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## A dual-rate production inventory model for deteriorating items: Addressing shortages with variable production cycles

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### Abstract

The dual-rate production inventory model for deteriorating items addresses the complexities of managing inventory for products that have a finite shelf life and are subject to deterioration. This model incorporates two production rates: a higher rate during periods of high demand and a lower rate when demand decreases, allowing for more flexible inventory management. It also accounts for variable production cycles, enabling firms to adjust production schedules dynamically to minimize costs associated with holding, ordering, and deterioration, while also considering potential shortages. By optimizing the switching time between production rates and the length of production cycles, the model aims to balance inventory levels, reduce wastage, and improve service levels. The model's approach to handling shortages through variable production cycles provides a strategic framework for businesses to enhance inventory control, particularly in industries dealing with perishable goods, ensuring better responsiveness to market demand and cost efficiency.

**Keywords:** Dual rate production, inventory model, deteriorating items, variable production cycles, inventory management, shortages, production rates, cost optimization

### Introduction

Overseeing stock for deteriorating items presents special difficulties because of the items' limited time span of usability and the potential for wastage, deterioration, and oldness. Traditional inventory models often fail to address the complexities associated with variable demand and the need for flexible production rates. To tackle these challenges, a dual-rate production inventory model is proposed, which utilizes two distinct production rates—high and low—to adapt to changing demand patterns and minimize costs related to holding, ordering, and deterioration. The model also incorporates variable production cycles, allowing firms to dynamically adjust production schedules to respond more effectively to demand fluctuations and potential shortages. This approach not only enhances inventory control by optimizing the timing of switching between production rates but also helps in reducing waste and improving service levels. Such a model is especially important for ventures managing short-lived merchandise, where effective stock administration is pivotal for keeping up with item quality, limiting misfortunes, and fulfilling client need. The dual-rate production inventory model provides a comprehensive framework for optimizing inventory management in complex and dynamic environments by incorporating considerations for both cycle variability and rate adjustments.

Jaggi and Tiwari (2014) <sup>[4]</sup> addressed the challenge of managing inventories under dynamic conditions, where both demand and holding costs fluctuate over time. They provided insights into how varying holding costs and demand affect optimal order quantities and total inventory costs. This model is particularly relevant for businesses facing uncertain market conditions and helps in understanding how to balance inventory levels and costs efficiently. Patel and Parekh (2014) <sup>[9]</sup> proposed a wide range of realistic factors, including inflation and credit terms, providing a comprehensive approach to inventory management in uncertain economic environments. This study is particularly useful for organizations that need to balance between holding costs and cash flow management. Yadav *et al.* (2019) <sup>[14]</sup> introduced the intricacies of time-shifting interest and holding costs, which are critical for organizations confronting fluctuating economic situations and stock levels. Rangarajan and Karthikeyan (2017) <sup>[11]</sup> highlighted the importance of considering both demand and cost variations over time,

which are often observed in real-world scenarios. The study helps in understanding how these variations impact inventory policies, aiding in more accurate inventory decision-making. In an ensuing report, Rangarajan and Karthikeyan (2017) [11] stretched out their work to EOQ models for both non-momentary and immediate decaying things with a cubic interest rate under expansion and reasonable postpone in installments. This model is particularly useful for scenarios where the demand rate is non-linear, and it incorporates financial elements such as inflation and credit policies, providing a more holistic approach to inventory management. Rangarajan and Karthikeyan (2018) [12] showed how inventory decisions could be optimized by strategically allocating items between warehouses with varying deterioration rates and costs.

Bishi *et al.* (2019) [3] consolidated outstanding interest to represent fast changes in market interest and featured the significance of picking ideal distribution center settings to limit costs. Industries that deal with perishable goods, where demand patterns can change significantly over time, can benefit from this model. Yadav and Swami (2019) [14] analyzed the complexities of managing items in two different storage locations, each with its holding costs, providing valuable insights into optimizing storage and inventory costs under such conditions.

Ali *et al.* (2021) [2] investigated a Economic Production Quantity (EPQ) model for crumbling things under a fractional exchange acknowledge strategy for cost subordinate interest, taking into account the impacts of expansion and unwavering quality. The creators examined what different exchange credit terms mean for stock levels and all out costs, underlining the meaning of consolidating cost subordinate interest and expansion factors in stock administration. Managers can now make well-informed decisions about credit policies and inventory replenishment in volatile market conditions thanks to the study's comprehensive model that incorporates financial and economic considerations. Manna *et al.* (2021) [7] focused on optimizing production and inventory costs for systems where production imperfections are a factor. The hybrid optimization technique demonstrated its capability to provide superior solutions compared to traditional methods, offering an innovative approach to dealing with complex production and inventory optimization problems where uncertainties in production quality exist.

Nath and Nabendu (2021) [8] addressed the complexities of managing inventories when demand is fully backlogged, and both time and price influence the demand. Supakar *et al.* (2022) [13] investigated the use of a counterfeit honey bee state calculation on a green creation stock issue for decaying things in a neutrosophic fuzzy climate. This study stands out for its focus on sustainability and the use of soft computing techniques to address inventory problems in an uncertain environment. The neutrosophic fuzzy approach provides a novel way to deal with ambiguity in demand and supply chain parameters.

Akhtar *et al.* (2023) [1] emphasized the impact of considering both time and price dependencies, highlighting how this hybrid algorithm outperforms other algorithms in finding optimal solutions for complex inventory models. Limi *et al.* (2024) [6] presented intricacy by considering numerous stockrooms and a quadratic interest design, making it exceptionally material to situations including various capacity areas and shifting interest rates. The review's accentuation on accumulating and its expense suggestions give significant

experiences to upgrading stock administration methodologies in multi-distribution center settings.

Overall, these studies highlight the importance of incorporating various real-world factors such as demand patterns, holding costs, deterioration rates, credit policies, and sustainability considerations into inventory models. The use of advanced optimization algorithms further enhances the applicability of these models in complex and uncertain environments, providing valuable tools for inventory management professionals and researchers.

### Key Assumptions

- **Two Rates of Production:** There are two different production rates - a high rate and a low rate. The production switches from high to low at a certain point in time.
- **Deterioration Rate:** Items are subject to deterioration over time, often modeled as an exponential decay.
- **Shortages:** Shortages are allowed, and they are backordered. That is, demand not met during a shortage period will be fulfilled when inventory becomes available.
- **Variable Production Cycle:** The production cycle time is a decision variable that affects the total cost.
- **Demand Rate:** Demand is constant over time.
- **Inventory Costs:** Includes holding costs, shortage costs, and setup costs for production.

### 1.3 Notations

T: Length of the production cycle.

$t_1$ : Time when production switches from high rate to low rate.

$t_2$ : Time when production stops and the inventory level reaches zero.

Q(t): Inventory level at time t.

$P_1$ : High production rate.

$P_2$ : Low production rate.

D: Constant demand rate.

$\theta$ : Deterioration rate of items.

$C_h$ : Holding cost per unit per time.

$C_s$ : Shortage cost per unit.

$C_p$ : Production cost per unit.

$C_o$  Setup cost per production run.

### Mathematical Formulation

(i) Production Phase with High Rate:  $0 \leq t \leq t_1$

$$\frac{dQ}{dt} + \theta Q(t) = P_1 - D, Q(0) = 0 \quad (1.1)$$

$$IF = e^{\theta t} \quad (1.2)$$

Multiplying (7.1) through by the integrating factor:

$$e^{\theta t} \frac{dQ}{dt} + \theta e^{\theta t} Q(t) = (P_1 - D)e^{\theta t}$$

$$\frac{d}{dt} [e^{\theta t} Q(t)] = (P_1 - D)e^{\theta t} \quad (1.3)$$

Integrating equation (7.3) on both sides with respect to t:

$$e^{\theta t} Q(t) = \frac{P_1 - D}{\theta} e^{\theta t} + C_1$$

Using the initial condition  $Q(0) = 0$ :

$$Q(0) = \frac{P_1 - D}{\theta} + C_1 = 0 \Rightarrow C_1 = -\frac{P_1 - D}{\theta}$$

$$Q(t) = \frac{P_1 - D}{\theta} (1 - e^{-\theta t}) \tag{1.4}$$

(ii) Production Phase with Low Rate:  $t_1 \leq t \leq t_2$

$$\frac{dQ}{dt} + \theta Q(t) = P_2 - D, Q(t_1) = Q_1 \tag{1.5}$$

where  $Q_1$  is the inventory level at  $t_1$ .  
Using the same integrating factor, the general solution is:

$$Q(t) = \left( Q_1 - \frac{P_2 - D}{\theta} \right) e^{-\theta(t-t_1)} + \frac{P_2 - D}{\theta} \tag{1.6}$$

Where

$$Q_1 = Q_1(t) = \frac{P_1 - D}{\theta} (1 - e^{-\theta t_1})$$

Substituting  $Q_1$  into the solution (7.6)

$$Q(t) = \left[ \frac{P_1 - D}{\theta} (1 - e^{-\theta t_1}) - \frac{P_2 - D}{\theta} \right] e^{-\theta(t-t_1)} + \frac{P_2 - D}{\theta} \tag{1.7}$$

(iii) Shortage Phase:  $t_2 \leq t \leq T$

During the shortage phase, the production stops, and only demand and deterioration affect the inventory level:

$$\frac{dQ}{dt} + \theta Q(t) = -D, Q(t_2) = 0, Q(T) = 0 \tag{1.8}$$

The solution of equation (7.8) reduces to

$$Q(t) = \frac{D}{\theta} [1 - e^{-\theta(t-t_2)}] \tag{1.9}$$

**Cost Analysis**

(i) Holding Cost:

$$\begin{aligned} TC_h &= C_h \left[ \int_0^{t_1} Q(t) dt + \int_{t_1}^{t_2} Q(t) dt \right] \tag{1.10} \\ &= C_h \left[ \frac{P_1 - D}{\theta} \left( t_1 + \frac{e^{-\theta t_1} - 1}{-\theta} \right) + \left\{ \frac{P_1 - D}{\theta} (1 - e^{-\theta t_1}) - \frac{P_2 - D}{\theta} \right\} \frac{1 - e^{-\theta(t_2-t_1)}}{\theta} + \frac{P_2 - D}{\theta} (t_2 - t_1) \right] \tag{7.11} \end{aligned}$$

(ii) Shortage Cost: The shortage cost is incurred during the period when there is a stockout:

$$\begin{aligned} TC_s &= C_s \int_{t_2}^T Q(t) dt \\ &= C_s \left[ \frac{D}{\theta} (T - t_2) - \frac{D}{\theta^2} \{1 - e^{-\theta(T-t_2)}\} \right] \tag{1.12} \end{aligned}$$

(iii) Production Cost: The production cost depends on the total units produced at the high and low rates:

$$TC_p = C_p [P_1 t_1 + P_2 (t_2 - t_1)] \tag{1.13}$$

Where  $P_1$  and  $P_2$  are high and low production rate.

(iv) Setup Cost:

$$TC_o = \frac{C_o}{T} \tag{1.14}$$

The absolute expense capability (TC) is the amount of the multitude of cost parts:

$$TC = TC_h + TC_s + TC_p + TC_o \tag{1.15}$$

Substituting the expressions for each cost component:

$$\begin{aligned} TC &= C_h \left[ \frac{P_1 - D}{\theta} \left( t_1 + \frac{e^{-\theta t_1} - 1}{-\theta} \right) + \left\{ \frac{P_1 - D}{\theta} (1 - e^{-\theta t_1}) - \frac{P_2 - D}{\theta} \right\} \frac{1 - e^{-\theta(t_2-t_1)}}{\theta} + \frac{P_2 - D}{\theta} (t_2 - t_1) \right] \\ &+ C_s \left[ \frac{D}{\theta} (T - t_2) - \frac{D}{\theta^2} \{1 - e^{-\theta(T-t_2)}\} \right] + C_p [P_1 t_1 + P_2 (t_2 - t_1)] + \frac{C_o}{T} \tag{1.16} \end{aligned}$$

The optimization problem can be formulated as follows: By determining the optimal values of  $t_1, t_2$  and  $T$ , the goal is to reduce the total cost (TC).

$$\min_{t_1, t_2, T} TC(t_1, t_2, T) \tag{1.17}$$

subject to:  $0 \leq t_1 \leq t_2 \leq T, Q(t) \geq 0$  for  $0 \leq t \leq t_2$  (1.18)

To find the stationary points, we need to solve the system of three nonlinear equations:

$$\frac{\partial TC}{\partial t_1} = 0, \frac{\partial TC}{\partial t_2} = 0, \frac{\partial TC}{\partial T} = 0 \tag{1.19}$$

This system is generally complex and we solve these equations numerically.

**Sensitivity Analysis:** Let's define the parameters' values as follows

- Holding Cost per Unit per Time ( $C_h$ ): 2
- Shortage Cost per Unit ( $C_s$ ): 5
- Production Cost per Unit ( $C_p$ ): 3
- Setup Cost per Production Run ( $C_o$ ): 50
- High Production Rate ( $P_1$ ): 50 units per time period
- Low Production Rate ( $P_2$ ): 20 units per time period
- Constant Demand Rate ( $D$ ): 30 units per time period
- Deterioration Rate ( $\theta$ ): 0.05

We want to find the optimal values of  
 $t_1$ : Time when production switches from high to low rate.  
 $t_2$ : Time when production stops, and the inventory level reaches zero.  
 $T$ : Total length of the production cycle.  
 We'll use Python and the optimize library to solve this optimization problem numerically.  
 Optimal  $t_1$ : 0.6963  
 Production switches from the high rate to the low rate after approximately 0.70 time units.  
 Optimal  $t_2$ : 2.0108  
 Production stops, and inventory reaches zero after approximately 2.01 time units.  
 Optimal  $T$ : 5.2231

The total production cycle length is approximately 5.22 time units. Minimum Total Cost: 336.4912

The minimum total cost for this configuration is approximately 336.49 units of cost.

**Table 1:** Sensitivity analysis of total cost with respect to key parameters

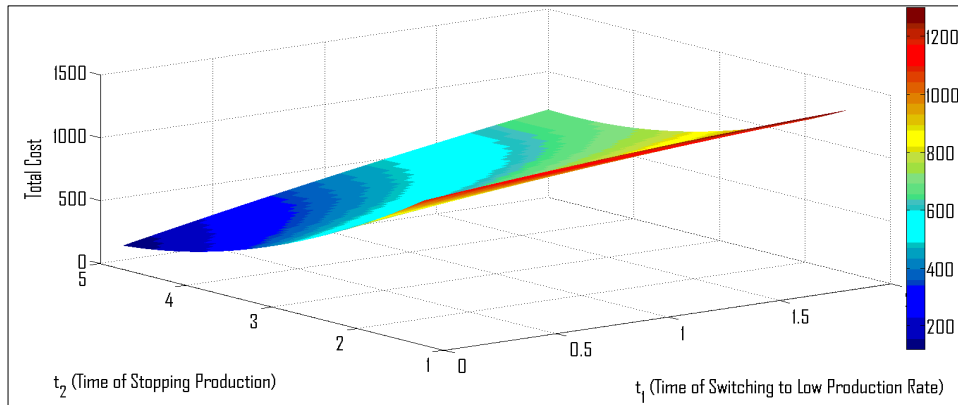
Parameter	Baseline Value	Base Cost	+10% Cost	-10% Cost
$C_h$	2	336.4278	345.0404	327.8151
$C_s$	5	336.4278	342.2074	330.6482
$C_p$	3	336.4278	339.7433	333.1123
$C_o$	50	336.4278	335.4527	337.4028
$P_1$	50	336.4278	335.8755	336.9801
$P_2$	20	336.4278	336.8885	335.9672
$D$	30	336.4278	337.3753	335.4804
$\theta$	0.05	336.4278	334.3248	338.5307

The table (1) shows the impact on total cost when each parameter is increased or decreased by 10% (+10% Cost and -

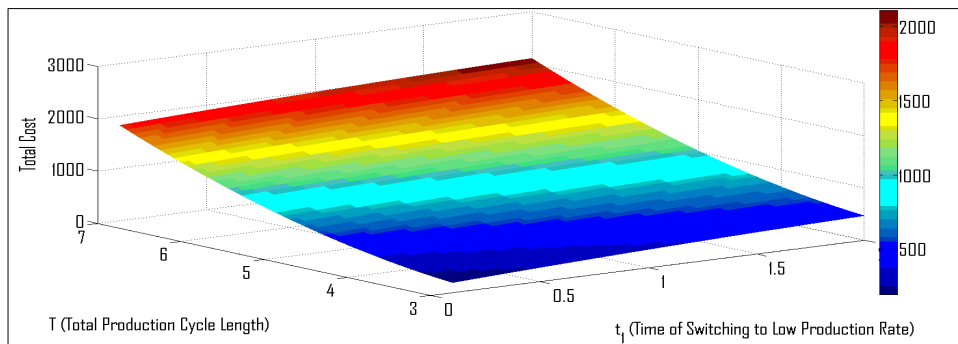
10% Cost, respectively), while keeping the other parameters constant. The results indicate the sensitivity of the total cost to each parameter, highlighting that changes in  $C_h$  and  $C_s$  have a more significant impact on total cost compared to other parameters. This analysis helps in understanding which

factors most influence the total cost, aiding decision-makers in optimizing inventory management under varying conditions.

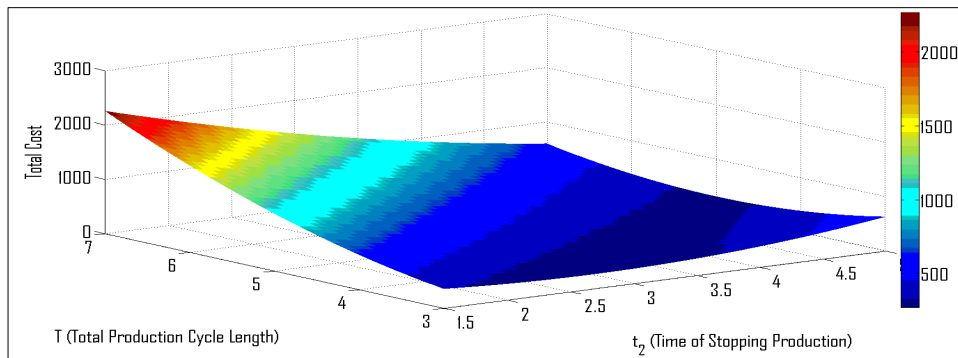
**Results and Discussion**



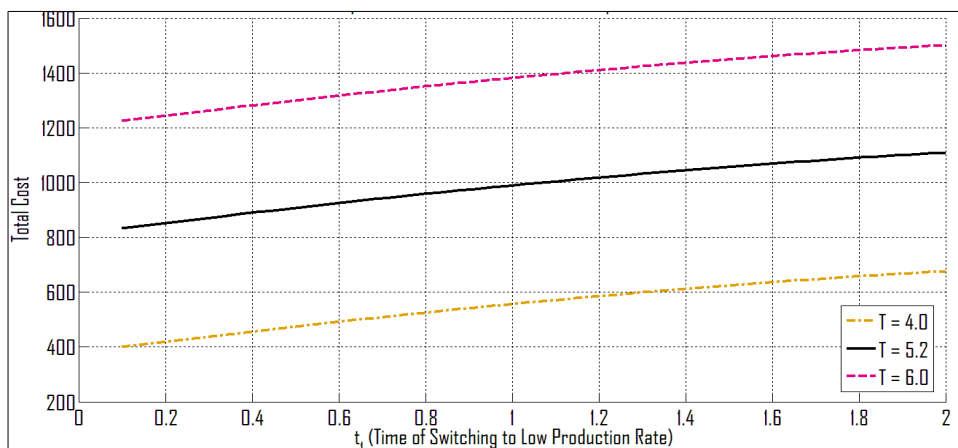
**Graph 1:** Total cost surface plot



**Graph 2:** Total cost surface plot ( $t_1$  vs.  $T$ ,  $t_2$  fixed)

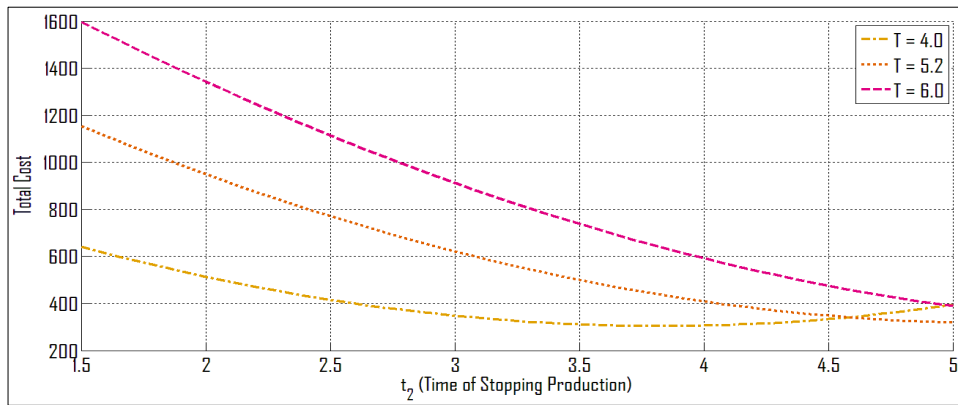


**Graph 3:** Total cost surface plot ( $t_1$ , vs.  $T$   $t_1$  fixed)

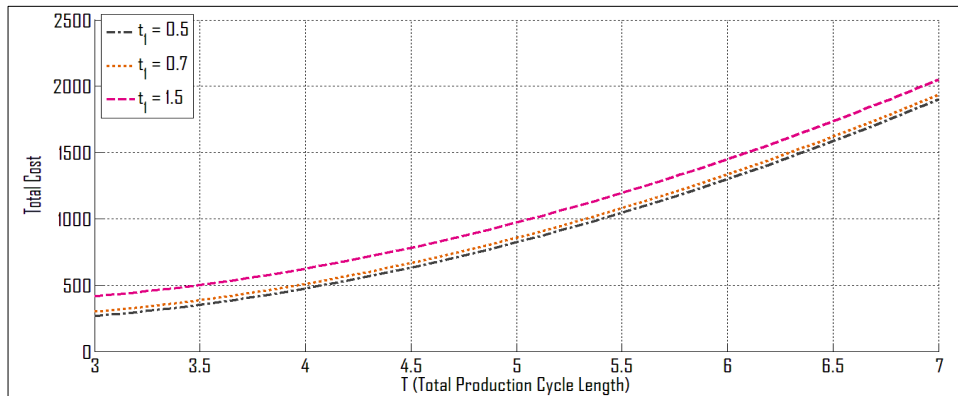


**Graph 4:** Total cost vs.  $t_1$  ( $t_2$  fixed, multiple  $T$  curves)





**Graph 5:** Total cost vs.  $t_2$  ( $t_1$  fixed, multiple  $T$  curves)



**Graph 6:** Total cost vs.  $T$  ( $t_2$  fixed, multiple  $t_1$  curves)

The graph (1) displayed is a 3D surface plot depicting the total cost as a function of two time variables:  $t_1$  (the time of switching to a low production rate) and  $t_2$  (the time of stopping production). The color gradient on the surface plot indicates different cost levels, with blue representing lower costs and red indicating higher costs. The plot reveals a steady increase in total costs as both  $t_1$  and  $t_2$  increase, suggesting that earlier switching to a low production rate and a delayed stopping time can result in lower total costs. Conversely, delaying the switch and stopping time tends to increase the total cost significantly. The visualization provides a comprehensive understanding of the cost dynamics based on different production time strategies.

The graph (2) displayed is a 3D surface plot that shows the relationship between total cost and two variables:  $t_1$  (the time of switching to a low production rate) and  $T$  (the total production cycle length), while keeping  $t_2$  (the time of stopping production) fixed. The z-axis shows the total cost, which ranges from 0 to 3000. The color gradient on the surface indicates varying levels of total cost, with blue representing lower costs and red indicating higher costs. The plot illustrates that the total cost decreases as  $t_1$  (time to switch to a low production rate) increases, particularly for shorter production cycle lengths  $T$ . For larger values of  $T$ , the total cost remains relatively high across all  $t_1$  values, indicating that a longer production cycle results in higher costs regardless of when the switch to a low production rate is made. Conversely, for shorter production cycles, the earlier switch to a low production rate can significantly reduce the total cost. This surface plot highlights the importance of optimizing both the time of switching to a lower production rate and the length of the production cycle to minimize total costs. The graph (3) is a 3D surface plot that depicts the total cost as a function of two variables:  $t_2$  (the time of stopping

production) and  $T$  (the total production cycle length), with  $t_1$  (the time of switching to a low production rate) held fixed. The color gradient on the surface indicates the total cost levels, with blue representing lower costs and red indicating higher costs. The plot shows a distinct pattern where the total cost generally decreases as  $t_2$  (the time of stopping production) increases, particularly for shorter production cycle lengths  $T$ . This trend suggests that for shorter production cycles, stopping production later (higher  $t_2$ ) can lead to significantly lower total costs. However, for longer production cycles (higher  $T$ ), the total cost remains relatively high regardless of the value of  $t_2$ . This implies that extending the production cycle length  $T$  results in increased costs, whereas optimizing the time to stop production  $t_2$  can effectively minimize costs within shorter production cycles. This visualization helps in understanding the cost dynamics involved in adjusting both the production cycle length and the stopping time for an optimal cost outcome.

The graph (4) is a 2D plot that illustrates the total cost as a function of  $t_1$  (the time of switching to a low production rate) with  $t_2$  (the time of stopping production) held fixed, for different values of  $T$  (the total production cycle length). Three different curves represent three different values of  $T$ : 4.0, 5.2, and 6.0. The curves are distinguished by different line styles and colors: a dashed orange line for  $T = 4.0$ , a dotted orange line for  $T = 5.2$  and a dashed magenta line for  $T = 6.0$ . The plot demonstrates that the total cost increases as  $t_1$  increases for all values of  $T$ . Additionally, the total cost is higher for longer production cycle lengths. For instance, the curve for  $T = 6.0$  (magenta) shows consistently higher costs compared to the curves for  $T = 5.2$  and  $T = 4.0$ . This indicates that delaying the switch to a low production rate ( $t_1$ ) and having longer production cycles ( $T$ ) both contribute to higher total costs. The graph provides insights into how adjusting the time

to switch to a lower production rate and the production cycle length can impact the total cost, emphasizing the need for an optimal balance between these parameters to minimize costs. The graph (5) is a 2D plot that shows the total cost as a function of  $t_2$  (the time of stopping production) with  $t_1$  (the time of switching to a low production rate) held fixed, for different values of  $T$  (the total production cycle length). Three different curves correspond to three different values of  $T$ : 4.0, 5.2, and 6.0. The curves are differentiated by different line styles and colors: a dashed orange line for  $T = 4.0$ , a dotted orange line for  $T = 5.2$ , and a dashed magenta line for  $T = 6.0$ . The plot indicates that the total cost decreases as  $t_2$  increases for all values of  $T$ . This suggests that delaying the time of stopping production ( $t_2$ ) leads to a reduction in total cost. Additionally, the total cost is consistently higher for longer production cycle lengths. For instance, the curve for  $T = 6.0$  (magenta) starts at a higher cost and remains above the other curves throughout the range of  $t_2$ . The plot shows that for shorter production cycles, the total cost can be reduced more significantly by increasing  $t_2$ . The graph emphasizes the importance of optimizing the stopping time and the production cycle length to minimize total costs effectively.

The graph (6) is a 2D plot that illustrates the total cost as a function of  $T$  (the total production cycle length) with  $t_2$  (the time of stopping production) held fixed, for different values of  $t_1$  (the time of switching to a low production rate). Three different curves represent three different values of  $t_1$ : 0.5, 0.7, and 1.5. The curves are distinguished by different line styles and colors: a dashed black line for  $t_1 = 0.5$ , a dotted orange line for  $t_1 = 0.7$ , and a dashed magenta line for  $t_1 = 1.5$ . The plot shows that the total cost increases as  $T$  (the production cycle length) increases for all values of  $t_1$ . Additionally, the total cost is higher for larger values of  $t_1$ . For example, the curve for  $t_1 = 1.5$  (magenta) is consistently above the curves for  $t_1 = 0.7$  and  $t_1 = 0.5$ . This indicates that delaying the switch to a low production rate (higher  $t_1$ ) results in higher costs, especially as the production cycle length ( $T$ ) becomes longer. The graph highlights the importance of both the timing of switching to a low production rate and the overall production cycle length in determining the total cost, suggesting that minimizing both parameters could help reduce costs.

### Concluding Remarks

A comprehensive framework for managing inventory in situations where items deteriorate over time is provided by the dual-rate production inventory model for deteriorating items, which addresses shortages with variable production cycles. By incorporating two production rates—one high and one low—alongside variable production cycles, the model effectively balances production and inventory holding costs while accommodating the complexities of item deterioration and demand. The model's applicability in real-world settings, where demand fluctuations and stockouts are common, is further enhanced by the inclusion of shortages. Sensitivity analysis reveals how changes in key parameters, such as production rates, holding costs, and deterioration rates, impact total cost, enabling decision-makers to optimize inventory strategies under various conditions. Overall, this model offers a robust approach to minimizing costs and managing resources efficiently in inventory systems where deterioration and shortages are critical considerations.

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