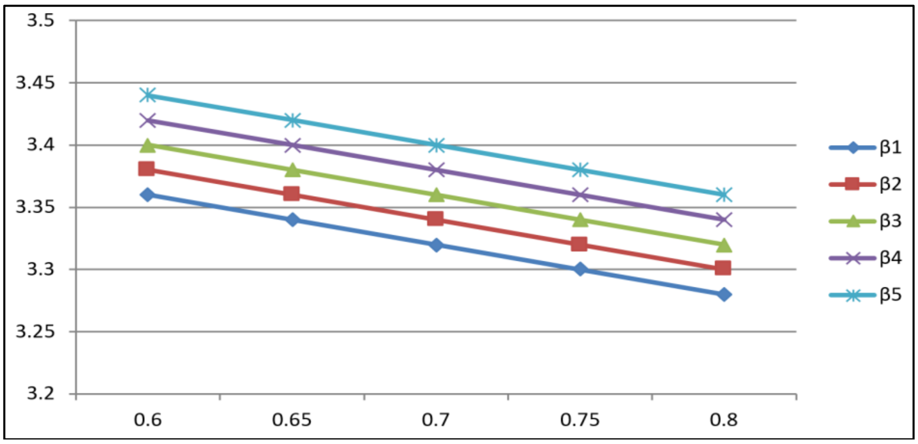


Fig 3: Average Period of the Repairman (B_0)

We can perceive commencing table 4 and graph 3 that B_0 be at its least when fix rate of both unit is 0.80 and its value is 3.28 and at its least when fix rate of in cooperation unit is 0.80.Maintaining a minimal importance of B_0 on behalf of a functional organism might lead to low maintenance costs.

Table 5: Average Period of the Repairman (B_0)

B_0	α_1	α_2	α_3	α_4	α_5
0.10	1.24	1.21	1.18	1.15	1.12
0.15	1.27	1.24	1.21	1.18	1.15
0.20	1.30	1.27	1.24	1.21	1.18
0.25	1.33	1.30	1.27	1.24	1.21
0.30	1.36	1.33	1.30	1.27	1.24

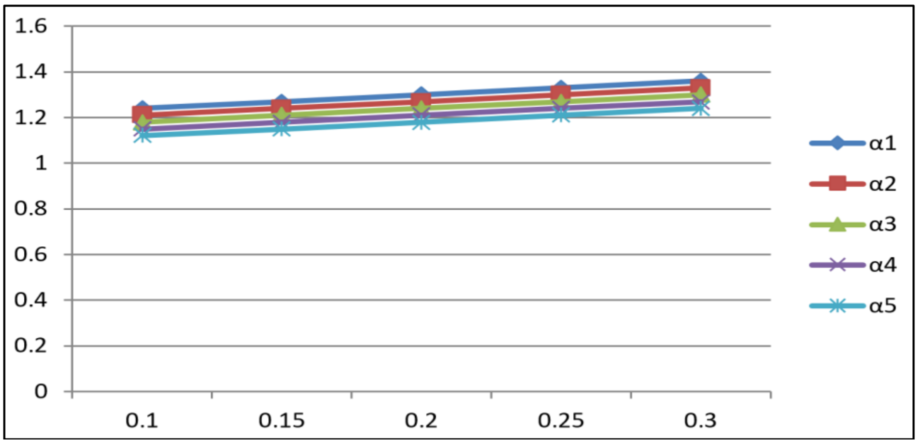


Fig 4: Average Period of the Repairman (B_0)

B_0 should be outsized for a first-class system, as revealed by means of table 5 and grid 4, which illustrate that B_0 is upper limit for incredibly low unit malfunction rates we realize that the most favorable value of the Average epoch is 1.36, which submits that the not a success rate of unit is highest.

7. Conclusion

Methods of controlling unit failure along with repair rates relating to financial resources but instead market conditions to obtain optimum values for system parameters. A degraded state is a state of the system in which the system or units perform a function continuously up to a satisfactory but lower (lower) limit than specified due to its required functions. By following the optimal settings of the framework parameters at a certain benchmark, stakeholders can determine whether to use RPGT to select an efficient repairman with a greater repair rate. Organizational parameters are evaluated just by contrast with alternative methods. Since stakeholders have no control over failure rates, the only thing that can be improved is the sub-unit's repair rates. Depending on financial resources and market conditions, management can regulate the unit's repair and failure rates to achieve the best possible value for the system characteristics. We come to the conclusion that preventive maintenance is essential to system performance optimization. The research propose and tactic employed in this chapter are replicable to supplementary industries with the assumptions and limitations taken into consideration. Since the industries have to made a fix investment to set up the system of any type of industry. The system's availability are all observed to decrease with an increase in stoppage rate and to amplify through the revamp rate, based on the analytical and figure discussions. The system cannot reach production after a limit, i.e. a recession occurs. A degraded state is a state of the system in which the system or units perform a function continuously up to a satisfactory but lower (lower) limit than specified due to its required functions.

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