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Proof of Beal's Conjecture

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Abstract
The following article presents a straight-forward proof to Beal's Conjecture. It uses mathematics commonly taught in the fourth and fifth grades of elementary/primary school: Fractions and factors. It begins with proving that there exists a factor common to all three terms comprising Beal's Conjecture then, using properties of fractions, proves that the common factor must be greater than 1, proving the three terms are not coprime and thus proving Beal's Conjecture.

Keywords: Beal's Conjecture, Fermat's Last Theorem, Greatest Common Factor (GCF), Least common Multiple (LCM), Least Common Denominator (LCD), Euclid's Algorithm, Mihailescu's Theorem, Catalan's Conjecture

Introduction
In December 1997, The American Mathematical Society (AMS) published a challenge to the mathematics community [1]. For that challenge, D. Andrew Beal, a banker and mathematics enthusiast, conjectured a generalization to Fermat's Last Theorem that for any solution of $A^x + B^y = C^z$, where A, B, C, x, y, and z are natural numbers and x, y, and z > 2, then A, B, and C must have a common factor. His challenge, Beal's Conjecture, was published in an article authored by R. Daniel Mauldin, titled "A Generalization of Fermat's Last Theorem: The Beal Conjecture and Prize Problem". *Notices of the AMS*, Vol. 44, No. 11 (p. 1436-37). This article proposes a proof of Beal's Conjecture.

Materials and Methods
This article is strictly a mathematical proof involving no data sets nor methodology.

Results
See Proof (below).

Discussion/Proof
Since the call of the conjecture is for a common factor, the obvious starting point is the Greatest Common Factor (GCF) and Euclid's Algorithm. Euclid's Algorithm traces its originally-published roots to Euclid's Elements, Book VII, Propositions 1 and 2 [2].
To prove Beal's Conjecture, we will show that A^x, B^y, and C^z have a greatest common factor and then we will show that the greatest common factor is greater than.

1. There exists a greatest common factor
To prove the existence of a greatest factor common to all three numbers, we start by comparing two of the terms at a time.

Case 1: Find the greatest common factor for A^x and C^z.
To find the common factor between A^x and C^z, we attempt to find their greatest common factor.

GCF (A^x, C^z).

Applying the original Euclidean algorithm, which provides that we can substitute the greater number with the difference between the greater and the lesser number without changing the GCF, we get:

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GCF (A^x, C^z) = GCF (A^x, C^z - A^x)

By substituting $C^z - A^x = B^y$, our equation becomes

GCF (A^x, C^z) = GCF (A^x, B^y).

Case 2: Find the greatest common factor for B^y and C^z.
To find the common factor between B^y and C^z, we attempt to find their greatest common factor.

GCF (B^y, C^z).

Again, applying the Euclidean algorithm, we get:

GCF (B^y, C^z) = GCF (B^y, C^z - B^y)

And by substituting $C^z - B^y = A^x$, our equation becomes

GCF (B^y, C^z) = GCF (B^y, A^x).

Because the solutions to both cases are equal, we combine the equations to get

GCF (A^x, C^z) = GCF (A^x, B^y) = GCF (B^y, C^z)

Because the greatest common factor between A^x and C^z is equal to (1) the greatest common factor between B^y and C^z and (2) the greatest common factor between A^x and B^y, we have proved that there exists a greatest common factor for A^x, B^y, and C^z. However, at this point, we have not proven that the greatest common factor is greater than 1.

2. The greatest common factor is greater than 1
Now that we have proven that there exists a greatest common factor between for A^x, B^y, and C^z, we need to prove that the common factor is greater than 1 (because if the GCF = 1, then A^x, B^y, and C^z would be coprime).
We begin by assuming that A^x, B^y, and C^z are coprime. We know that

GCF (a, b) * LCM (a, b) = a*b for any a, b >0, and by substituting our terms we have

GCF (A^x, B^y) * LCM (A^x, B^y) = $A^x * B^y$.

Since we have assumed that A^x, B^y, and C^z are coprime, we can substitute GCF (A^x, B^y) = 1 to get

1 * LCM (A^x, B^y) = $A^x * B^y$.

We know that if A^x and B^y are coprime, then LCM (A^x, B^y) = $A^x * B^y$. Looking back at our original equation
$A^x + B^y = C^z$, we see that by diving both sides by C^z, we get
$(A^x + B^y) / C^z = C^z / C^z = 1$, and separating them becomes
$A^x/C^z + B^y/C^z = 1$. The fractions created for A^x and B^y have a common denominator, C^z and we know that if A^x and B^y are coprime, then $C^z >=$ LCM (A^x, B^y), we also know that LCD (A^x, B^y) = ($A^x * B^y$).

It is at this point where we have a mathematical contradiction, that is, there exists a least common denominator which is a lesser value than the LCM (A^x, B^y) = $A^x * B^y$ created by our assumption that GFC (A^x, B^y)=1. Here's how we arrive at that conclusion.

If we assume that GFC (A^x, B^y)=1, then whatever fraction we derive from our given equation must have a LCD (A^x, B^y) = $A^x * B^y$. From Beal's conjecture's original equation
$A^x + B^y = C^z$, where A, B, C, x, y, and z are natural numbers and x, y, and z > 2, we can divide by C^z and get the following: $(A^x + B^y) / C^z = 1$. By substituting $A^x + B^y = C^z$, we get

$(A^x + B^y) / (A^x + B^y) = 1$.

Now $(A^x + B^y) >=$ LCD (A^x, B^y). So if $(A^x + B^y) < (A^x * B^y)$, then we have a LCD whose value is less than the predicted LCD/LCM with the assumption that GCF = 1. We examine to see if $(A^x + B^y)$ is actually less than $(A^x * B^y)$.
Generally, for natural numbers, this is not always true. For example, if A^x and B^y were equal to 1, then we would have (1+1) < (1*1), which is not true. To eliminate this possibility (of A^x and B^y being equal to 1) we look to Mihailescu's Theorem [3] (the proof of Catalan's Conjecture) [4] which proved that there are only two consecutive power numbers, 8 = (2^3) and 9 = (3^2). Because there are no other consecutive power numbers, and in particular under the constraints given by Beal where all powers are > 2, then, neither A^x (as the difference of two power numbers, i.e., $C^z - B^y$) nor B^y (also as the difference of two power numbers, i.e., $C^z - A^x$) can equal 1 under any circumstance.
Now we examine the next lowest possibility of (2+2) < (2*2), which is not true. To eliminate this possibility (of A^x and B^y being equal to 2) we look no further than Beal's constraints which requires that all powers are greater than 2. Rewriting this possibility, we get $(2^1+2^1) < (2^1*2^1)$, which is not true, however, looking at the minimum, the power of 3, we get $(2^3+2^3) < (2^3*2^3)$=16/64, which is true. We see that for all other numbers whose base is greater than 1 and whose power is greater than 2 that $(A^x + B^y) < (A^x * B^y)$ will always be true under Beal's constraints. Since there is always a common denominator, $(A^x + B^y)$, which is always less than $A^x * B^y$, then GFC (A^x, B^y) must always be greater than 1.

Conclusion
Having proven that A^x, B^y, and C^z have a common factor and that the common factor greater than 1, we have proven Beal's Conjecture. QED

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