Journal of Mathematical Problems, Equations and Statistics

E-ISSN: 2709-9407
P-ISSN: 2709-9393
JMPES 2021; 2(1): 101-105

www.mathematicaljournal.com
Received: 25-02-2021
Accepted: 25-04-2021

Alok Kumar Srivastava
Department of Mathematics, Government Degree College Barakhal, Santkabirnagar, Uttar Pradesh, India

Kunjlal Singh
Department of Mathematics, Government Degree College Dhadha Bujurg Hata, Kushinagar, Uttar Pradesh, India

Correspondence Author;
Alok Kumar Srivastava
Department of Mathematics, Government Degree College Barakhal, Santkabirnagar, Uttar Pradesh, India

Study of specific curvature tensor on the noninvariant hypersurfaces with (f, g, u, v, λ)-structure of an affinely cosymplectic manifold

Alok Kumar Srivastava and Kunjlal Singh

DOI: https://dx.doi.org/10.22271/math.2021.v2.i1a.156

Abstract
In this paper, we have stuied about noninvariant hyper surface of an affinely cosymlectic manifold. Further, we have obtained some interesting results on projective curvature tensor, conformal curvature tensor, concircular curvature tensor and conharmonic curvature tensor under certain condition.
Mathematics subject classification 2010: 53 D10, 53C25, 53C21

Keywords: Cosymlectic manifold, projective, conformal, concircular; conharmonic

Introduction

Let M_{2n+1} be a (2n+1) dimensional almost contact manifold with (1, 1) type tensor F, a fundamental vector field T and a contact form A. Let us consider a 2n-dimensional manifold M_{2n} embedded in M_{2n+1} with embedding b: $M_{2n} \rightarrow M_{2n+1}$. Let us choose an affine normal N on M_{2n} in such a way that FN is always tangent to hypersurface and satisfy the following linear transformations.

$FBX = BfX + u(X)N$ (1.1)
$FN = -BU$ (1.2)
$T = BV + \lambda N$ (1.3)
$A(BX) = v(X)$ (1.4)

Where f is a (1, 1) type tensor; U, V are vector fields; u, v are 1– *forms* and λ a C^∞ -function. If u ≠ 0, M_{2n} is called a noninvariant hypersurface of $\overline{M}_{2n+1}$.
From (1.1), (1.2), (1.3), (1.4) and using properties of alm~~ost~~ contact structure (F, T, A), we have the following induced structure on M_{2n}

a) $f^2 X = -X + u(X)U + v(X)V$ (1.5)
b) $u(fX) = \lambda v(X)$, $v(fX) = -A(N)u(X)$
c) $fU = -A(N)V$, $fV = \lambda U$
d) $u(U) = 1 - \lambda A(N)$, $u(V) = 0$
e) $v(U) = 0$, $v(V) = 1 - \lambda A(N)$

If the vector fields T and N are distinct affine normals on M_{2n}, the M_{2n} has a quartic structure.

$f^4 + (1+\lambda A(N)) f^2 + \lambda A(N)I = 0$ (1.6)

The above equation may be factorized as

$(f^2 + \lambda A(N)I)(f^2 + I) = 0$ (1.7)

Here we have three case, namely $A(N) = \lambda$; $\lambda = 1$, $A(N) \neq 1$; $\lambda A(N) = 1$. *If*
We put
(i) A(N) = λ in (1.5), we have
a) $f^2 = -I + u \otimes U + v \otimes V$ (1.8)
b) $fU = -\lambda V$, $fV = \lambda U$
c) $uo f = \lambda v$, $vo f = -\lambda u$
d) $u(U) = 1 - \lambda^2$, $u(V) = 0$

e) $v(U) = 0, v(V) = 1-\lambda^2$

(ii) If $\lambda = 1$, $A(N) \neq 1$, Then we have
a) $f^2 = -I + (1-A(N))(u \otimes U + v \otimes V)$ (1.9)
b) $u o f = v$, $v o f = -A(N)u$
c) $fU = -A(N)V$, $fV = U$
d) $u(U) = 1, u(V) = 0$
e) $v(U) = 0, v(V) = 1$

(iii) If $\lambda A(N) = 1$, we have
a) $f^2X = -X + u(X)U + v(X)V$ (1.10)
b) $u(fX) = \lambda v(X)$, $v(fX) = -\frac{1}{\lambda}u(X)$
c) $fU = -\frac{1}{\lambda}V$, $fV = \lambda U$
d) $u(U) = 0, u(V) = 0$
e) $v(U) = 0, v(V) = 0$

The equation (1.8) gives on M_{2n} (f, U, V, u, v, λ)-structure. Now if we introduce a metric g on the (f, U, V, u, v, λ)-structure, such that

$$\begin{cases} g(U, X) = u(X),\ g(V, X) = v(X) \\ g(fX, fY) = g(X, Y) - u(X)u(Y) - v(X)v(Y) \end{cases} \quad (1.11)$$

The above structure reduces to an (f, g, u, v, λ)-structure on M_{2n}

2. Properties of Induced Curvature Tensor on the noninvariant Hypersurface with (f, g, u, v, λ)-structure of cosymplectic manifold

Let $'\tilde{K}$ be the curvature tensor on $\overline{M}_{2n+1}$ and 'K be the induced curvature tensor, the Gauss characteristic equation and Mainardi codazzi equations are given by

$$'\tilde{K}(BX, BY, BZ, BW)\, ob = {'K}(X, Y, Z, W) - h(Y, Z)h(X, W) \quad (2.1)$$

$$+h(X, Z)h(Y, W)$$

$$'\tilde{K}(BX, BY, BZ, N)\, o\, b = (D_Xh)(Y, Z) - (D_Yh)(X, Z) \quad (2.2)$$

Let M_{2n} be noninvariant hypersurface of a cosymplectic manifold $\overline{M}_{2n+1}$, then the induced curvature tensor $'K$ on M_{2n}, is given by

$$'K(X, Y, Z, V) = -h(Y, Z)(X\lambda) - \lambda w(X)h(Y, Z) + h(X, Z)(Y\lambda) \quad (2.3)$$

$$+\lambda w(Y)h(X, Z) - \{(D_X h)(Y, Z) - (D_Yh)(X, Z)\}$$

$$(D_Xh)(Y, V) = (D_Yh)(X, V) \quad (2.4)$$

Theorem (2.1): On the noninvariant hypersurface of an affinely cosymplectic manifold with (f, g, u, v, λ)-structure, if H is closed and
$\mu(Y) = -\lambda w(Y)$, then we have
a) $K(X, Y, V) = 0$ (2.5)
b) $Ric(Y, V) = 0$; where $\mu = d\lambda$.

Proof: If H is closed, then we have

$$(D_XH)(Y) - (D_YH)(X) = 0 \quad (2.6)$$

Now from (2.3), we have

$$'K(X, Y, Z, V) = -h(Y, Z)(d\lambda)(X) - \lambda w(X)h(Y, Z) + h(X, Z)(d\lambda)(Y)$$

$+\lambda w(Y)h(X, Z) - \lambda\{(D_X h)(Y, Z) - (D_Y h)(X, Z)\}$

$K(X, Y, V) = -(d\lambda)(X)HY - \lambda w(X)HY + HX(d\lambda)Y + \lambda w(Y)HX$

$-\lambda\{(D_X H)(Y) - (D_Y H)(X)\}$,

using equation (2.6), we get

$K(X, Y, V) = -\mu(X)HY - \lambda w(X)HY + HX\mu(Y) + \lambda w(Y)HX$

where $d\lambda = \mu$, taking $\mu(Y) = -\lambda w(Y)$, we get

$K(X, Y, V) = 0$.

Contracting with respect to X, we get

$Ric(Y, V) = 0$.

Theorem (2.2): On the noninvariant hypersurface M_{2n}, with (f, g, u, v, λ) structure of an affinely cosymplectic manifold $\overline{M}_{2n+1}$, if H is closed and $\mu(Y) = -\lambda w(Y)$, then the projective curvature tensor is given by

$$'P(X, Y, Z, V) = 0. \tag{2.7}$$

Proof: The projective curvature tensor is given by

$$P(X, Y, Z) = K(X, Y, Z) + \frac{1}{n-1}\{Ric(X, Z)Y - Ric(Y, Z)X\}$$

Putting Z = V and using equation (2.5), we get

$P(X, Y, V) = 0$

or

$'P(X, Y, Z, V) = 0$.

Theorem (2.3): On the noninvariant hypersurface M_{2n}, with (f, g, u, v, λ) - structure of an affinely cosymplectic manifold $\overline{M}_{2n+1}$, if H is closed and

$\mu(Y) = -\lambda w(Y)$, then the conformal curvature tensor is given by

$$'W(X, Y, Z, V) = \frac{1}{n-2}\{v(Y)Ric(X, Z) - v(X)Ric(Y, Z)\} \tag{2.8}$$

$$+\frac{r}{(n-1)(n-2)}\{v(X)g(Y, Z) - v(Y)g(X, Z)\}$$

$$'W(U, Y, Z, V) = \frac{1}{n-2}v(Y)Ric(U, Z) - \frac{r}{(n-1)(n-2)}v(Y)u(Z) \tag{2.9}$$

$$'W(V, Y, Z, V) = -\frac{1}{n-2}(1-\lambda^2)Ric(Y, Z) + \frac{r}{(n-1)(n-2)} \tag{2.10}$$

$$\{1-\lambda^2)g(Y, Z) - v(Y)v(Z)\}$$

$'W(X, Y, U, V) = \frac{1}{n-2}\{v(Y)\, Ric(X, U) - v(X) Ric(Y, U)\}$ (2.11)

$+ \frac{r}{(n-1)(n-2)}\{v(X)u(Y) - v(Y)u(X)\}$

$'W(X, Y, V, V) = 0.$ (2.12)

Proof: The conformal curvature tensor is defined as

$W(X, Y, Z) = K(X, Y, Z) - \frac{1}{n-2}\{Ric(Y, Z)X - Ric(X, Z)Y$

$- g(X, Z)RY + g(Y, Z)RX\}$

$+ \frac{r}{(n-1)(n-2)}\{g(Y, Z)X - g(X, Z)Y\}$

Putting $Z = V$ and using equation (2.5), we get

$W(X, Y, V) = \frac{1}{n-2}\{v(X)RY - v(Y)RX\} + \frac{r}{(n-1)(n-2)}\{v(Y)X - v(X)Y\}$

or

$'W(X, Y, Z, V) = \frac{1}{n-2}\{v(Y)\, Ric\,(X, Z) - v\,(X)\, Ric(X, Z)\}$

$+ \frac{r}{(n-1)(n-2)}\{v(X)g\,(Y, Z) - v(Y)g(X, Z)\}$

from above equation, we can easily find the equations (2.9), (2.10), (2.11) and (2.12) after putting $X = U$, $X = V$, $Z = U$ and $Z = V$ respectively.

Theorem (2.4): On the noninvariant hypersurface M_{2n}, with (f, g, u, v, λ) – structure of an affinely cosymplectic manifold $\overline{M}_{2n+1}$, if H is closed and

$\mu(Y) = -\lambda w(Y)$, then the concircular curvature tensor is given by

$'C(X, Y, Z, V) = \frac{r}{n(n-1)}\{v(Y)g(X, Z) - v(X)g(Y, Z)\}$ (2.13)

$'C(U, Y, Z, V) = \frac{r}{n(n-1)}\, v(Y)u\,(Z)$ (2.14) (2.14)

$'C(V, Y, Z, V) = \frac{r}{n(n-1)}\{v(Y)v(Z) - (1-\lambda^2)g(Y, Z)$ (2.15)

$'C(X, Y, U, V) = \frac{r}{n(n-1)}\{v(Y)u(X) - v(X)u(Y)\}$ (2.16)

$'C(X, Y, V, V) = 0.$ (2.17)

Proof: The concircular curvature tensor is defined as

$C(X, Y, Z) = K(X, Y, Z) - \frac{r}{n(n-1)}\{g(Y, Z)X - g(X, Z)Y\}$

Putting $Z = V$ and using equation (2.5), we get

$C(X, Y, V) = -\frac{r}{n(n-1)}\{v(Y)X - v(X)Y\}$

$'C(X, Y, Z, V) = \frac{r}{n(n-1)}\{v(Y)g(X, Z) - v(X)g(Y, Z)\}$

Putting $X = U$, $X = V$, $Z = U$ and $Z = V$ respectively in above equation, we will find equations (2.14), (2.15), (2.16) and (2.17) easily.

Theorem (2.5): On the noninvariant hypersurface M_{2n}, with (f, g, u, v, λ) - structure of an affinely cosymplectic manifold $\overline{M}_{2n+1}$, if H is closed and

$\mu(Y) = -\lambda w(Y)$, then the conharmonic curvature tensor is given by

$$'L(X, Y, Z, V) = \frac{1}{n-2}\{v(Y)Ric(X, Z) - v(X)Ric(Y, Z)\} \quad (2.18)$$

$$'L(U, Y, Z, V) = \frac{1}{n-2} v(Y)Ric(U, Z) \quad (2.19)$$

$$'L(V, Y, Z, V) = -\frac{1}{n-2}(1-\lambda^2)Ric(Y, Z) \quad (2.20)$$

$$'L(X, Y, U, V) = \frac{1}{n-2}\{v(Y)Ric(X, U) - v(X)Ric(Y, U)\} \quad (2.21)$$

$$'L(X, Y, V, V) = 0. \quad (2.22)$$

Proof: The conharmonic curvature tensor is defined as

$L(X, Y, Z) = K(X, Y, Z) - \frac{1}{n-2}\{Ric(Y, Z)X - Ric(X, Z)Y + g(Y, Z)RX - g(X, Z)RY\}$

Putting $Z = V$ and using (2.5), we get

$L(X, Y, V) = -\frac{1}{n-2}\{v(Y)RX - v(X)RY\}$

$'L(X, Y, Z, V) = \frac{1}{n-2}\{v(Y)Ric(X, Z) - v(X)Ric(Y, Z)\}$

Putting $X = U$, $X = V$, $Z = U$ and $Z = V$ respectively in above equation, we will find equations (2.19), (2.20), (2.21) and (2.22).

References

1. Blair DE, Ludden D. Hypersurfaces in almost contact manifolds. Tohoku Mathematical Journal. 1969;22:354-362.
2. Chaudhary OP, Narain D. Properties of curvature tensor on a C-Sasakian manifold. Journal of the National Academy of Mathematics. 1997;11:191-199.
3. Blair DE. The theory of quasi-Sasakian structure. Journal of Differential Geometry. 1970;4:155-161.
4. Narain D. Hypersurface with (f, g, u, v, λ)-structure of an affinely cosymplectic manifold. Indian Journal of Pure and Applied Mathematics. 1989;2068:799-803.
5. Goldberg SI, Yano K. Noninvariant hypersurface of almost contact manifolds. Journal of the Mathematical Society of Japan. 1970;22:25-34.
6. Yano K, Okumura M. On (f, g, u, v, λ)–structure. Kodai Mathematical Seminar Reports. 1970;22:401-423.
7. Srivastava AK, Narain D. Quasi-umbilical hypersurfaces of Sasakian manifold. Journal of the Tensor Society of India. 2006;24:77-90.
8. Srivastava AK, Narain D. Properties of induced curvature tensor of the noninvariant hypersurface with (f, g, u, v, λ)-structure of a Sasakian manifold. Journal of Rajasthan Academy of Physical Sciences. 2007;6(1):77-86.
9. Srivastava AK, Narain D. Induced connections on the noninvariant hypersurface with covariant almost analytic vector fields of a Sasakian manifold. Journal of the National Academy of Mathematics. 2003;17:43-49.
10. Srivastava SK, Srivastava AK. A note on quasi-umbilical hypersurface of a Sasakian manifold with (ϕ, g, u, v, λ)-structure. International Journal of Mathematics Research. 2013;5(2):283-288.
11. Srivastava AK, Srivastava SK. Parallel vector fields on the noninvariant hypersurface of a Sasakian manifold. Annals of Multi-Disciplinary Research. 2016;6(1):181-184.