Journal of Mathematical Problems, Equations and Statistics

E-ISSN: 2709-9407
P-ISSN: 2709-9393
JMPES 2020; 1(1): 04-09

www.mathematicaljournal.com
Received: 04-11-2019
Accepted: 09-12-2019

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Mean delay model of the router under self-similar variable packet input traffic

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Abstract
Internet Protocol (IP) packet traffic exhibits self-similarity or LRD property and causes the degradation of switch performance. Therefore, it is crucial for an appropriate buffer design of a switch. In this paper, by taking voids into consideration we investigate delay behavior of the switch under self-similar variable length packet traffic by modeling it as $MMPP/M/1/K$ queuing system wherein MMPP (Markov-Modulated Poisson Process) is fitted by equating the variance of MMPP and that of self-similar traffic. MMPP model is already validated one to emulate the self-similar characteristics. We investigate mean delay against system parameters, traffic parameters and fitting parameters. Numerical results show that analysis presented in this paper is useful in dimensioning the switch.

Keywords: MMPP, self-similar traffic, voids, mean delay

Introduction
Background
It is evident from seminal studies that local and wide area network traffic is self-similar, that is, long range dependent [1-3]. Such traffic has impact on performance of routers or switches. Hence the Quality of Service (QoS) metrics such as packet delay and packet loss probability are greatly affected. The Markovian Arrival Process (MAP) has been widely adopted to emulate self-similar traffic [4, 5]. These works involve Markov Modulated Poisson Process (2^d-MMPP), which is a superposition of d 2-state Interrupted Poisson Processes (IPPs) and a Poisson process. MMPP is fitted by equating the second order statistics of resultant MMPP and that of self-similar traffic over desired time-scales [4, 5]. MMPP is a particular case of MAP and IPP is a particular case of MMPP.

The quality of service (QoS) metrics such as packet loss probability and mean packet delay are significant in dimensioning the switch. In the papers [6, 7], first, switch is modeled as $MMPP/M/1/K$ queueing system and then the impact of Hurst parameter and traffic intensity are investigated by means of both analytical and simulation results. Packet lengths are assumed to follow exponential distribution. But, when the packet length is variable, voids will occur in the switch buffer and performance of the switch degrades as voids will incur excess loads [8]. This issue is not addressed in the papers [6, 7]. In the paper [8], packet arrivals are assumed to follow Poisson distribution which is proved to be unrealistic [1]. Here we are concerned with the performance analysis of the switch by means of $MMPP/M/1/K$ queueing system taking voids into consideration.

The rest of the paper is organized as follows. We first overview the definitions of self-similar processes and MMPP. We then present the analytical results of $MMPP/M/1/K$ system. In next section, we present some numerical results pertaining to packet loss probability and mean waiting time of $MMPP/M/1/K$ queueing system and illustrate the effects of parameters such as Hurst parameter, traffic intensity, number of components in superposition, and time scales. Finally some conclusions are given in section V.

Self-Similar Traffic and Markov Modulated Poisson Process (MMPP)
Self-Similar Traffic
The definition of exact second-order self-similar processes is given as follows. If we consider X as a second -order stationary process with variance σ^2, and divide time axis into disjoint intervals of unit length, we could define $X=\{X_t/t=1,2,3........\}$ to be the number of

points (packet arrivals) in the t^{th} interval. A new sequence $X^{(m)} = \{X_t^{(m)}\}$, where $X_t^{(m)} = \frac{1}{m}\sum_{i=1}^{m} X_{(t-1)m+i}$, $t = 1,2,3.....$, is the average of the original sequence in m non-overlapping blocks. Then the process X is defined as an exact second order self-similar process with the Hurst parameter, $H = 1-\beta/2$, if $Var(X^{(m)}) = \sigma^2 m^{-\beta}, \quad \forall m \geq 1.$

Markov Modulated Poisson Process

Now we present the fundamentals of MMPP. MMPP is a doubly stochastic process in which arrival rate is given by $\lambda[J_t]$, where $J_t, t \geq 0$ is an m-state Markov process. The arrival rate can therefore take on only m values, namely $\lambda_1, \lambda_2, ..., \lambda_m$. It is equal to λ_j whenever the Markov process is in the state j, $1 \leq j \leq m.$ The MMPP is fully parameterized by the infinitesimal generator Q of the Markov process and the vector $\lambda = (\lambda_1, \lambda_2, ..., \lambda_m)$ of the arrival rates. Let Λ be the diagonal matrix with $\Lambda_{jj} = \lambda_j$, $1 \leq j \leq m.$ In the case of two states, Q and Λ are given as follows:

$$Q = \begin{bmatrix} -c_1 & c_1 \\ c_2 & -c_2 \end{bmatrix}, \quad \Lambda = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}. \tag{2}$$

The mean arrival rate λ of MMPP is given by $\lambda = \vec{\pi}\Lambda e$, where $\vec{\pi}$ is the stationary probability vector of Q, i.e. $\vec{\pi}Q = 0, \vec{\pi}e = 1$ and e is an all -1 column vector with designated dimension. If we let N_t, t ≥0, be the number of arrivals in (0, t], for the stationary MMPP, the Mean of N_t is

$$E[N_t] = \frac{c_2\lambda_1 + c_1\lambda_2}{c_1 + c_2} t. \tag{3}$$

The Variance of N_t is

$$Var\,[N_t] = E[N_t] + \frac{2c_1c_2(\lambda_1-\lambda_2)^2}{(c_1+c_2)^3} t - \frac{2c_1c_2(\lambda_1-\lambda_2)^2}{(c_1+c_2)^4}[1-e^{-(c_1+c_2)t}]. \tag{4}$$

The interesting feature of MMPP is that a superposition of MMPPs is still MMPP.

MMPP/M/1/K Queueing System

Asynchronous switch with self-similar variable length packet input traffic is modeled as $MMPP/M/1/K$ queuing system. In $MMPP/M/1/K$ system, the packets arrive according to the MMPP of states m and is characterized by the matrices Q, Λ, where Q, Λ are $m \times m$ matrices. The service time is exponential with service rate μ. Let D_k, $k \geq 0$ denotes the matrix of order $m \times m$ whose (i, j) element is the probability that given departure at time 0, which left at least one packet in the system and the process is in state i, the next departure occurs when the arrival process in j, and during that service time there were k arrivals. Then D_k satisfies the following equation:

$$\sum_{k=0}^{\infty} D_k z^k = \mu \int_0^{\infty} e^{[Q-\Lambda+\Lambda z]x} e^{-\mu x} dx \tag{5}$$

$$i.e. \sum_{k=0}^{\infty} D_k z^k = \mu\left(\mu I - \left(Q - \Lambda + \Lambda z\right)\right)^{-1}$$

$$i.e. \sum_{k=0}^{\infty} D_k z^k = \mu^2 \sum_{k=0}^{\infty} \left(\frac{Q-\Lambda+\Lambda z}{\mu}\right)^k, \tag{6}$$

where I is the unit matrix of designated dimension. Now we compute the D_k s by extending the methodology [9, 10] from the deterministic service time distribution to the exponential service time distribution. For $k=0$ in (6), we have

$$D_0 = \mu^2\left(I+\sum_{r=0}^{\infty}\left(\frac{Q-\Lambda}{\mu}\right)^r\right). \tag{7}$$

For $l,n \geq 1$, let $T(n,l)$ be the coefficient of z^{l-1} in $\mu^2\left(\frac{Q-\Lambda+\Lambda z}{\mu}\right)^n$, n^{th} term of the series on right hand side of (6). From (6), we have

$$T(1,1)=\mu^2\left(\frac{Q-\Lambda}{\mu}\right), \qquad T(1,2)=\mu^2\frac{\Lambda}{\mu}$$

$$T(n,1)=\mu^2\left(\frac{Q-\Lambda}{\mu}\right)^n, \quad T(n,l)=0, \text{ if } l>n+1,$$

and

$$T(n,1)+T(n,2)z+T(n,3)z^2+\ldots\ldots\ldots\ldots+T(n,l)z^{l-1}+\ldots\ldots.+T(n,n+1)=\mu^2\left(\frac{Q-\Lambda+\Lambda z}{\mu}\right)^2$$

Multiplying both sides by $\mu^2\left(\frac{Q-\Lambda+\Lambda z}{\mu}\right)$, we obtain

$$[T(n,1)+T(n,2)z+T(n,3)z^2+\ldots..+T(n,l)z^{l-1}+\ldots..+T(n,n+1)z^n]\mu^2\left(\frac{Q-\Lambda+\Lambda z}{\mu}\right)$$

$$=T(n+1,1)+T(n+1,2)z+\ldots\ldots\ldots$$

Equating the coefficients of like powers of z, we obtain,

$$T(n+1,1)=T(n,1)\mu^2\left(\frac{Q-\Lambda}{\mu}\right),$$

$$T(n+1,2)=T(n,2)\mu^2\left(\frac{Q-\Lambda}{\mu}\right)+T(n,1)\mu^2\frac{\Lambda}{\mu},$$

$$T(n+1,q)=T(n,q)\mu^2\left(\frac{Q-\Lambda}{\mu}\right)+T(n,q-1)\mu^2\frac{\Lambda}{\mu}\ q\in \mathrm{N}. \tag{8}$$

From (6), we have

$$D_s=\sum_{k=s}^{\infty}T(k,s+1), \ s=1,2,.. \tag{9}$$

We compute the matrices D_s s using the recurrence formulae (7) - (9). Now we consider the embedded Markov chain $\{L(n),J(n)/n\geq 0\}$ at the departure epochs of the queueing system $MMPP/M/1/K$ on the state space $S=\{(b,i)/0\leq b\leq K-1,1\leq i\leq m\}$, where $L(n)$ denotes buffer occupancy and $J(n)$ denotes the state of MMPP. Then the pertaining embedded Markov chain has transition probability matrix:

$$P = \begin{bmatrix} GD_0 & GD_1 & \ldots & GD_{K-2} & GE_{K-1} \\ D_0 & D_1 & \ldots & D_{K-2} & E_{K-1} \\ 0 & D_0 & \ldots & D_{K-3} & E_{K-2} \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & D_1 & E_2 \\ 0 & 0 & \ldots & D_0 & E_1 \end{bmatrix}, \tag{10}$$

where $G = (\Lambda - Q)^{-1}\Lambda,$ consisting of conditional probabilities that system is not busy and $E_i = \sum_{k=i}^{\infty} D_k$. Let $\overrightarrow{y_k}$, $(0 \le k \le K-1)$ be an $1 \times m$ vector whose i^{th} element is the stationary conditional probability that the number of packets in the system is k and the state of underlying arrival process is in i at an arbitrary time. The packet loss probability (PLP) as given in the paper [11] is

$$PLP = 1 - \frac{\left(1 - \overrightarrow{y_0}e\right)}{\rho}, \quad (11)$$

where $\rho = \frac{\lambda}{\mu}$, traffic intensity, and λ is the mean arrival rate of MMPP and is given by $\lambda = \vec{\pi}\Lambda e$.

Numerical Results

In this section, we present some numerical results of mean delay and investigate its behavior in terms of traffic intensity, time-scale, Hurst parameter H, and DLU. First, transition rate matrix Q and arrival rate matrix Λ of MMPP are fitted according to the generalized variance based method [5] for the self-similar traffic pertaining to the values $H = 0.7, 0.8, 0.9,$ variance $\sigma^2 = 0.6,$ arrival rate $\lambda = 1$ over the time-scales $[10^2, 10^5], [10^2, 10^6],$ and $[10^2, 10^7]$ as in the paper [5]. Next, the stationary probability vector $\vec{y}$ is computed, and then mean delay W is computed against traffic intensity without taking voids into consideration as in the paper [IAENG] using the formula $w = \bar{\nu} \sum_{k=1}^{\infty} k(\bar{y}_k\, e)$ to realize the effect of voids consideration. Numerical calculations are performed using MATLAB and are compared with that of Q-SQUARED for the validation of MATLAB program and results are depicted in Fig.2. The difference between the analytical results and that of Q-SQUARED is found to be very small. From the Fig.2, it is clear that as traffic intensity increases mean delay increases. First, at $\rho = 0.4,$ the packet loss probability is computed as in the paper [11], then $\bar{\nu}_e$ is computed using the above eqn, and then mean delay is computed against D using the eqn (15). Results are depicted in the Fig. 3. From the Fig 3. It is clear that mean delay increases as D increases and the values are higher when H and time-scale are higher. Fig.4. depicts the variation of mean delay against traffic intensity ρ when the DLU $D = 0.1$. From the Fig. 4, one can infer that mean delay increases as ρ increases. From the Fig.2 and Fig 4, it is clear that trend in the case of voids is different from the without voids case, and values are higher in the case of voids.

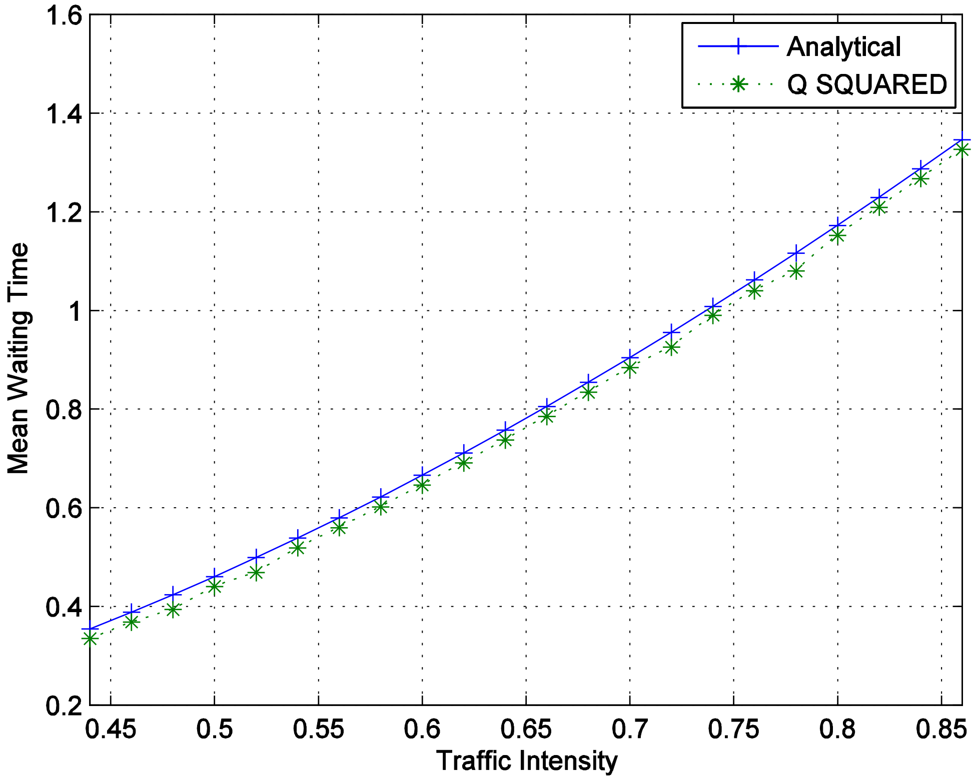

Fig 2: Variation of Mean Waiting Time with Traffic Intensity when H=0.7 and d=4

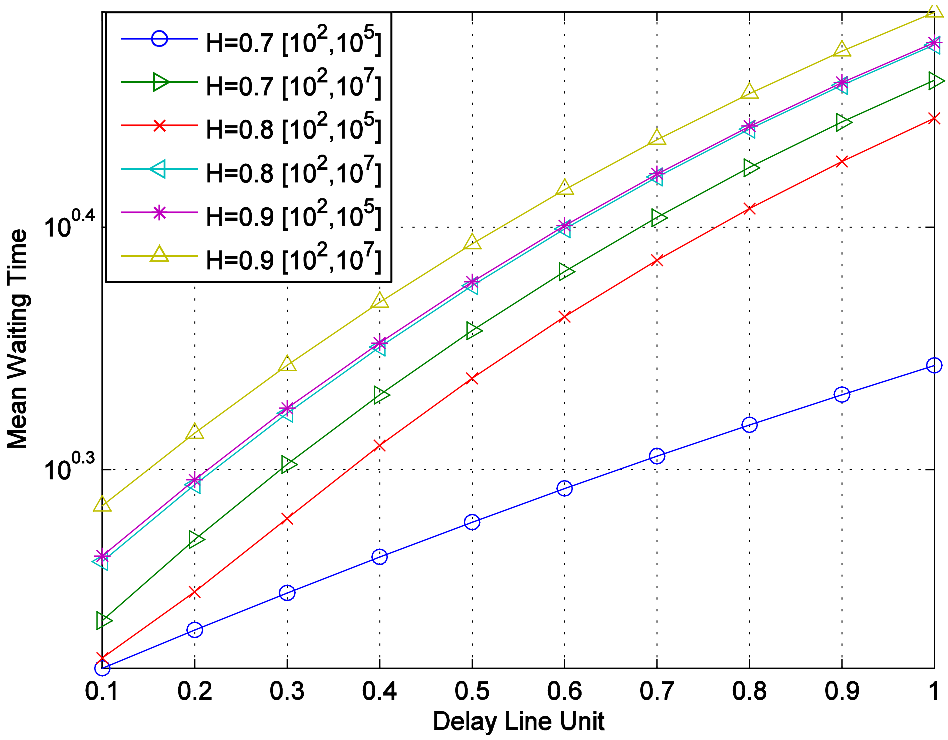

Fig 3: Variation of mean waiting time with delay line unit at different H

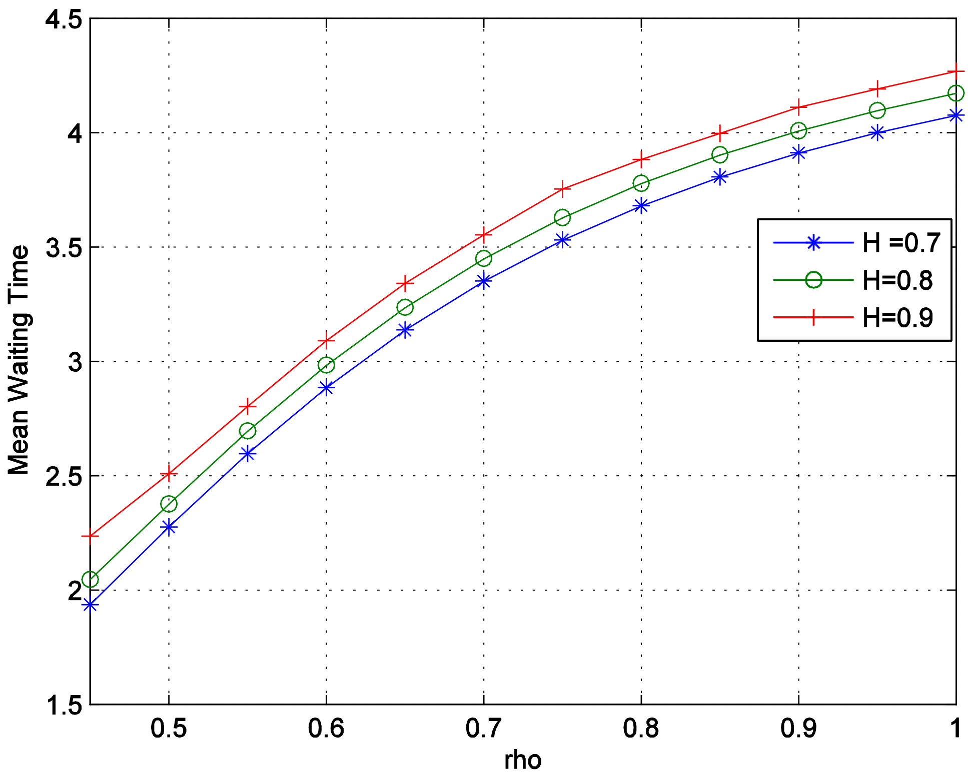

Fig 4: Variation of Mean Delay with Traffic Intensity at various H.

Conclusions

1. In this model, packet lengths and voids is assumed to follow exponential distribution and uniform. Distribution our numerical results reveal that time-scale and Hurst parameter do have impact on the mean delay.
2. Mean delay increases as H, ρ and DLU (D) increase. Based on the analysis presented in this paper, one could select the appropriate time-scale and D to meet the QoS requirement.
3. This kind of analysis is useful in dimensioning the switch under self-similar variable length packet traffic.

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