Journal of Mathematical Problems, Equations and Statistics

E-ISSN: 2709-9407
P-ISSN: 2709-9393
JMPES 2025; 6(1): 109-111

www.mathematicaljournal.com
Received: 12-11-2024
Accepted: 17-12-2024

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On expansion nano T-continuity

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Abstract
The purpose of this work is to study a new class of maps called T-continuous mapping in nano topological space. This was done based on T-open set, which was defined at the beginning. During the study, we made use of the concept of expansion to give the most important properties of T-continuous mapping.

Keywords: Nano pen set, nano topological space and nano continuity

Introduction
Nano topological space (Briefly NTS) was first introduced by Thivagar and Carmel [1], where they introduced the concepts of Nano Open set, nano closed set, nano interior set and closure set. Also, they defined nano continuous mapping [2]. The notion of almost continuous mapping was studied by Singal and Sungal [3]. Saad introduce the notion of T- open set and T-closed set [4].

In this work, the concepts nano T-open set, nano T-closed set and nano T-contin-uous mapping are initiated. The important objective is studying the characteristics of nano T-continuous mapping.

Definition 1.1 [1]: An approximation space is the pair(U, R), where U is an universe set and R is an equivalence-relation on U.

i) The lower approximation of $\mathbb{A} \subseteq U$ defined as $L_R(\mathbb{A}) = \bigcup_{x\in U}\{R_x : R_x \subseteq \mathbb{A}\}$, where R_x is an equivalence class with respect to x.
ii) The upper approximation of $\mathbb{A} \subseteq U$ defined as $U_R(\mathbb{A}) = \bigcup_{x\in U}\{R_x : R_x \cap \mathbb{A} \neq \emptyset\}$.
iii) The boundary region of $\mathbb{A} \subseteq U$ with respect to R defined as $B_R(\mathbb{A}) = U_R(\mathbb{A}) - L_R(\mathbb{A})$.

Definition 1.2 [1]: Let U be the universe and R be an equivalence-relation on U and $\tau_R(\mathbb{A}) = \{U, \emptyset, L_R(\mathbb{A}), U_R(\mathbb{A}), B_R(\mathbb{A})\}$, where $\mathbb{A} \subseteq U$ and $\tau_R(\mathbb{A})$ satisfies the following axioms:

- U and $\emptyset \in \tau_R(\mathbb{A})$,
- The union of elements of any sub $-$ collection of $\tau_R(\mathbb{A})$ is in $\tau_R(\mathbb{A})$;
- The intersection of the elements of any finite sub-collection of $\tau_R(\mathbb{A})$ is in $\tau_R(\mathbb{A})$.

The pair $(U, \tau_R(\mathbb{A}))$is called a nano topological space (for short NTS) and any element in $\tau_R(\mathbb{A})$ is called nano open set. The complement of nano open set is called nano closed set.

Definition 1.3 [5]: Let $(U, \tau_R(\mathbb{A}))$ be NTS, $P(U)$ be power set of U. A mapping $\mathcal{F}: \tau_R(\mathbb{A}) \longrightarrow P(U)$ is said to be an expansion on $(U, \tau_R(\mathbb{A}))$ if $D \subseteq \mathcal{F}D$, for each $D \in \tau_R(\mathbb{A})$.

Example 1.4: Let $(U, \tau_R(\mathbb{A}))$ be NTS. Then:

i) A mapping $\mathcal{F}_C: \tau_R(\mathbb{A}) \longrightarrow P(U)$ such that $\mathcal{F}_C(D) = Cl_N(D)$, $\forall D \in \tau_R(\mathbb{A})$ is an expansion on $(U, \tau_R(\mathbb{A}))$, due to $D \subseteq \mathcal{F}_C(D)$.
ii) A mapping $\mathcal{F}_{IC}: \tau_R(\mathbb{A}) \longrightarrow P(U)$ such that $\mathcal{F}_{IC}(D) = Int_N(Cl_N(D))$, $\forall D \in \tau_R(\mathbb{A})$ is an expansion on $(U, \tau_R(\mathbb{A}))$, because $D = Int_N(D)$ and $D \subseteq N - Cl(D)$, thus $D \subseteq \mathcal{F}_C(D)$.
iii) A mapping $\mathcal{F}_U: \tau_R(\mathbb{A}) \longrightarrow P(U)$ such that $\mathcal{F}_U(D) = D \cup (Int_N(Cl_N(D))^c$, $\forall D \in \tau_R(\mathbb{A})$ is an expansion on $(U, \tau_R(\mathbb{A}))$, it is clear.

Definition 1.5 [5]: Let $(U, \tau_R(\mathbb{A}))$ be NTS. A pair expansion $\mathcal{F}$, $\mathcal{E}$ on $(U, \tau_R(\mathbb{A}))$ is said to be mutually dual if $\mathcal{F}D \cap \mathcal{E}D = D$, for each $D \in \tau_R(\mathbb{A})$.

Lemma 1.6: The pair expansion $\mathcal{F}_{IC}, \mathcal{F}_U$ on $(U, \tau_R(\mathbb{A}))$ is mutually dual.

Proof: $\mathcal{F}_{IC}D \cap \mathcal{F}_U D = Int_N(Cl_N) \cap D \cup (N - Int(Cl_N(D))^c = Int_N\big(Cl_N(D)\big) \cup (Int_N(Cl_N(D))^c \cap D = U \cap D = D.$

2 Nano T-Continuous mapping

Definition 2.1: Let $(U, \tau_R(\mathbb{A}))$ be NTS and D be subset of U, then:

i) D is called nano t-set if $Int_N(D) = Int_N(Cl_N(D))$

ii) D is called nano regular open set if $D = Int_N(Cl_N(D))$

Definition 2.2: Let $(U, \tau_R(\mathbb{A}))$ be NTS and D be subset of U. A point $x \in U$ is called nano T-cluster point of D if for any nano t-set K contain x implies $K \cap D \neq \emptyset$.

The set of all nano T-cluster points of D is called nano T-closure set of D and its denoted by $TCl_N(D)$.

Definition 2.3: A subset D of nano topological space $(U, \tau_R(\mathbb{A}))$ is said to be nano T-closed if $D = TCl_N(D)$ (Respectively, D^c is called nano T-open set).

Remark 2.4: Let $(U, \tau_R(\mathbb{A}))$ be NTS and D be subset of U. Then:

i) If $D = TInt_N(D)$, then D is nano T-open.

ii) If D is nano t-set, then D is nano T-open set.

iii) if D is nano closed set, then D is nano T-open set.

Example 2.5: Let $U = \{x, y, z, w, s\}$ and $U/R = \{\{x\}, \{y, z, w, s\}\}$. Take $\mathbb{A} = \{x, y, z\}$, then the nano topology on U is $\tau_R(\mathbb{A}) = \{U, \emptyset, \{x\}, \{y, z, w, s\}\}$, since {x},{y,z,w,s}are clopen, so they are nano t-sets and nano T-open sets.

Definition 2.6: Let $(U, \tau_R(\mathbb{A}))$ and $(V, \dot{\tau_R}(\mathbb{B}))$be two NTS. A mapping $\mathcal{L}: (U, \tau_R(\mathbb{A})) \longrightarrow (V, \dot{\tau_R}(\mathbb{B}))$ is called nano T-continuous if the inverse image of any nano open set in V is nano T-open set in U.

Example 2.7: Let $U = \{x_1, x_2, x_3, x_4, x_5\}, V = \{y_1, y_2, y_3, y_4, y_5\}$ and $U/R = \{\{x_1\}, \{x_2, x_3, x_4, x_5\}$, $V/R = \{\{y_1\}, \{y_2, y_3\}, \{y_4, y_5\}\}$ Take $\mathbb{A} = \{x_1, x_2, x_3\}$and $\mathbb{B} = \{y_1, y_2, y_3\}$, so the nano topologies on U and V are $\tau_R(\mathbb{A})$ $=\{U, \emptyset, \{x_1\}, \{x_2, x_3, x_4, x_5\}\}$, $\dot{\tau_R}(\mathbb{B}) = \{V, \emptyset, \{y_1, y_2, y_3\}\}$. Define $\mathcal{L}: (U, \tau_R(\mathbb{A})) \longrightarrow (V, \dot{\tau_R}(\mathbb{B}))$ such that $\mathcal{L}(x_1) = \{y_5\}$, $\mathcal{L}(x_2) = \mathcal{L}(x_3) = \{y_1\}, \mathcal{L}(x_4) = \{y_2\}, \mathcal{L}(x_5) = \{y_3\}$. The invers images of nano open sets in V are $U, \emptyset$ and $\{x_2, x_3, x_4, x_5\}$, but they are nano T-open sets in U. Hence, $\mathcal{L}$ is nano T-continuous mapping.

Definition 2.8: Let $(U, \tau_R(\mathbb{A}))$ and $(V, \dot{\tau_R}(\mathbb{B}))$be two NTS. A mapping $\mathcal{L}: (U, \tau_R(\mathbb{A})) \longrightarrow (V, \dot{\tau_R}(\mathbb{B}))$ is called nano almost T-continuous if for each nano open set D in V containing $\mathcal{L}(w)$, there exists nano T-open set G in U containing w such that $\mathcal{L}(G) \subseteq Int_N(Cl_N(D))$.

Theorem 2.9: A mapping $\mathcal{L}: (U, \tau_R(\mathbb{A})) \longrightarrow (V, \dot{\tau_R}(\mathbb{B}))$ is nano almost T-continuous if and only if $\mathcal{L}^{-1}(D) \subseteq TInt_N(\mathcal{L}^{-1}(Int_N\big(Cl_N(D)\big)))$, $\forall D \in \dot{\tau_R}(\mathbb{B})$.

Proof: Suppose that $\mathcal{L}$ is nano almost T-continuous mapping and D is any nano open set in V. Let $w \in \mathcal{L}^{-1}(D)$so, $\mathcal{L}(w) \in D$, since D is a nano open, then it is neighborhood open of $\mathcal{L}(w)$ in V. There exists a nano T-open neighborhood G of w in U such that $\mathcal{L}(G) \subseteq Int_N(Cl_N(D))$. Thus $w \in G \subseteq \mathcal{L}^{-1}(Int_N\big(Cl_N(D)\big)$ and this implies $w \in TInt_N(\mathcal{L}^{-1}(Int_N(Cl_N(D)))$. Hence, $\mathcal{L}^{-1}(D) \subseteq TInt_N(\mathcal{L}^{-1}(Int_N(Cl_N\ (D))))$.

Conversely, suppose D be any nano open set in V such that $\mathcal{L}(w) \in D$, then $w \in \mathcal{L}^{-1}(D) \subseteq TInt_N(\mathcal{L}^{-1}(Int_N\big(Cl_N(D)\big)))$. Let $G = TInt_N(\mathcal{L}^{-1}(Int_N\big(Cl_N(D)\big))) \subseteq \mathcal{L}^{-1}(Int_N\big(Cl_N(D)\big))$, this implies $\mathcal{L}(G) \subseteq \mathcal{L}(\mathcal{L}^{-1}(Int_N\big(Cl_N(D)\big))$, therefore, $\mathcal{L}(G) \subseteq Int_N(Cl_N(D))$ and G is nano T-open set. Hence, $\mathcal{L}$ is almost continuous mapping.

Definition 2.10: Let $(U, \tau_R(\mathbb{A}))$ and $(V, \dot{\tau_R}(\mathbb{B}))$be two NTS. A mapping $\mathcal{L}: (U, \tau_R(\mathbb{A})) \longrightarrow (V, \dot{\tau_R}(\mathbb{B}))$ is called $\mathcal{F}$ expansion nano T-continuous if $\mathcal{L}^{-1}(D) \subseteq TInt_N(\mathcal{L}^{-1}(\mathcal{F}D))$, for each $D \in \dot{\tau_R}(\mathbb{B})$.

Theorem 2.11: Let $(U, \tau_R(\mathbb{A}))$ and $(V, \dot{\tau_R}(\mathbb{B}))$be two NTS and Let $\mathcal{F}$ and $\mathcal{E}$ two mutually dual expansion on V. Then a mapping $\mathcal{L}: (U, \tau_R(\mathbb{A})) \longrightarrow (V, \dot{\tau_R}(\mathbb{B}))$ is T-continuous if and only if $\mathcal{L}$ is $\mathcal{F}$ expansion nano T-continuous and $\mathcal{E}$ expansion nano T-continuous.

Proof: Let $\mathcal{F}$ and $\mathcal{E}$ be two mutually dual expansion on V and $D \in \dot{\tau_R}(\mathbb{B})$. Then $\mathcal{F}D \cap \mathcal{E}D = D$ and $\mathcal{L}^{-1}(\mathcal{F}D) \cap \mathcal{L}^{-1}(\mathcal{E}D) = \mathcal{L}^{-1}(D)$. But $\mathcal{L}$ is nano T-continuous, then $\mathcal{L}^{-1}(D) = TInt_N(\mathcal{L}^{-1}(D)$=$TInt_N\big(\mathcal{L}^{-1}(\mathcal{F}D) \cap \mathcal{L}^{-1}(\mathcal{E}D)\big) = TInt_N\big(\mathcal{L}^{-1}(\mathcal{F}D)\big)$ $\cap TInt_N(\mathcal{L}^{-1}(\mathcal{E}D))$, therefore $\mathcal{L}^{-1}(D) \subseteq TInt_N(\mathcal{L}^{-1}(\mathcal{F}D))$ and $\mathcal{L}^{-1}(D) \subseteq TInt_N(\mathcal{L}^{-1}(\mathcal{E}D))$for each $D \in \dot{\tau_R}(\mathbb{B})$. Hence, $\mathcal{L}$ is $\mathcal{F}$ T-expansion nano T-continuous and $\mathcal{E}$ T-expansion nano T-continuous.

Conversely, suppose that $\mathcal{L}$ is $\mathcal{F}$ expansion nano T-continuous and $\mathcal{E}$ expansion nano T-continuous such that $\mathcal{F}$ and $\mathcal{E}$ two mutually dual expansion on V. Then $\mathcal{L}^{-1}(D) \subseteq TInt_N(\mathcal{L}^{-1}(\mathcal{F}D))$and $\mathcal{L}^{-1}(D) \subseteq TInt_N(\mathcal{L}^{-1}(\mathcal{E}D))$for each $D \in \dot{\tau_R}(\mathbb{B})$, also $\mathcal{F}D \cap \mathcal{E}D = D$. Thus, $\mathcal{L}^{-1}(\mathcal{F}D) \cap \mathcal{L}^{-1}(\mathcal{E}D) = \mathcal{L}^{-1}(D)$, now $TInt_N(\mathcal{L}^{-1}(D)) = TInt_N\big(\mathcal{L}^{-1}(\mathcal{F}D)\big) \cap TInt_N\big(\mathcal{L}^{-1}(\mathcal{E}D)\big) \supseteq$

$\mathcal{L}^{-1}(\mathcal{F}D) \cap \mathcal{L}^{-1}(\mathcal{E}D) = \mathcal{L}^{-1}(D)$, this implies $\mathcal{L}^{-1}(D) \subseteq TInt_N(\mathcal{L}^{-1}(D))$. But $TInt_N(\mathcal{L}^{-1}(D)) \subseteq \mathcal{L}^{-1}(D)$, so $\mathcal{L}^{-1}(D) = TInt_N(\mathcal{L}^{-1}(D))$, that is $\mathcal{L}^{-1}(D)$ is nano T-open set in U. Hence, $\mathcal{L}$ is nano T-continuous.

Theorem 2.12: An nano T-continuous mapping $\mathcal{L}: (U, \tau_R(\mathbb{A})) \longrightarrow (V, \dot{\tau_R}(\mathbb{B}))$ is almost T-continuous.
Proof: Suppose $\mathcal{L}$ is An nano T-continuous and $D \in \dot{\tau_R}(\mathbb{B})$, then $\mathcal{L}^{-1}(D)$ is nano T-open set in U and so, $\mathcal{L}^{-1}(D) = TInt_N(\mathcal{L}^{-1}(D))$. We have $D \subseteq Int_N(Cl_N(D))$ and so, $D \subseteq \mathcal{L}(\mathcal{L}^{-1}Int_N(Cl_N(D)))$, thus $\mathcal{L}^{-1}(D) \subseteq TInt_N(\mathcal{L}^{-1}(\mathcal{L}\mathcal{L}^{-1}Int_N(Cl_N(D)))) \subseteq TInt_N(\mathcal{L}^{-1}(Int_N(Cl_N(D))))$. By Theorem 2.9, $\mathcal{L}$ is A nano almost T-continuous.

Theorem 2.13: A mapping $\mathcal{L}: (U, \tau_R(\mathbb{A})) \longrightarrow (V, \dot{\tau_R}(\mathbb{B}))$ is nano T-continuous if and only if $\mathcal{L}$ is nano almost T-continuous, where $\mathcal{L}$ is $\mathcal{F}_U$ expansion nano continuous.

Proof: Let $\mathcal{L}$ be nano T-continuous mapping. Thus, $\mathcal{L}$ is nano almost T-continuous from Theorem 2.12. Conversely, since $\mathcal{L}$ is nano almost T-continuous, then $\mathcal{L}^{-1}(D) \subseteq TInt_N(\mathcal{L}^{-1}(Int_N(Cl_N(D))))$ by Theorem 2.9, which equivalent to $\mathcal{L}$ is $\mathcal{F}_{IC}$ expansion nano continuous. Also, $\mathcal{L}$ is $\mathcal{F}_U$ expansion nano continuous, but we have $\mathcal{F}_{IC}$ and $\mathcal{F}_U$ are mutually dual by Lemma 1.6. Hence, $\mathcal{L}$ is nano T-continuous mapping due to Theorem 2.11.

Definition 2.14: Let $(U, \tau_R(\mathbb{A}))$ and $(V, \dot{\tau_R}(\mathbb{B}))$be two NTS and $\mathcal{F}$ expansion on $(V, \dot{\tau_R}(\mathbb{B}))$. A mapping $\mathcal{L}: (U, \tau_R(\mathbb{A})) \longrightarrow (V, \dot{\tau_R}(\mathbb{B}))$ is said to be T-closed $\mathcal{F}$ expansion nano T-continuous if $\mathcal{L}^{-1}(\mathcal{F}D)^c$ is T-closed for each $D \in \dot{\tau_R}(\mathbb{B})$.

Definition 2.15: Let $(U, \tau_R(\mathbb{A}))$ be NTS and $\mathcal{F}$ is nano open expansion on $(U, \tau_R(\mathbb{A}))$, then $\mathcal{F}$ is called idempotent if $\mathcal{F}(\mathcal{F}(D)) = \mathcal{F}(D)$, for each $D \in \tau_R(\mathbb{A})$.

Theorem 2.16: Let $\mathcal{L}$ be a mapping from $(U, \tau_R(\mathbb{A}))$ into $(V, \dot{\tau_R}(\mathbb{B}))$ and $\mathcal{F}$ be idempotent, then $\mathcal{L}$ is $\mathcal{F}$ expansion nano T-continuous if and only if $\mathcal{L}$ is closed $\mathcal{F}$ nano T-continuous.

Proof: Let $\mathcal{L}$ be $\mathcal{F}$ expansion nano T-continuous mapping and D be nano open set in V. We have $\mathcal{F}(\mathcal{F}(D)) = \mathcal{F}(D)$ because $\mathcal{F}$ is an nano open expansion and $\mathcal{F}$ is idempotent, thus $\mathcal{L}^{-1}(\mathcal{F}D) \subseteq TInt_N\mathcal{L}^{-1}(\mathcal{F}(\mathcal{F}D)) = TInt_N\mathcal{L}^{-1}(\mathcal{F}D)$ and so, $\mathcal{L}^{-1}(\mathcal{F}D)$ is nano T-open set in U. Hence is $\mathcal{L}$ is $\mathcal{F}$ expansion nano T-continuous
Conversely, Let $\mathcal{L}$ be $\mathcal{F}$ expansion nano T-continuous. Since $[\mathcal{L}^{-1}(\mathcal{F}D)^c]^c = [\mathcal{L}^{-1}(\mathcal{F}D)]^{c^c} = \mathcal{L}^{-1}(\mathcal{F}D)$ and $\mathcal{L}^{-1}(\mathcal{F}D)^c$ is nano T-closed in $(U, \tau_R(\mathbb{A}))$, then $\mathcal{L}^{-1}(\mathcal{F}D)$ is nano T–open set. Thus $\mathcal{L}^{-1}(\mathcal{F}D) \subseteq TInt_N\mathcal{L}^{-1}(\mathcal{F}D)$for each $D \in \dot{\tau_R}(\mathbb{B})$. Hence $\mathcal{L}$ is closed $\mathcal{F}$ nano T-continuous.

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