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A simple proof on the Fermat's last theorem in cases of n=3k (k=1,2,3……)

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Abstract
Using simple mathematics of middle school and "the disprove method", the paper discusses a special case of x=y of the Fermat's last theorem, but strangely, if x=y, from Diophantine equation it derives an impossible equality 2=1, therefore, it is reasonable to suggest that with respect to Fermat's last theorem, it should supplement a restriction of $x \neq y \neq z$. Moreover, all the positive integers which is bigger than 2 can be represented by n=3k; n=3k+1; and n=3k+2 (k=1,2,3, ……), on the basis of the proof on the Fermat's last theorem in case of n=3, using the mathematical induction and "the disprove method" the paper proves that the theorem is also true for all cases of n=3k (k=1,2,3,……).

Keywords: diophantine equation, Fermat's last theorem, x=y, disprove method, restriction, n=3k (k=1,2,3,……)

1. Introduction
In 1637 years, Pierre de Fermat claimed that there are no positive integer solutions for the Diophantine equation $x^n + y^n = z^n$ when n>2, all of x, y, z and n are positive integers. This statement originally proposed as a conjecture, death-likely passed about 350 years. During the long period of years, proving Fermat's last theorem has become an important topic in the mathematics fields and attracted a lot of researchers [1-11]. Until 1994 years, Andrew Wiles successfully proved Fermat's last theorem, and won the Abel prize with 700000USD for this contribution.

Although that, people continuously studied Fermat's last theorem, perhaps they hope to find simpler and better proofs [2-3, 8-9]. Moreover, some people like to study a series of special cases of Fermat's last theorem, such as the cases for n=3, 4, and so on. The Fermat's last theorem in case of n=3 has been proven elsewhere [12], on the proof, this paper will prove that the Fermat's last theorem is also true in cases of n=3k (k=1,2,3,……).

2. Fermat's Last Theorem
In 1637 years, Pierre de Fermat in the form of a note scribbled in the margin of his copy of ancient Greek text Arithmetica written by Diophantus. Fermat claimed that the Diophantine equation $x^n + y^n = z^n$ has no integer solution for n>2. But Pierre de Fermat has never proved his conjecture by himself.

Later, this theorem was concluded as [6-7]: For all n greater than 2, there do not exist x, y, z such that $x^n + y^n = z^n$, where x, y, z, n are positive integers.

Concerning Fermat's last theorem, many researchers like to discuss the special cases for such as n=3, n=4, 6, et al., in the following, the paper will firstly discuss another special case for x=y which is similar to the case of y=z or x=z.

3. Theorem 1
Theorem 1: In the Fermat's last theorem, besides the restriction of that x, y, z and n are positive integers, it is still required that x≠y≠z.

In consideration of the Diophantine equation

$$x^n + y^n = z^n \qquad (1),$$

using "the disprove method ", if x=y in eq.(1), thus, when n=k+1, from eq.(1) we obtain

$x^{k+1} + y^{k+1} = x \cdot x^k + y \cdot y^k = x \cdot (x^k + y^k) = z^{k+1} = z \cdot z^k$ (2), because $x^k + y^k = z^k$, hence, eq.(2) demonstrates x=z. Therefore, eq.(1) can be written as

$$x^n + y^n = 2x^n = z^n = x^n, \quad (3),$$

it results in a wrong equality

$$2=1 \quad (4).$$

Eq.(4) demonstrates that in Fermat's last theorem, x can't be equal to y or z, and y can't be equal to z as well.

4. The Cases of n=3k (k=1,2,3……)

Considering the Diophantine equation

$$x^{3n} + y^{3n} = z^{3n} \quad (5),$$

with the mathematical induction, when n=1, eq.(5) is written

$$x^3 + y^3 = z^3 \quad (6),$$

this case has been proven in my another paper [12].
Suppose that when n=k, there are no positive integers x, y, and z to fit the equation

$$x^{3k} + y^{3k} = z^{3k} \quad (7),$$

thus, when n=k+1, eq. (5) becomes

$$x^{3(k+1)} + y^{3(k+1)} = z^{3(k+1)} \quad (8).$$

Using "the disprove method", if there are positive integers x, y, z and k to fit eq. (8), eq. (8) is written:

$$x^3 \cdot x^{3k} + y^3 \cdot y^{3k} = z^3 \cdot z^{3k} \quad (9),$$

according to the supposition, if x, y and z are not required to be positive integers, eq. (7) can be regarded as an identity, substituting it into eq. (9), it arrives

$$(x^3 - z^3) \cdot x^{3k} + (y^3 - z^3) \cdot y^{3k} = 0 \quad (10).$$

In eq. (10), because x > 0, y > 0, z > 0, and according to the theorem1, x≠y≠z, therefore, z > x, and z > y as well, so the inequality $(x^3 - z^3) \cdot x^{3k} + (y^3 - z^3) \cdot y^{3k} < 0$ must be true, eq. (10) is wrong! In terms of "the disprove method", when n=k+1, there are also no integers of x, y, and z to make them fit eq. (8). The Fermat's last theorem in all of the cases of n=3k (k=1,2,3……) is proven.

5. Conclusions

The paper discussed Fermat's last theorem for a special case of x=y, but if x=y(or y=z, z=x) it will results in the wrong result 2=1 from the Diophantine equation. Therefore, with respect to Fermat's last theorem, the paper suggests to supplement a restriction of $x \neq y \neq z$ besides the restriction of x, y, z and n are positive integers. Moreover, the Fermat's last theorem in the simplest case of n=3 has been proven elsewhere, on the basis of it, using the mathematical induction this paper proved that the Fermat's last theorem is also true for all of the infinite cases of n=3k(k=1,2,3……). If we want to satisfyingly prove that the Fermat's last theorem is true for all the positive integers, we merely need to prove the Fermat's last theorem in cases of n=3k+1 and n=3k+2 (k=1,2,3,……). In the present proof, equation $x^{3k} + y^{3k} = z^{3k}$ was substituted into eq.(9) because it can be regarded as an identity if x, y and z are not required to be positive integers.

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