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Pulsating Poiseuille micro-gas flow with MHD inclination effect

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Abstract
The Pulsating Poiseuille micro-gas flow with MHD inclination has been analytically investigated in this study. The plates are assumed stationary and the flow is derived by sinusoidal pressure gradient in the laminar flow regime. We solved the governing equations using a transformation technique, which resulted in a system of ordinary differential equations that could be solved analytically. The impact of MHD inclination on the dimensionless velocity is thoroughly examined and graphically displayed. In addition, numerical results of skin friction have been presented. It is found from the study that velocity increases with small values of angle of inclination & magnetism and then decreases with higher values. Increasing values of phase angle and the Knudsen number are found to increase the velocity while increasing the frequency of fluctuating driven force results in a decrease in the velocity of the fluid.

Keywords: MHD inclination, Pulsating Poiseuille flow, basic gaseous micro-flow, fluctuating flows, frequency of fluctuating driven force

Introduction

Applications for fluid flows in micro-scale devices can be found in many areas of human activity, including the industrial and medical fields. Electronic cooling, environmental testing, and inkjet printing are applications for micro-pumps. It has been discovered that micro-ducts have significant functions in high-frequency fluidic control systems, diode lasers, and infrared detectors. Biological cell reactors, blood analyzers, automobile airbag accelerators, keyless entry systems, and thick micro-mirror arrays for high-definition optical displays are just a few examples of equipment that contain micro-devices. Small pumps are employed in the medical profession to manufacture nano-liters of substances, monitor and manage the administration of minute amounts of medication, and construct artificial pancreas. Because of such numerous applications for micro-fluidic systems, hydrodynamics research on micro-fluidic flow is becoming a booming area of research.

Some recent works of literature on micro-gas and fluctuating micro-gas flows include the work of Haddad and Al-Nimr in . The transient Couette flow, the pulsing Poiseuille flow, the Stokes second problem, and natural convention were the four flow instances that were examined as they investigated the effects of variable driving force frequency on fundamental gaseous micro-flows in the slip flow regime. It was determined that the temperature jump and slip in velocity rise as the driving forces' frequency and Knudsen number improve. However, their effect becomes insignificant when the driving forces' frequency and Knudsen number are suitably small. The impact of recurrently fluctuating driving forces on fundamental micro-flows in porous media is examined in . The unsteady transverse magnetic field-induced periodic flow of a viscous fluid with heat generation through a porous planer channel has been considered in . Momentum analysis of complex time-periodic flows is discussed in . The impact of MHD inclination and unstable heat transfer on the laminar, turbulent, and transitional flow of a fundamental gaseous micro-flow across a vertically moving oscillating plate is examined analytically in . The impact of injection and suction on Transient fluctuating Couette micro-gas flow has been analyzed in · It is concluded in the study that both suction and injection retard the velocity of the fluid. Oscillatory magnetohydrodynamic (MHD) Stoke's flow past a flat plate with induced magnetic field effects has been studied in . Idowu and Olabode studied unsteady MHD Poiseuille flow between infinite plates in the presence of heat transfer and inclined magnetic field. They considered negligible pressure gradient and found from their study that the Hartman number decreases the velocity and skin friction, while the Prandlt number decreases the temperature profile.

To the best of the authors' knowledge, the effects of MHD inclination on Poiseuille fluctuating micro-gas flow have not been considered in the literature.

Mathematical Formulation

Consider the Pulsating Poiseuille micro-gas flow, which occurs between two infinitely long parallel plates, with a sinusoidal driving pressure gradient (i.e. $\partial p/\partial x = \sin(\varpi t)$). The distance between the two plates is chosen to be $2L$. As shown in *Fig. 1*, a coordinate system is taken such that the x-axis is parallel and the y-axis is orthogonal to the plates. The flow is assumed to be hydro-dynamically fully developed, and hence $\partial u / \partial x = 0$.

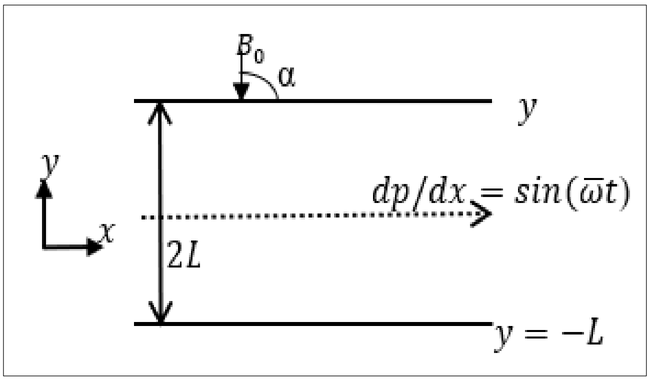

Fig 1: Schematic diagram of the flow

The governing momentum equation and the slip boundary conditions can be presented in dimensional form as:

Governing Equation

$$\frac{\partial u}{\partial t} = \frac{1}{\rho}\frac{\partial p}{\partial x}\sin(\varpi t) + \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_o^2 \sin^2\alpha}{\rho}u \tag{1}$$

Boundary Conditions

$$u(t,-1) = \frac{2-\sigma_v}{\sigma_v}\lambda\left.\frac{\partial u}{\partial y}\right|_{y=-1} \tag{2}$$

$$u(t,1) = -\frac{2-\sigma_v}{\sigma_v}\lambda\left.\frac{\partial u}{\partial y}\right|_{y=1} \tag{3}$$

We now introduce the following dimensionless variables:

$$Y = \frac{y}{L}, \tau = \frac{t}{t_r}, U = \frac{u}{u_r}, u_r = \frac{-L^2(\partial p/\partial x)}{\mu}, \quad M^2 = \frac{\sigma L^2 B_o^2}{\mu}, \omega = \frac{\varpi}{\omega_r}, Kn = \frac{\lambda}{L} \tag{4}$$

Substituting Eq. (4) into Eq. (1-3), we obtained the

Dimensionless Governing Equation

$$\frac{\partial U}{\partial \tau} = \frac{\partial^2 U}{\partial Y^2} - M^2\sin^2\alpha U \tag{5}$$

Dimensionless Boundary Conditions

$$U(\tau,-1) = \frac{2-\sigma_v}{\sigma_v}Kn\left.\frac{\partial U}{\partial Y}\right|_{Y=-1} \tag{6}$$

$$U(\tau,-1) = -\frac{2-\sigma_v}{\sigma_v}Kn\left.\frac{\partial U}{\partial Y}\right|_{Y=1} \tag{7}$$

Where α is the angle between β_0 and $u(t,y)$ for $0 \le \alpha \le \pi/2$, M is the magnetic force, and Kn is the Knudsen number.

Fluid flow regimes have been divided into four groups based on the Knudsen number values: continuum flow $Kn < 10^{-3}$, slip flow regime $10^{-3} \le Kn < 10^{-1}$, transition flow $10^{-1} Kn \le 10^{1}$, and free molecular flow $Kn > 10^{1}$; [9, 10]. The values of Kn were taken between 10^{-3} and 10^{-1} for the slip flow regime, as documented in the literature. The degree of slip in the system is determined by the tangential momentum accommodation coefficient σ_υ, which is dependent on the working fluid and boundary surface conditions. Experimental results have shown that it is between the ranges of $0.2-0.8$. Extreme slip conditions that fall under its lower application limit, while practical and experimental uses fall under its upper limit.

An exact solution to this flow problem is obtained by using the separation of variables used by Haddad *et al* in [1].

i.e. $$U(\tau,Y) = \mathrm{Im}\{\exp(i\omega\tau)V(Y)\} \tag{8}$$

Where Im represents the imaginary part of the complex solution and $i = \sqrt{-1}$. By differentiating Eq. (8) and substituting into Eq. (5), we obtained the following ODE:

$$V^{11}(Y) - \left(M^2\sin^2\alpha + i\omega\right)V(Y) = -1 \tag{9}$$

The Eq. (9) is then solved analytically and substituted into Eq. (8) to obtain:

$$U(\tau,Y) = \mathrm{Im}\left\{\exp(i\omega\tau)\left(c_1\cosh\left(Y\sqrt{K}\right) + c_2\sinh\left(Y\sqrt{K}\right) + \frac{1}{k}\right)\right\} \tag{10}$$

Using boundary conditions (6) and (7) in equation (10), the solution becomes:

$$U(\tau,Y)=\text{Im}\left\{\exp(i\omega\tau)\left(\left(\frac{1}{K}\right)\left(1-\frac{\cosh\left(Y\sqrt{K}\right)}{\cosh(K)+a\sinh\left(\sqrt{K}\right)}\right)\right)\right\} \tag{11}$$

where $K=M^2\sin^2\alpha+i\omega$ and

$$a=\frac{2-\sigma_v}{\sigma_v}Kn\sqrt{K} \tag{12}$$

The skin friction is obtained by differentiating Eq. (11) with respect to Y.

$$\tau_Y=\frac{\partial U}{\partial Y}=\text{Im}\left\{\exp(i\omega\tau)\left[\frac{-\sqrt{K}\sinh\left(Y\sqrt{K}\right)}{K\left[\cosh\left(\sqrt{K}\right)+a\sinh\left(\sqrt{K}\right)\right]}\right]\right\} \tag{13}$$

The skin friction at the lower and upper plates are respectively obtained as:

$$\tau_{-1}=-\left.\frac{\partial U}{\partial Y}\right|_{Y=-1}=\text{Im}\left\{\exp(i\omega\tau)\left[\frac{\sqrt{K}\sinh\left(\sqrt{K}\right)}{K\left[\cosh\left(\sqrt{K}\right)+a\sinh\left(\sqrt{K}\right)\right]}\right]\right\} \tag{14}$$

$$\tau_{1}=\left.\frac{\partial U}{\partial Y}\right|_{Y=1}=\text{Im}\left\{\exp(i\omega\tau)\left[\frac{-\sqrt{K}\sinh\left(\sqrt{K}\right)}{K\left[\cosh\left(\sqrt{K}\right)+a\sinh\left(\sqrt{K}\right)\right]}\right]\right\} \tag{15}$$

3. Results and Discussion

To facilitate a physical interpretation, graphical discussions have been conducted using arbitrary values of α (angle of inclination), M (magnetic force), ω (frequency of the fluctuating driven force), $\omega\tau$ (phase angle), and Kn (Knudsen number) in Fig. (2-6). The values $\omega=1$, $Kn=0.1$, $\omega\tau=\pi/4$, $\alpha=\pi/6$, $M=2$, and $\sigma_{\upsilon}=0.7$ are used throughout the discussion whenever values are not specified.

The Effect of the angle of inclination of the magnetic field is seen in Fig. 2. It is observed that the velocity increases with an increase in α for values of α from 0 to $\pi/8$ and then decreases with an increase in α. Fig. 3 illustrating the effect of Magnetism on the flow. An increase in magnetism results in to increase in velocity for the values of M ranging from 0 – 1.5, and then decreases with increasing M. Fig. 4 depicts the effect of the increase in the Knudsen number on the flow. It is found that the velocity of the fluid increases with an increase in the values of the Knudsen number.

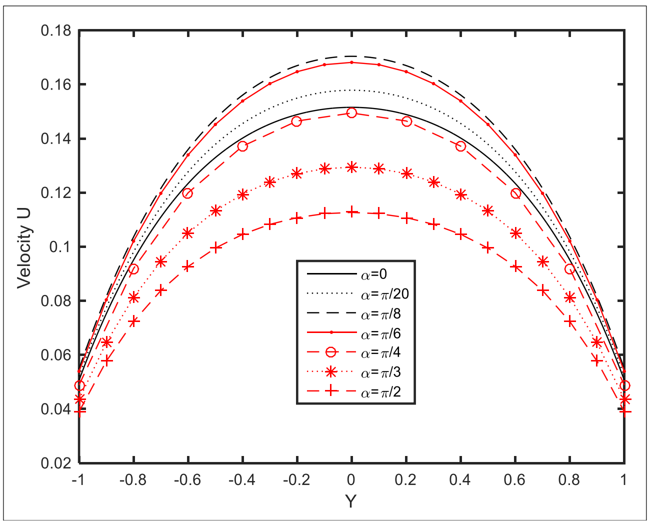

Fig 2: Velocity profile when varying α.

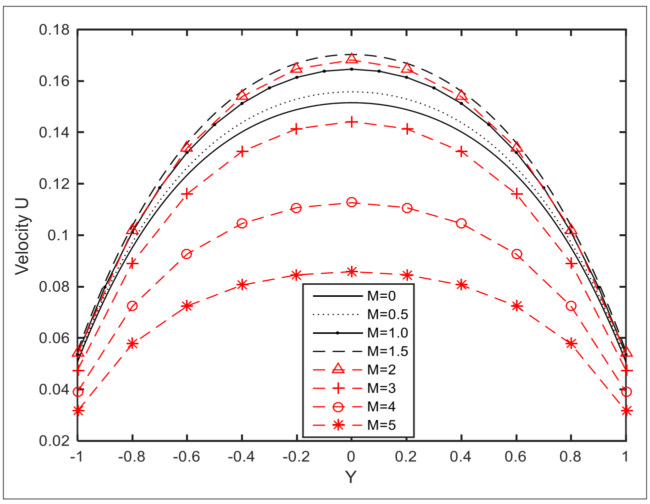

Fig 3: Velocity profile when varying M

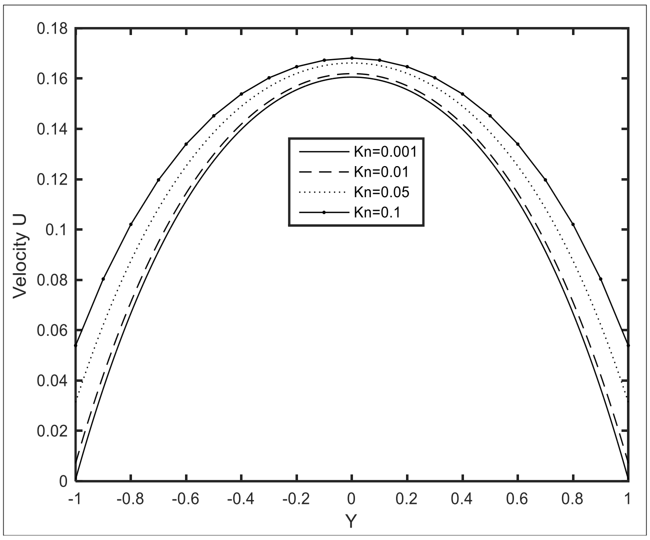

Fig 4: Velocity profile when varying Kn

The velocity profile resulting from varying driving force frequency values, ω, is depicted in Fig. 5. It is discovered that the local velocity of the moving fluid decreases as the driving force's frequency increases.

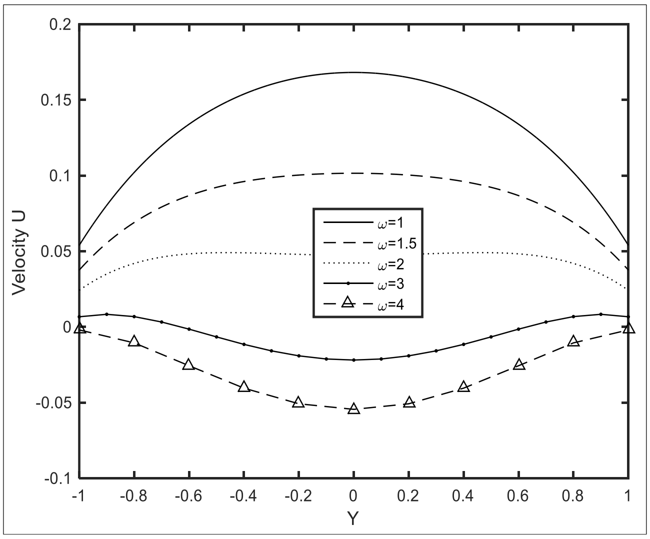

Fig 5: Velocity profile when varying ω.

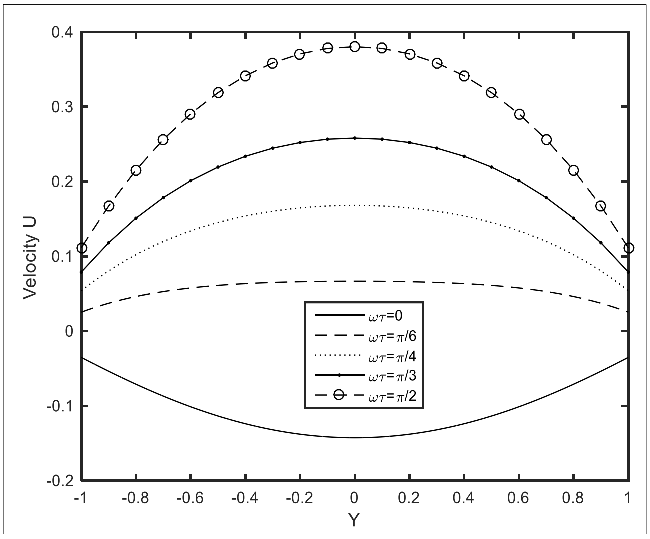

Fig 6: Velocity profile when varying $\omega\tau$.

Table 1: Numerical values for skin friction

α	τ_{-1}	τ_1	M	τ_{-1}	τ_1
0	0.2725181	0.2725181	0	0.2725181	0.2725181
π/18	0.2818028	0.2818028	1	0.2884256	0.2884256
π/9	0.2938995	0.2938995	2	0.2903762	0.2903762
π/6	0.2903762	0.2903762	3	0.2554722	0.2554722
π/3	0.2342300	0.2342300	4	0.2098972	0.2098972
π/2	0.2099614	0.2099614	5	0.1700084	0.1700084

Fig. 6 shows velocity distribution when varying . It is found that increasing the values of increases the velocity of the moving fluid. Table 1 displays the numerical values of skin friction at the plates. The skin friction is found to decrease with an increase in values of α and

Conclusions

It is found from the study that velocity increases with lower values of α and M and decreases with higher values of α and M. Increasing values of $\omega\tau$ and Kn is found to increase the velocity while increasing the values of ω lead to a decrease in velocity of the moving fluid. The skin friction is found to decrease with an increase in values of α and M.

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Nomenclature

Kn Knudsen number $(=\lambda/L)$
L Reference length
P Pressure
t Time
t_r Reference time $(=L^2/\upsilon)$
U Dimensionless axial velocity $(=u/u_r)$
u Axial velocity (in x-direction)
u_r Reference velocity $(=-L^2(\partial p/\partial x)/\mu$
V Complex solution function for velocity
X Dimensionless axial coordinate $(=x/L)$
x Axial coordinate
Y Dimensionless transverse coordinate $(=y/L)$
y Transverse coordinate

Greek symbols

λ Mean-free-path length
ρ Density
μ Dynamic viscosity
υ Kinematic viscosity
σ_υ Tangetial momentum accommodation coefficient $(=0.7)$
ϖ Frequency
ω Dimensionless frequency $(=\varpi/\omega_r)$
ω_r Reference frequency $(=\upsilon/L^2)$
τ Dimensionless time $(=t/t_r)$

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