Journal of Mathematical Problems, Equations and Statistics

E-ISSN: 2709-9407
P-ISSN: 2709-9393
JMPES 2024; 5(2): 12-18

www.mathematicaljournal.com
Received: 14-06-2024
Accepted: 22-07-2024

R Deepa
Assistant Professor, Department of Mathematics, Excel Engineering College, Komarapalayam, Tamil Nadu, India

Dr. P Sundararajan
Associate Professor, Department of Mathematics, Arignar Anna Govt. Arts College, Namakkal. Tamil Nadu, India

Corresponding Author:
R Deepa
Assistant Professor, Department of Mathematics, Excel Engineering College, Komarapalayam, Tamil Nadu, India

The adjoint of invers of K regular super fuzzy matrix

R Deepa and Dr. P Sundararajan

DOI: https://dx.doi.org/10.22271/math.2024.v5.i2a.143

Abstract
In this paper the adjoint theory of inverse of k-regualr super fuzzy matrix (IKRSFM) will be studied. We state a formula for the adjoint matrix of a square fuzzy matrix, and this formula establishes the relationship between the adjoints of two IKRS fuzzy matrices. Also, we shall find the relationship between the adjoints of two IKRS fuzzy matrices corresponding the relationship between the IKRS fuzzy matrices. In this paper we define the symmetric, reflexive and transitive fuzzy matrices.

Keywords: Inverse of K regular, fuzzy matrix, adjoint, symmetric, reflexive, and transitive

Introduction
Super fuzzy matrix is an interesting topic of fuzzy mathematics. Fuzzy matrices have become an essential part of contemporary mathematics. There are now various innovative models for fuzzy matrices.

Definition 1: Fuzzy Matrix
Let us consider a matrix
A=[*aij*] *m x n*
where $a_{ij} \in [0,1]$, $1 \le i \le m$ and $1 \le j \le n$. Then A is fuzzy matrix.

Definition 2: Fuzzy super Matrix
A super matrix is one whose elements are also matrices, and whose members can be other matrices or scalars.
In general, we will work with super matrices whose members include any scalar. A super matrix refers to a global rectangle or square array of matrices.

Definition 3: Square IKRSFM
The adjoint matrix $B = [b_{ij}]$ of a IKRSFM $A = [a_{ij}]$ of order n, is a square matrix of IKRSFM of same order n, denoted by adjA, is defined as, $b_{ij} = |A_{ij}|$ where $|A_{ij}|$ is the determinant of the square fuzzy matrix of order $(n-1)$ obtained from a square IKRSFM A of order n by deleting row j and column i and $B = [b_{ij}] = adj$A.

Definition 4: Fuzzy Matrix
A fuzzy matrix is a matrix with entries ranging from 0 to 1. We frequently only have fuzzy matrices in the following configurations: fuzzy square, fuzzy row, fuzzy column, and fuzzy rectangle.
The fuzzy matrix (A) may be represented as follows:

$$A = \begin{bmatrix} A_{11} & A_{12} & . & . & . & A_{1n} \\ A_{21} & A_{22} & . & . & . & A_{2n} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ A_{m1} & A_{m2} & . & . & . & A_{mn} \end{bmatrix}$$

Definition 5: Fuzzy Super row matrix

Let $A_s = [A_1/A_2/ \ldots./A_n]$ (t>1) where, each A_i is a fuzzy row vector, i=1,2,…,t. we call A_S as the fuzzy super row matrix.

$A_S = [A_1,A_2, \ldots.,A_n]$

2. The Adjoint of Inverse of K-regular super fuzzy matrix

Theorem 1

For $n \times n$ fuzzy IKRSFM A and B,

5.1.1 $A \leq B \Rightarrow adj\ A \leq adj\ B$

5.1.2 $adj\ A + adj\ B \leq adj\ (A+B)$ Proof: Let $A = [\ a\ ij\ ]$ and $B = \lfloor bij \rfloor$ where $i, j \in \{1,2,\ldots,n\}$.

(i) since $A \leq B$

$\Rightarrow a_{ij} \leq b_{ij}\ \forall\ i, j \in \{1.2,\ldots,n\}$

$\Rightarrow a_{t\sigma(t)} \leq b_{t\sigma(t)}$ for every $t \neq j$, $\sigma(t) \neq i \Rightarrow adj A \leq adj B$

(ii) since $A, B \leq A + B \quad [\because A + B = \max\{A, B\}]$

$\Rightarrow adj A, adj B \leq adj(A+B)\ [\because A \leq B \Rightarrow adj A \leq adj B]$

$\Rightarrow adj A + adj B \leq adj(A+B)$

Theorem 2 The adjoint of the transpose of a matrix is the transpose of the adjoint of the matrix. i.e., for a square IKRSFM A of order n, $adj\ A' = (adj\ A)'$.

Proof: Let $A = \left[a_{ij}\right]$ be a $n \times n$ fuzzy matrix.

Let $B = \left[b_{ij}\right] = adj\ A$ and $C = (adj\ A)'$

$$b_{ij} = \sum_{\sigma \in S_{n_j n_i}} \prod_{t \in n_j} a_{t\sigma(t)}$$

$$c_{ij} = \sum_{\sigma \in S_{n_j n_i}} \prod_{\sigma(t) \in n_i} a_{t\sigma(t)}$$

Which is element b_{ji}

Hence $adj\ A' = (adj\ A)'$, which proves the assertion

Definition 6 Let A be a square fuzzy matrix (IKRSFM) of order n, then following hold:

i. A is said to be **reflexive** fuzzy matrix iff $A \geq I_n$ i.e., iff all diagonal elements in IKRSFM A are unity i.e., iff $a_{ii} = 1\ \forall\ i$.

ii. A is said to be **symmetric** iff $A' = A$ i.e., iff the square IKRSFM A remains unaltered by interchanging its rows and columns i.e., iff $a_{ij} = a_{ji}\ \forall i,j \in \{1,2,\ldots,n\}$.

iii. A is said to be **transitive** iff $A^2 \leq A$ i.e., iff the square IKRSFM A multiplied by itself gives the elements less than or equal to the corresponding elements of the square IKRSFM A. i.e., iff $a_{ik}\ a_{kj} \leq a_{ij}$ for every $k = 1,2,\ldots,n$.

A square IKRSFM is **similarity (equivalence relation)** iff it is reflexive, symmetric, and transitive.

Example 1 Let A be a square IKRSFM of order 3.

i. Consider a square IKRSFM of order 3X3,

$$A = \begin{bmatrix} 1 & 0.2 & 0 \\ 0.3 & 1 & 0.4 \\ 0.9 & 0.3 & 1 \end{bmatrix}$$

Since all the diagonal elements in square fuzzy matrix are unity, then A is a reflexive fuzzy matrix.

ii. Consider a square IKRSFM,

$$A = \begin{bmatrix} 0.3 & 0.4 & 0.5 \\ 0.4 & 0.6 & 0.1 \\ 0.5 & 0.1 & 1 \end{bmatrix}$$

Then,

$$A' = \begin{bmatrix} 0.3 & 0.4 & 0.5 \\ 0.4 & 0.6 & 0.1 \\ 0.5 & 0.1 & 1 \end{bmatrix} = A$$

Thus, A is a symmetric fuzzy matrix.

iii. Consider a square fuzzy matrix,

$$A = \begin{bmatrix} 0.6 & 0.7 & 0.6 \\ 0.5 & 0.6 & 0.5 \\ 0.6 & 0.7 & 0.6 \end{bmatrix}$$

Then,

$$A^2 = \begin{bmatrix} 0.6 & 0.7 & 0.6 \\ 0.5 & 0.6 & 0.5 \\ 0.6 & 0.7 & 0.6 \end{bmatrix} \begin{bmatrix} 0.6 & 0.7 & 0.6 \\ 0.5 & 0.6 & 0.5 \\ 0.6 & 0.7 & 0.6 \end{bmatrix}$$

$$= \begin{bmatrix} 0.6 & 0.6 & 0.6 \\ 0.5 & 0.5 & 0.5 \\ 0.6 & 0.6 & 0.6 \end{bmatrix} \leq A$$

as the element in A^2 are $\leq$ the corresponding elements in A. Thus, A is a transitive IKRSFM.
Next, consider the square IKRSFM.

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

All diagonal elements in A equal to 1 implies that A is reflxive. Further, observe that $A' = A$ so A is symmetric. Also, A^2=A leads to the conclusion that A is transitive. Hence A is similar.
Let us see how the properties of a square IKRSFM are carried over to its adjoint.

Theorem 3 Let A be a square IKRSFM of order *n*, Then we have the following properties:

i. If A is reflexive, then *adj* A is reflexive.
ii. If A is symmetric, then *adj* A is symmetric.

If A is transitive, then *adj* A is transitive.

Proof: (i) Let $B = [b_{ij}] = adj\ A$

Then,

$$b_{ij} = \sum_{\sigma \in S_{n_j n_i}} \prod_{t \in n_j} a_{t\sigma(t)}$$

$$b_{ii} = \sum_{\sigma \in S_{n_i}} \prod_{t \in n_j} a_{t\sigma(t)}$$

Taking only the identity permutation $\sigma(t) = t$; we get

$b_{ii} \geq a_{11}a_{22} \ldots a_{(i-1)(i-1)}a_{(i+1)(i+1)} \ldots a_{nn}$ i.e; $b_{ii} \geq 1 \; \forall \; i$ $(\because a_{ii} = 1 \; \forall \; i)$ and so $b_{ii} = 1 \; \forall \; i$
Hence *adj* A is reflexive.

Since A is symmetric, then $a_{ij} = a_{ji} \; \forall \; i, j$.

Let,

$$B = \left[b_{ij} \right] = adj A$$

$$b_{ij} = \sum_{\sigma \in S_{n_j n_i}} \prod_{t \in n_j} a_{t\sigma(t)} = \sum_{\sigma \in S_{n_i n_j}} \prod_{t \in n_i} a_{t\sigma(t)} \quad [\because a_{ij} = a_{ji} \forall i, j]$$

$= b_{ji}$. Hence *adj* A is symmetric.

$$d_{hk} = \begin{cases} a_{hk} & \text{if } h < i, k < j, \\ a_{(h+1)k} & \text{if } h \geq i, k < j, \\ a_{h(k+1)} & \text{if } h < i, k \geq j, \\ a_{(h+1)(k+1)} & \text{if } h \geq i, k \geq j. \end{cases}$$

(iii). Since A is transitive, then $a_{ik} \, a_{kj} \leq a_{ij} \forall \; i,j$ .et $B = \lceil bij \rceil = adjA$.
Let $D = A_{ij}$, we can determine the elements of D in terms of the elements of A as follows:
where A_{ij} denotes the $(n-1)\times(n-1)$ fuzzy matrix obtained from A by deleting *ith* row and *jth* column. Now we show that $A_{st} \, A_{tu} \leq A_{su}$ for every $t \in \{1,2,\ldots,n\}$.
Let $R = A_{st}$, $C = A_{tu}$, $F = A_{su}$ and $W = A_{st}A_{tu}$.

Now $w_{ij} = \sum_{k=1}^{n-1} r_{ik} c_{kj}$

$$= \sum_{k=1}^{n-1} a_{ik}a_{kj} \leq a_{ij} = f_{ij} \text{ if } i < s, k < t, j < u,$$

$$= \sum_{k=1}^{n-1} a_{ik}a_{k(j+1)} \leq a_{i(j+1)} = f_{ij} \text{ if } i < s, k < t, j \geq u,$$

$$= \sum_{k=1}^{n-1} a_{i(k+1)} a_{(k+1)\,j} \leq a_{ij} = f_{ij} \text{ if } i < s, k \geq t, j < u,$$

$$= \sum_{k=1}^{n-1} a_{(i+1)\,(k+1)} a_{(k+1)j} \leq a_{(i+1)j} = f_{ij} \text{ if } i \geq s, k \geq t, j < u,$$

$$= \sum_{k=1}^{n-1} a_{(i+1)\,(k+1)}\, a_{(k+1)\,(j+1)} \leq a_{(i+1)\,(j+1)} = f_{ij} \text{ if } i \geq s,\ k \geq t, j \geq u,$$

$$= \sum_{k=1}^{n-1} a_{(i+1)\,k}\, a_{k\,(j+1)} \leq a_{(i+1)\,(j+1)} = f_{ij} \text{ if } i \geq s,\ k < t, j \geq u.$$

Thus $w_{ij} \leq f_{ij}$ in every case and therefore $A_{st}\, A_{tu} \leq A_{su}$ for every $t \in \{1,2,..,n\}$
Since, we know that $|AB| \geq |A||B|$ we have,

$$A_{st} A_{tu} \leq A_{st} A_{tu} \leq A_{su}.$$

This means $b_{ts} b_{ut} \leq b_{us}$ i.e., $b_{ut} b_{ts} \leq b_{us}$ for every $t \in \{1,2,..,n\}$

Hence, B = *adj(*A)is transitive.

Example 2 Consider a square IKRSFM of A

Let

$$A = \begin{bmatrix} 1 & 0 & 0.3 \\ 0.1 & 1 & 0 \\ 0.4 & 0.5 & 1 \end{bmatrix}$$ be a reflexive fuzzy matrix, then

$$adj A = \begin{bmatrix} 1 & 0.3 & 0.3 \\ 0.1 & 1 & 0.1 \\ 0.4 & 0.5 & 1 \end{bmatrix}$$

Since all the diagonal elements in *adj(*A) are unity, then *adj(*A) is a reflexive IKRSFM.
(ii). Let, the given matrix A be a symmetric IKRSFM, then

$$A = \begin{bmatrix} 0.2 & 0 & 0.6 \\ 0 & 1 & 0.1 \\ 0.6 & 0.1 & 0.9 \end{bmatrix}$$

$$adjA = \begin{bmatrix} 0.9 & 0.1 & 0.6 \\ 0.1 & 0.6 & 0.1 \\ 0.6 & 0.1 & 0.2 \end{bmatrix}$$

Since $(adjA)^1 = adjA$, then *adjA* is a symmetric IKRSFM.
(iii). Let, the given matrix A be a transitive IKRSFM, then

Now

$$A = \begin{bmatrix} 0.6 & 0.7 & 0.6 \\ 0.5 & 0.6 & 0.5 \\ 0.6 & 0.6 & 0.6 \end{bmatrix} \quad adjA = \begin{bmatrix} 0.6 & 0.6 & 0.6 \\ 0.5 & 0.6 & 0.5 \\ 0.6 & 0.7 & 0.6 \end{bmatrix}$$

$$(adjA)^2 = \begin{bmatrix} 0.6 & 0.6 & 0.6 \\ 0.5 & 0.6 & 0.5 \\ 0.6 & 0.7 & 0.6 \end{bmatrix} \begin{bmatrix} 0.6 & 0.6 & 0.6 \\ 0.5 & 0.6 & 0.5 \\ 0.6 & 0.7 & 0.6 \end{bmatrix}$$

$$= \begin{bmatrix} 0.6 & 0.6 & 0.6 \\ 0.5 & 0.6 & 0.5 \\ 0.6 & 0.6 & 0.6 \end{bmatrix}$$

$\leq$ *adj(*A), then *adj(*A) is a transitive IKRSFM.
Next, consider the similarity IKRSFM

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Then A is reflexive, symmetric, and transitive. Now

$$adjA = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Then *adj(*A) is reflexive, symmetric, and transitive. Hence *adj(*A) is like IKRSFM.

Conclusion

The adjoint theory of inverse of k-regualr super fuzzy matrix (AIKRSFM) will be studied. Also state an adjoint formula for the square fuzzy matrix, and this formula established the relationship between the adjoints of two IKRS fuzzy matrices.

References

1. Abbasbandy S, Ezzatti R, Jafarian A. LU decomposition method for solving fuzzy system of linear equations. Applied Mathematics and Computation. 2006;172:633-643.
2. Adlassing KP. Fuzzy set theory in medical diagnosis. IEEE Transactions on Systems, Man, and Cybernetics. 1982;16:260-265.
3. Adlassing KP. A survey on medical diagnosis and fuzzy subsets. In: Gupta MM, Sanchez E, editors. Approximate Reasoning in Decision Analysis. New York: North Holland; c1982. p. 203-217.
4. Bandler W, Kohout L. Fuzzy power sets and fuzzy implication operators. Fuzzy Sets and Systems. 1980;4:13-30.
5. Bellman R, Zadeh L, Giertz M. On the analytic formalism of theory of fuzzy sets. Information Sciences. 1973;5:149-156.
6. Bell DA. Information Theory and Its Engineering Applications. New York: Pitman; c1953.
7. Dubois D, Prade H. Fuzzy Sets and Systems: Theory and Application. New York: Academic Press; c1986.

8. Ben-Israel A, Greville TN. Generalized Inverses: Theory and Applications. New York: John Wiley & Sons; c1974.
9. Blin JM. Fuzzy relations in group decision theory. Journal of Cybernetics. 1974;4:17-24.
10. Buckles BP, Petry FE. A fuzzy representation of data for relational databases. Fuzzy Sets and Systems. 1982;7:213-226.
11. Ravi J. Fuzzy graph and their applications: A review. International Journal for Science and Advance Research in Technology. 2022;8(1):107-111.