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Inventory D2C model where demand depends on price and social media advertisement

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Abstract
The factory-to-consumer model is a business model where manufacturers sell directly to consumers, without using intermediaries like retailers or wholesalers. This model is also known as direct-to-consumer (D2C) model. The D2C e-Commerce business model allow more control over your business and product. Without intermediaries, brands can quickly launch new product, test marketing strategies, and adjust pricing based on real- time customer insights. This paper presents inventory D2C model where demand depends on price and social media advertisement. Shortages are not allowed. The deterioration rate has been considered here to be constant. A mathematical model is developed by using differential equations and solve analytically by minimizing the total inventory cost. The result is illustrated with the help of numerical example of the model. Sensitivity analysis is carried out to analyze the effect of changes in the optimal solution with respect to change in various parameters.

Keywords: Inventory, D2C (direct-to-consumer) model, deterioration, social media advertisement, price dependent demand

1. Introduction

Advertising and Pricing are pivotal factors influencing consumer demand for products and services. Advertising influences demand by shaping consumer perceptions and preferences, while pricing directly affects demand through its impact on consumer purchasing decisions. This proposed model is based on two concept first is Direct–to-consumer business strategy where businesses sell their products directly to consumers and other is a digital platform that lets business owners to sell their goods and services online is called e- commerce website. Social media plays a crucial role in the success of direct-to-consumer businesses by providing a platform to engage directly with customers, build brand awareness, and drive sales. Social media ads like Facebook, Instagram are generally more affordable than traditional advertising. These media allows businesses to interact directly with potential customers through comments, likes and shares, which helps build relationships and trust.

The D2C model allows companies to control their brand, marketing and customer data. These companies can keep more profit because they don't have to pay wholesalers and retailers. Selling directly to customers allows company to collect valuable data on customer behavior, preferences, and feedback, helping improve products and marketing strategies. This model has direct visibility into sales data, allowing for better demand forecasting, inventory control, and reduced wastage. This model is suitable for small enterprises.

2. Background of research

In classical inventory models the demand rate is assumed to be constant but demand for physical goods may be time dependent, stock dependent and also depend on marketing of the product. Shah and Pandey (2009) [7] designed a deteriorating inventory model in which demand depends on advertisement and stock display. Lin-na (2016) developed model to explore the agricultural retail e-commerce marketing channels. Hasan *et al*. (2019) [4] designed an inventory model for decaying products with consideration of price, frequency of advertisement and continuous time dependent demand. Kumar *et al*. (2021) [3] developed an inventory model for deteriorating items when demand depends on advertisement and stock with partial backlogging. Wenji (2021) [7] studied on invention of fresh food in commercial aspects using e- commerce over internet. Cheng *et al*. (2023) [1] derived inventory model with advertisement and customer relationship management sensitive demand for product's life cycle. Rathore *et al*. (2023) [5] developed a model for "predictive analytics for inventory management in multi vendor e-commerce" focuses on using predictive analytics to improve inventory control in an environment where there are multiple vendors.

3. Notation
$I(t)$ = Inventory level with respect to time t,
D = Demand,
θ= Deterioration rate,
C_o = Ordering cost,
C_h = Holding cost,
C_d =Deterioration cost
C_A = Advertisement cost
P = Price
a = demand parameter
b = demand parameter
T = Length of each ordering cycle
A^α = Frequency of Advertisement

4. Assumptions
1. The inventory system deals with a single item.
2. The entire lot is delivered in one batch.
3. The demand rate D(A,P) is dependent on selling price and the frequency of advertisement. It is denoted by

$D(A,P) = A^\alpha (a\text{-}bp)$ where $a, b, \alpha > 0$.

4. Production is always greater than or equal to the demand rate.
5. The replenishment rate is infinite.
6. Lead time is negligible.
7. Shortages are not allowed in this model.
8. the deterioration rate is constant i.e. θ.

5. Mathematical model
In this mathematical model, The rate of change of inventory level due to demand & deterioration rate during a positive stock period is given by following differential equations

$$\frac{dI(t)}{dt} + \theta I(t) = -A^\alpha(a - bp)\ 0 < t < T \tag{1}$$

Solution of equation (1) is

$$I(t) = \frac{A^\alpha(a-bp)}{\theta}[e^{\theta(T-t)} - 1] \tag{2}$$

In boundary condition $I(t) = Q$ at $t=0$, $I(t) = 0$ at $t=T$

$$Q = \frac{A^\alpha(a-bp)}{\theta}[e^{\theta T} - 1] \tag{3}$$

1. Ordering cost = C_o (4)

2. Production cost = $C_p \int_0^T A^\alpha(a - bp)dt$

$=C_p A^\alpha(a - bp)T$ (5)

3. Holding cost = $C_h \int_0^T I(t)dt$

$= C_h \int_0^T \frac{A^\alpha(a-bp)}{\theta}[e^{\theta(T-t)} - 1]dt$

$= C_h \frac{A^\alpha(a-bp)}{\theta^2}[e^{\theta T} - \theta T - 1]$ (6)

4. Deterioration cost = θC_d (7)

Where θ = deterioration rate

5. Advertisement cost = $A^\alpha C_A$ (8)

Total cost (*TC*) - The total cost comprises the sum of ordering cost, the production cost, holding cost, deteriorating cost, and advertisement cost. They are grouped after evaluating the above cost individually.

Total cost = OC + PC + HC + DC + AC

$$\text{Total cost} = C_o + C_p A^{\alpha}(a - bp)T + C_h \frac{A^{\alpha}(a-bp)}{\theta^2}[e^{\theta T} - \theta T - 1] + \theta C_d + A^{\alpha} C_A \quad (9)$$

$$\text{Total cost per unit time is (TC)} = \frac{1}{T}\left[C_o + C_p A^{\alpha}(a - bp)T + C_h \frac{A^{\alpha}(a-bp)}{\theta^2}[e^{\theta T} - \theta T - 1] + \theta C_d + A^{\alpha} C_A\right] \quad (10)$$

5. Analysis and Optimization

Our objective is to minimize the total cost per unit time. The necessary and sufficient condition for total cost to be minimize are

$$\frac{\partial(TC)}{\partial T} = 0 \quad (11)$$

and

$$\frac{\partial^2(TC)}{\partial T^2} > 0, for\ all\ T > 0. \quad (12)$$

Therefore, The first order differential equation of *TC* with respect to *T* is as follows

$$\frac{\partial(TC)}{\partial T} = -\frac{C_o + \theta C_d + A^{\alpha} C_A}{T^2} + \frac{C_h A^{\alpha}(a-bp)}{2} = 0 \quad (13)$$

$$\text{So, } T = \sqrt{\frac{2(C_o + \theta C_d + A^{\alpha} C_A}{C_h A^{\alpha}(a-bp)}}$$

The second order differential equation of *TC* with respect to *T* is as follows

$$\frac{\partial^2 TC}{\partial T^2} = \frac{2(C_o + \theta C_d + A^{\alpha} C_a)}{T^3} > 0 \quad (14)$$

The solution of the equation (13) gives the optimal value of *T* (say T^*) and satisfies the condition of equation (14).

6. Numerical example

For the numerical illustration of the developed model, the values of various parameters in proper units can be taken as follows:
$C_o = 200$, $C_A = 25$, $C_d = 10$, $C_h = 2.0$, $C_p = 10$, $a = 90$, $b = 0.3$, $\theta = 0.1$, $P=100$, $A=12$, $\alpha = 0.2$
Solving Equation (11) with the above parameters, we obtain the value of $T^* = 1.566$. On substitution of the optimal value of T^* in Equation (10), we obtain the minimum total cost per unit time $TC^* = 1294.709$.

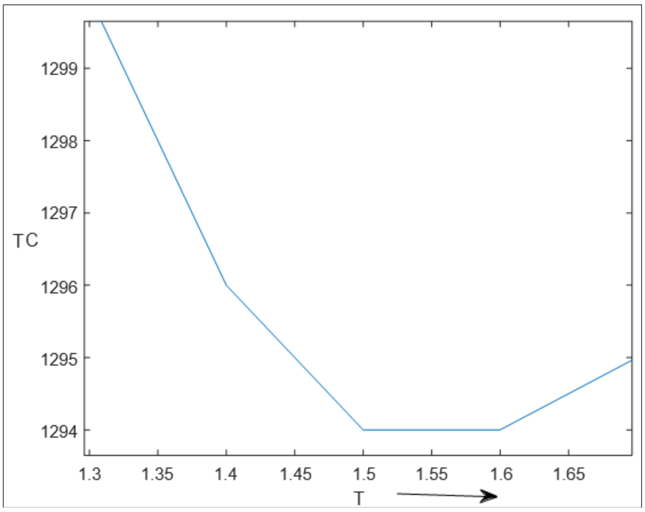

Fig 1: Graphical representation of T vs TC

7. Sensitivity analysis

We have performed sensitivity analysis by changing parameter C_o, C_h, A, P, C_d, C_A and keeping the remaining parameters at their original values. The original values are taken from above example. The results are obtained by changing one parameter at a time and keeping the other parameter at their original values. The analysis is carried out by changing parameter -20%, -10%, 10%, 20% and corresponding changes in the cycle time T and total cost (TC) are exhibited in the following table.

Table 1: Sensitivity analysis with different parameters

Parameter	Value	% change	T*	TC(Total cost)
C_o	160	-20%	1.431	1269
	180	-10%	1.500	1281
	200	0	1.566	1294
	220	10%	1.630	1307
	240	20%	1.691	1320
C_h	1.6	-20%	1.75	1263
	1.8	-10%	1.65	1279
	2.0	0	1.56	1294
	2.2	10%	1.49	1310
	2.4	20%	1.43	1325
C_p	8	-20%	1.566	1097
	9	-10%	1.566	1196
	10	0	1.566	1294
	11	10%	1.566	1393
	12	20%	1.566	1491
A	9.6	-20%	1.595	1244
	10.8	-10%	1.588	1257
	12	0	1.566	1294
	13.2	10%	1.553	1317
	14.4	20%	1.542	1339
P	80	-20%	1.493	1408
	90	-10%	1.529	1351
	100	0	1.566	1294
	110	10%	1.607	1237
	120	20%	1.651	1180

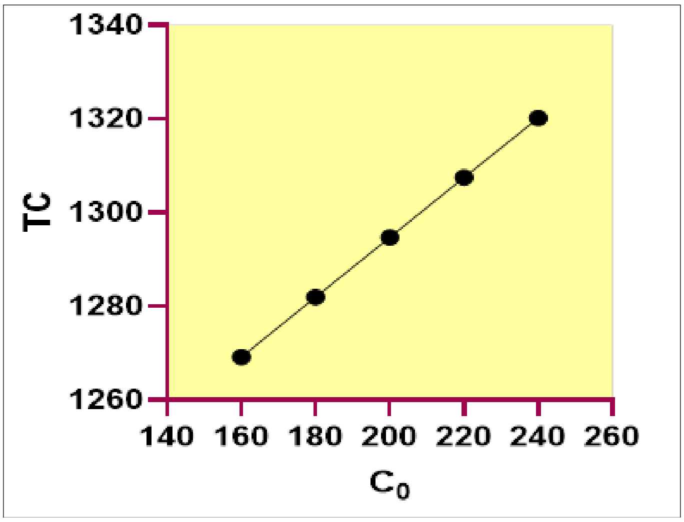

Fig 2: Graphical representation of total cost (TC) vs Ordering cost

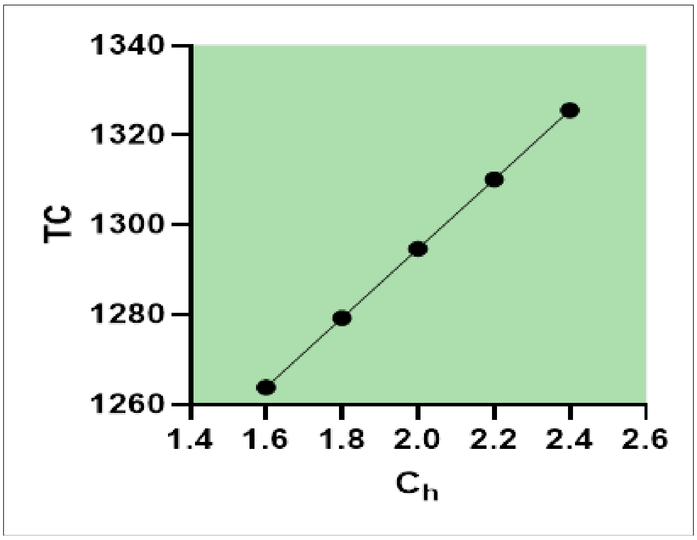

Fig 3: Graphical representation of total cost (TC) vs Holding cost

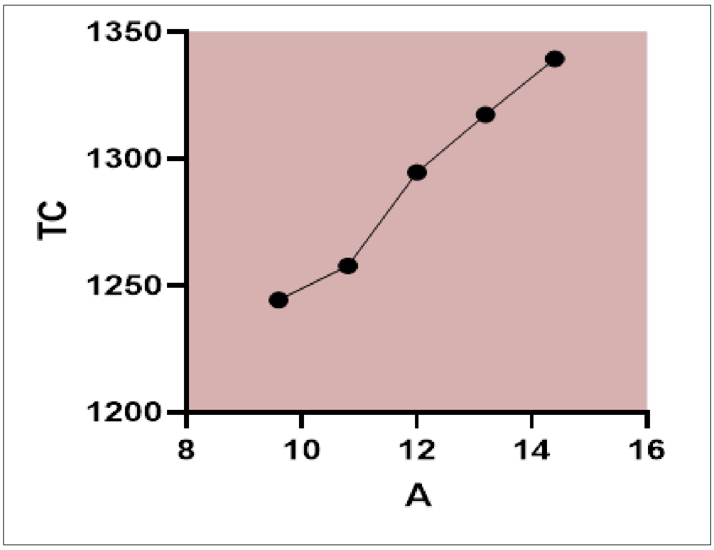

Fig 4: Graphical representation of total cost (TC) vs demand parameter (A)

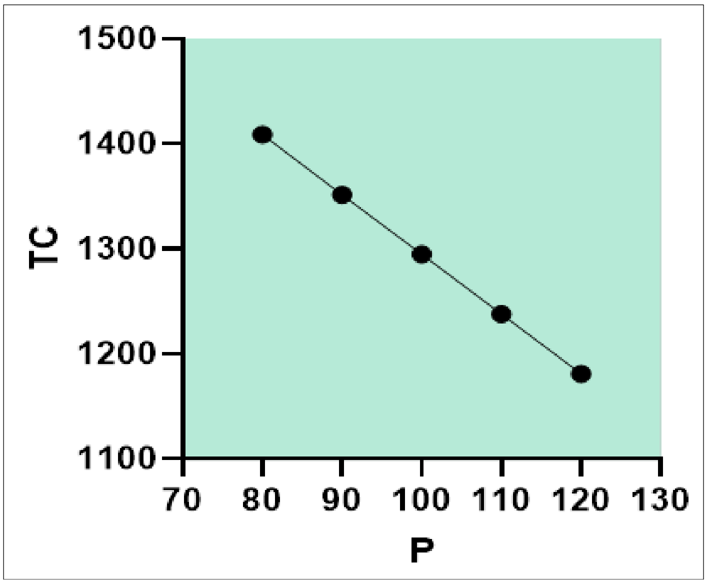

Fig 5: Graphical representation of total cost (TC) vs demand parameter (P)

From table 1, the following observations can be made

1. Increase in the value of holding cost, ordering cost and production cost results increase the value of total cost (TC) keeping all parameters same.
2. The effect on total cost with respect to the value of parameter P, is inversely proportional, keeping all parameter same.
3. The effect of value of parameter A^{α} is proportional to the value of total cost. So we increase the value of A result increase the value of total cost.

8. Conclusion

In this article, we described an inventory D2C model in which demand depends on selling price and social media advertisement. This model allowing companies to directly monitor customer demand, enabling them to adjust production and stock levels more precisely, minimizing overstocking and under stocking while optimizing inventory levels based on real-time data, leading to improved efficiency and reduced cost. In this model stock was depleted due not only to customers demand but also to deterioration. The rate of deterioration is constant. We found the optimal cycle length in order to minimize the total cost. Here we established mathematical model and its solution. To demonstrate the model numerical example and its sensitivity analysis are given.

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