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Alok Kumar Srivastava
Department of Mathematics, Government Degree College Barakhal, Santkabirnagar, Uttar Pradesh, India

Kunjlal Singh
Department of Mathematics, Government Degree College Dhadha Bujurg Hata, Kushinagar, Uttar Pradesh, India

Correspondence Author;
Alok Kumar Srivastava
Department of Mathematics, Government Degree College Barakhal, Santkabirnagar, Uttar Pradesh, India

Induced curvature tensor on the non-invariant hyper surfaces with (*f, g, u, v, λ*)-structure of an affinity cosymplectic manifold

Alok Kumar Srivastava and Kunjlal Singh

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Abstract
In this paper, we have studied about cosymplectic manifold and noninvariant hypersurface of an affinely cosymplectic manifold. Further we have obtained induced curvature tensor of an affinely cosymplectic manifold and obtain some results under certain conditions.
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Introduction

Let M_{2n+1} be a (2n+1) dimensional almost contact manifold with (1, 1) type tensor F, a fundamental vector field T and a contact form A. Let us consider a 2n-dimensional manifold M_{2n} embedded in M_{2n+1} with embedding b: $M_{2n} \rightarrow M_{2n+1}$. Let us choose an affine normal N on M_{2n} in such a way that FN is always tangent to hypersurface and satisfy the following linear transformations

$$FBX = BfX + u(X)N \tag{1.1}$$
$$FN = -BU \tag{1.2}$$
$$T = BV + \lambda N \tag{1.3}$$
$$A(BX) = v(X) \tag{1.4}$$

Where f is a (1,1) type tensor; U, V are vector fields; u, v are 1– *forms* and λ a C^∞ -function. If u ≠ 0, M_{2n} is called a noninvariant hypersurface of M_{2n+1}. From (1.1), (1.2), (1.3), (1.4) and using properties of almost contact structure (F, T, A), we have the following induced structure on M_{2n}.

a) $f^2 X = -X + u(X)U + v(X)V$ (1.5)
b) $u(fX) = \lambda v(X)$, $v(fX) = -A(N)u(X)$
c) $fU = -A(N)V$, $fV = \lambda U$
d) $u(U) = 1 - \lambda A(N)$, $u(V) = 0$
e) $v(U) = 0$, $v(V) = 1 - \lambda A(N)$

If the vector fields T and N are distinct affine normals on M_{2n}, the M_{2n} has a quartic structure.

$$f^4 + (1+\lambda A(N))\, f^2 + \lambda A(N) I = 0 \tag{1.6}$$

The above equation may be factorized as

$$(f^2 + \lambda A(N)I)(f^2 + I) = 0 \tag{1.7}$$

Here we have three case, namely $A(N) = \lambda$; $\lambda = 1$, $A(N) \neq 1$; $\lambda A(N) = 1$. *If*
We put

(i) A(N) = λ in (1.5), we have

a) $f^2 = -I + u \otimes U + v \otimes V$ (1.8)
b) $fU = -\lambda V,\ fV = \lambda U$
c) $u o f = \lambda v,\ v o f = -\lambda u$
d) $u(U) = 1 - \lambda^2,\ u(V) = 0$
e) $v(U) = 0,\ v(V) = 1 - \lambda^2$

(ii) If $\lambda = 1,\ A(N) \neq 1$, *Then we have* (1.9)
a) $f^2 = -I + (1-A(N))(u \otimes U + v \otimes V)$
b) $u o f = v,\ v o f = -A(N)u$
c) $fU = -A(N)V,\ fV = U$
d) $u(U) = 1,\ u(V) = 0$
e) $v(U) = 0,\ v(V) = 1$

(iii) If $\lambda A(N) = 1$, we have
a) $f^2X = -X + u(X)U + v(X)V$ (1.10)
b) $u(fX) = \lambda v(X),\ v(fX) = -\frac{1}{\lambda} u(X)$
c) $fU = -\frac{1}{\lambda} V,\ fV = \lambda U$
d) $u(U) = 0,\ u(V) = 0$
e) $v(U) = 0,\ v(V) = 0$

The equation (1.8) gives on M_{2n} (f, U, V, u, v, λ)-structure. Now if we introduce a metric g on the (f, U, V, u, v, λ)-structure, such that

$$\begin{cases} g(U, X) = u(X),\ g(V, X) = v(X) \\ g(fX, fY) = g(X, Y) - u(X)u(Y) - v(X)v(Y) \end{cases} \quad (1.11)$$

The above structure reduces to an (f, g, u, v, λ)-structure on M_{2n}

2. Hypersurfaces of an Affinely Cosymplectic Manifold

Let M_{2n+1} is an affinely cosymplectic manifold, i.e. $\overline{D}_x F = 0$ and $\overline{D}_x A = 0$, where $\overline{D}$ denote the connection on M_{2n+1}. Since $\overline{D}_x A = 0$ implies that $\overline{D}_x T = 0$, i.e. the vector field A is parallel. Let D be the induced connection on the hypersurface M_{2n} of the affine connection $\overline{D}$, now using Gauss and Weingarten's equations

$$\overline{D}_{BX} BY = BD_XY + h(X, Y)N \quad (2.1)$$

and

$$\overline{D}_{BX}N = BHX + w(X)N \quad (2.2)$$

Where h and H are the second fundamental tensor of type (0,2) and (1,1) respectively and w is a 1- *form*.
Now differentiating (1.1), (1.2), (1.3), (1.4) covariantly and using

$\overline{D}_x F = 0$, $\overline{D}_x A = 0$, $\overline{D}_x T = 0$ and (2.1), (2.2) and reusing (1.1), (1.2), (1.3) and (1.4), we get

a) $(D_X f)(Y) = u(Y)HX - h(X,Y)U$ (2.3)
b) $D_XV = \lambda HX,\ D_XU = fHX + w(X)U$
c) $(D_Xv)(Y) = \lambda h(X,Y),\ (D_Xu)(Y) = -h(X, fY) - w(X)u(Y)$
d) $h(X,V) = -X\lambda - \lambda w(X)$

In (2.3) b, if we take 1- *form* $w = u$, then

$$D_XU = fHX + u(X)U \quad (2.4)$$

Putting $X = U$, we get

$D_U U = fHU + (1 - \lambda^2)U$

If vector field U is auto parallel, then $D_U U = 0$, therefore we have

$fHU = -(1-\lambda^2)U$ (2.5)

Again Putting $X = V$ in (2.4), we get

$D_V U = fHV$

If vector field V is parallel to vector field U, then $D_V U = 0$, we get

$fHV = 0.$ (2.6)

In (2.3)b, if we take 1- *form* $w = v$, then we get

$D_X U = fHX + v(X)U$ (2.7)

Putting $X = U$, we get

$D_U U = fHU$

If vector field U is auto parallel, then $D_U U = 0$, therefore we have

$fHU = 0.$

Again Putting $X = V$ in (2.7), we get

$D_V U = fHV + (1 - \lambda^2)U$ (2.8)

If vector field V is parallel to vector field U, then $D_V U = 0$, which yields

$fHV = -(1 - \lambda^2)U.$ (2.9)

Also from (2.3)b, putting $X = U$, we get

$D_U U = fHU + w(U)U.$

If vector field U is auto parallel, i.e. $D_U U=0$ we get

$fHU + w(U)U = 0.$ (2.10)

Again Putting $X = V$ in (2.3)b, we get

$D_V U = fHV + w(V)U$

If vector field V is parallel to vector field U, then $D_V U = 0$, which yields

$fHV + w(V)U = 0.$ (2.11)

Theorem (2.1): On the noninvariant hypersurface of an affinely cosymplectic manifold with (f,g, u, v, λ)-structure, we have

$(D_X h)(Y,V) - (D_Y h)(X,V) = (Y\lambda)\, w(X) - (X\lambda)w(Y)$ (2.12)

$-\lambda\{(D_X w)(Y) - (D_Y w)(X)\}$

Proof: From (2.3)d, we have

$(D_Y h)(X,V)+h(D_Y X, V)+h(X,D_Y V) = -D_Y(X\lambda) - (D_Y \lambda)w(X)$

$-\lambda(D_Y w)(X) - \lambda w(D_Y X)$

which yields

$(D_Y h)(X,V) = (D_Y X)(\lambda) – h(X,\lambda HY) – YX\lambda – (Y\lambda)w(X) – \lambda(D_Y w)(X)$ (2:13)

Interchanging X and Y, we get

$(D_X h)(Y,V) = (D_X Y)(\lambda) - h(Y, \lambda HX) - XY\lambda - (X\lambda)w(Y) - \lambda(D_X w)(Y)$ (2.14)

Subtracting (2.13) from (2.14), we get

$(D_X h)(Y,V) - (D_Y h)(X,V) = (D_X Y)(\lambda) - (D_Y X)(\lambda) + \lambda g(HX, HY)$
$–\lambda g(HX, HY) - (XY\lambda - YX\lambda) + (Y\lambda)w(X)$
$–(X\lambda)w(Y) – \lambda\{(D_X w)(Y) - (D_Y w)(X)\}$
since $D_X Y - D_Y X = [X,Y]$ and $[X,Y] = XY – YX$, yields (2.12).

Corollary (2.1): If 1– *form w* is closed, then

$(D_X h)(Y,V) – (D_Y h)(X,V) = (Y\lambda)w(X) – (X\lambda)w(Y)$ (2.15)

Proof: Using $(D_X w)(Y) - (D_Y w)(X) = 0$ in (2.12), we get (2.15).

3. Properties of Induced Curvature Tensor on the noninvariant Hypersurface with (f, g, u, v, λ)-structure of cosymplectic manifold

Let $'\tilde{K}$ be the curvature tensor on $\overline{M}^{2n+1}$ and 'K be the induced curvature tensor, the Gauss characteristic equation and Mainardi codazzi equations are given by

$'\tilde{K}(BX, BY, BZ, BW)\, ob = 'K(X, Y, Z,W) – h(Y, Z)h(X,W)$ (3.1)

$+h(X,Z)h(Y,W)$

$'\tilde{K}(BX, BY, BZ, N)\, o\, b = (D_X h)(Y, Z) - (D_Y h)(X,Z)$ (3.2)

Theorem (3.1): Let M_{2n} be noninvariant hypersurface of a cosymplectic manifold $\overline{M}_{2n+1}$, then the induced curvature tensor $'K$ on M_{2n}, is given by

$'K(X,Y,Z,V) = –h(Y,Z)(X\lambda) – \lambda w(X)h(Y, Z)+h(X,Z)(Y\lambda)$
$+\lambda w(Y)h(X, Z) – \{(D_X h)(Y, Z)-(D_Y h)(X, Z)\}$ (3.3)

Proof: On cosymplectic manifold, we have
$'\tilde{K}(BX, BY, BZ,T) = 0$

$'\tilde{K}(BX, BY, BZ, BV + \lambda N) = 0$

$'K(BX, BY, BZ, BV)+\lambda'\tilde{K}(BX, BY, BZ, N) = 0$

Using (3.1) and (3.2), we have

$'K(X,Y,Z,V) – h(Y, Z)h(X,V)+h(X,Z)h(Y,V)$

$+\lambda\{(D_X h)(Y,Z) – (D_Y h)(X, Z)\}= 0$

$'K(X, Y, Z, V) = h(Y, Z)h(X,V) – h(X,Z)h(Y,V)$

$–\lambda\{(D_X h)(Y,Z) – (D_Y h)(X,Z)\}$

Using (2.3)d, we have

$'K(X, Y, Z, V) = h(Y,Z)-X\lambda – \lambda w(X)) – h(X,Z)(- Y\lambda- \lambda w(Y))$
$– \lambda\{(D_X h)(Y,Z) - (D_Y h)(X, Z)\}$

$'K(X,Y,Z,V) = -h(Y, Z)(X\lambda) - \lambda w(X)h(Y, Z) + h(X,Z)(Y\lambda)$

$+\lambda w(Y)h(X,Z) - \lambda\{(D_X h)(Y,Z) - (D_Y h)(X, Z)\}$

Corollary (3.1): On M_{2n}, we have

$$(D_X h)(Y,V) = (D_Y h)(X,V) \tag{3.4}$$

Putting $Z = V$ in (3.3), we get

$'K(X,Y,V,V) = -h(Y, V)(X\lambda) - \lambda w(X)h(Y, V) + h(X,V)(Y\lambda)$

$+\lambda w(Y)h(X,V) - \lambda\{(D_X h)(Y,V) - (D_Y h)(X,V)\}$

$= \{(Y\lambda) + \lambda w(Y)\}(X\lambda) + \lambda w(X)\{(Y\lambda) + \lambda w(Y)\} - \{(X\lambda) + \lambda w(X)\}(Y\lambda)$

$- \lambda w(Y)\{(X\lambda) + \lambda w(X)\} - \lambda\{(D_X h)(Y,V) - (D_Y h)(X,V)\}$

or

$'K(X, Y, V, V) = \lambda\{(D_Y h)(X,V) - (D_X h)(Y,V)\}$

Since $'K(X, Y, V, V) = 0$, we get

$(D_X h)(Y,V) = (D_Y h)(X,V)$.

Corollary (3.2): On the noninvariant hypersurface of an affinely cosymplectic manifold with (f,g,u,v, λ) - structure, if 1-*form* w is closed, we have

$$w \propto d\lambda \tag{3.5}$$

Proof: If 1 – *form* w is closed, we have

$(D_X w)(Y) - (D_Y w)(X) = 0$

From result (3.4), we get

$(D_Y h)(X,V) - (D_X h)(Y,V) = 0$

Using result (2.12), we get

$(X\lambda)w(Y) - (Y\lambda)w(X) = 0$ (As 1– *form* w is closed)

$$\frac{w(Y)}{Y\lambda} = \frac{w(X)}{X\lambda} = k$$

$\Rightarrow w(X) = k(X\lambda) = k(d\lambda)(X)$
$\Rightarrow w \propto d\lambda'$

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