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Convergence of secant method

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Abstract

Fourth roots of the natural numbers from 1 to 30 have been found by secant method and these values have been compared with the actual values. The minimum error 0.000000000758 and minimum percentage error 0.000000057558 have been found in the determination of fourth root of 3. The average value in the error is 0.000000091494. The maximum error 0.000000753232 and maximum percentage error 0.000063338985 have been found in the determination of fourth roots of 2.

Keywords: Secant method, convergence, iteration, algorithm, terminating condition, numerical accuracy, roots of equation

Introduction

Fourth roots of the natural numbers from 1 to 30 have been calculated earlier by "Convergence of the Bisection Method" [1], "Convergence of the Newton-Raphson Method" [2] and "Convergence of the Method of False Position,"[3]. In this research paper, we will explore the convergence of the Secant Method. The Secant Method replaces the need for calculating the derivative as in Newton's Method and instead uses a finite difference approximation based on the two most recent iterations. Instead of finding a tangent line to the function f (x) at a single point, the Secant Method uses a line that connects two points. The next iteration is determined by the point where this secant line intersects the x-axis. To start the iteration, two initial values, x_0 and x_1, are required [4-8].

$$s_n = \frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}},$$

$$x_{n+1} = x_n - \frac{f(x_n)}{s_n}.$$

This formulation clarifies the substitution of Newton's derivative $f'(x_n)$ with the slope of the secant, s_n. The Secant Method converges faster than a method with linear convergence but slower than a method with quadratic convergence. It's important to note that an algorithm that converges quickly but takes a few seconds per iteration may ultimately consume more time than an algorithm that converges more slowly but only takes a few milliseconds per iteration [9-15].

The Secant Method only necessitates one function evaluation per iteration, as the value of $f(x_n-1)$ can be retained from the previous iteration. In contrast, Newton's Method necessitates one function evaluation and one evaluation of the derivative per iteration. The computational cost of evaluating the derivative can vary significantly. In some instances, it may be relatively straightforward, while in others, it could be considerably more challenging or even impossible to compute [16-21].

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Material and Method

If two starting values are x_0 and x_1 then the subsequent iterates are given by [1-8].

$$s_n = \frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}},$$

$$x_{n+1} = x_n - \frac{f(x_n)}{s_n}.$$

Where n=0, 1, 2,

Computer program for the calculation of fourth roots of natural numbers from 1 to 30 with the help of secant method have been developed in C^{++} language and is given below-

```
#include<conio.h>
#include<stdio.h>
#include<math.h>
//Secant method
void main(void)
{FILE *fpt;
int n;
float delta,ar[1000],aa;
double f(float x);
clrscr();
//Filename to store the fourth roots in each iteration using secant method
fpt=fopen("lavsec1.txt", "w");
//ar[0] and ar[1] are the initial guesses
ar[0]=0.9; ar[1]=3.0; n=1;
//Value of function f(x)
fprintf(fpt,"f(x)=x^4-1\n");
//delta is tolerence
delta=0.00001;
fprintf(fpt," n ar[n] f(ar[n])\n");
printf(" n ar[n] f(ar[n])\n");
do
{ ar[n+1]=ar[n]-f(ar[n])*(ar[n]-ar[n-1])/(f(ar[n])-f(ar[n-1]));
aa=fabs(f(ar[n+1]));
n++;
fprintf(fpt,"%3d%15.12f%15.12f\n",n-1,ar[n],f(ar[n]));
printf("%3d%15.12f%15.12f\n",n-1,ar[n],f(ar[n]));
} while (aa > delta);
printf("Root=%20.12f\n",ar[n]);
printf("Value of function=%20.12f\n", f(ar[n]));
printf("No. of iterations=%3d\n",n-1);
getch();
fclose(fpt);
}/Function definition
double f(float x)
{double r;
r=x*x*x*x-1;
return(r);
}
```

Initial guesses are 0.9 and 3.0 in the calculation of fourth roots of natural numbers from 1 to 30 with the help of secant method. Terminating condition has been taken as

$f(x_n) < 0.00001$

For the calculation of fourth roots by secant method, the following functions have been taken.

$f(x) = x^4 - n$ where n = 1,2, 3, 30

Numerical accuracy of secant method has been measured by percentage error and defined as follows

Percentage error = error in the value of fourth root * 100/actual value of fourth root

Numerical accuracy of secant method is inversely proportional to percentage error.

Let us define numerical rate of convergence of secant method as follows

Numerical rate of convergence of secant method = 1 / (1000αβγ)

Where α = Total number of iterations
β = 30 * difference between two initial guesses of root
γ = Stopping tolerance

Result and Discussion

Calculation of fourth root of 1 by secant method with initial guesses 0.9 and 3.0

Secant method has been applied to calculate the root of equation.

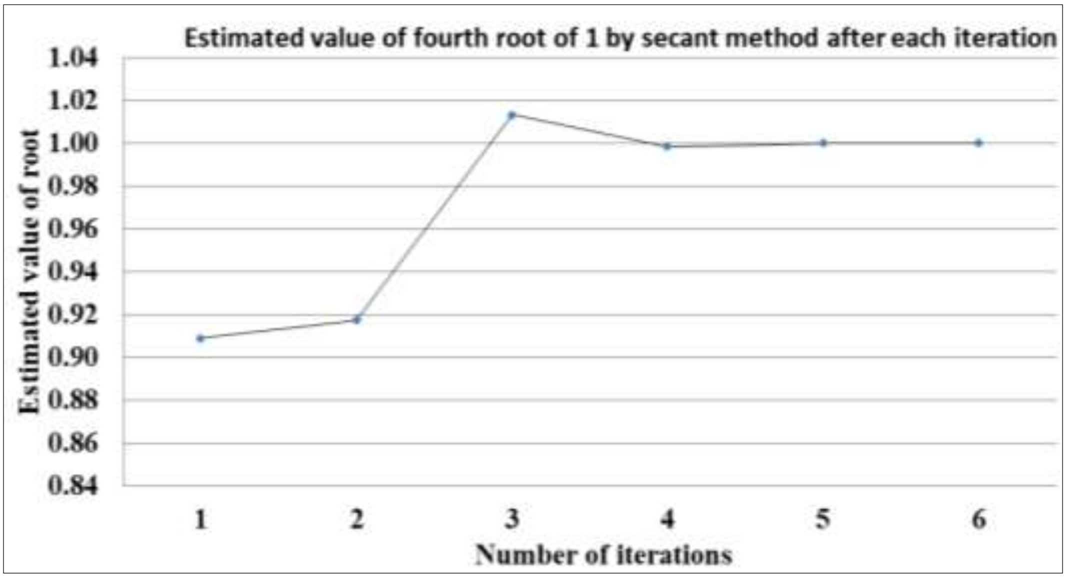
$F(x) = x^4 - 1 = 0$

With two initial guesses, $x_0 = 0.9$ and $x_1=3.0$ by using C++ computer program. No. of iterations, root guessed by secant method in each iteration (x_n) and value of function at $x = x_n$ are shown in Table-1. The value of root and the value of function after each iteration in the calculation of fourth root of equation $f(x) = x^4 - 1 = 0$ by secant method is shown in Graph-1 and Graph-2.

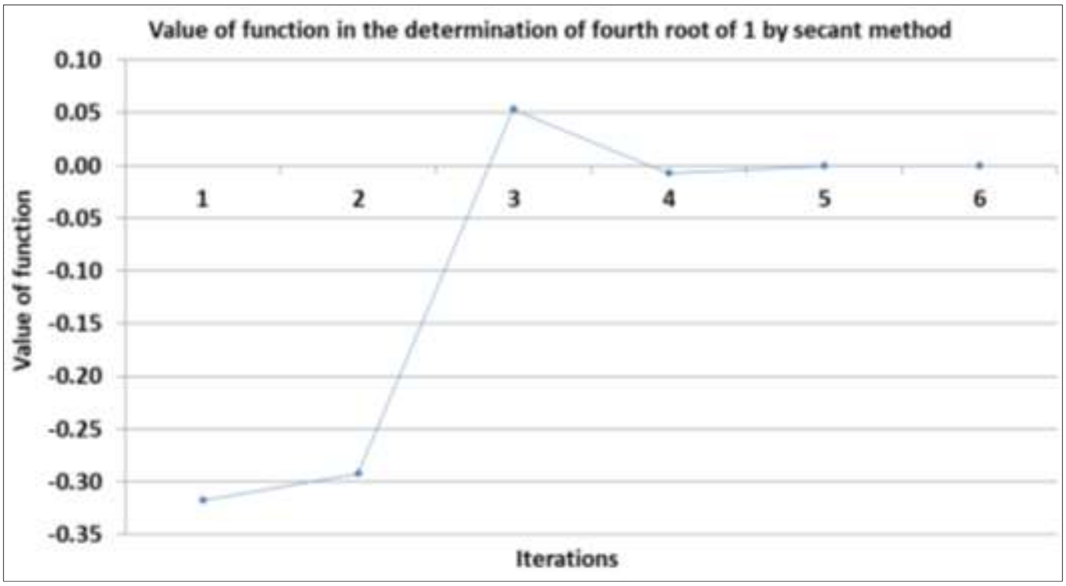
Table 1: No. of iterations, root of equation $f(x) = x^4 - 1 = 0$ guessed by secant method in each iteration (x_n) and value of function at $x = x_n$

No. of iterations	Root guessed by secant method (x_n)	Value of function at $x = x_n$
1.	0.908988714218	-0.317293614933
2.	0.917249262333	-0.292136556180
3.	1.013174891472	0.053750210081
4.	0.998268187046	-0.006909277526
5.	0.999966084957	-0.000135653270
6.	1.000000119209	0.000000476837

Actual value of fourth root of 1	1.000000000000
Calculated value of fourth root of 1 by secant method	1.000000119209
Difference between actual and calculated values of fourth root of 1 by secant method	0.000000119209
Percentage error in the value of fourth root of 1 calculated by secant method	0.000011920900
Numerical rate of convergence of secant method in the calculation of fourth root of 1	7.936507936508



Graph 1: Value of root after each iteration in the determination of root of equation $f(x) = x^4 - 1 = 0$ by secant method



Graph 2: Value of function after each iteration in the determination of roots of equation f(x) = x4 – 1 = 0 by secant method

Calculation of fourth root of 2 by secant method with initial guesses 0.9 and 3.0

Secant method has been applied to calculate the root of equation.

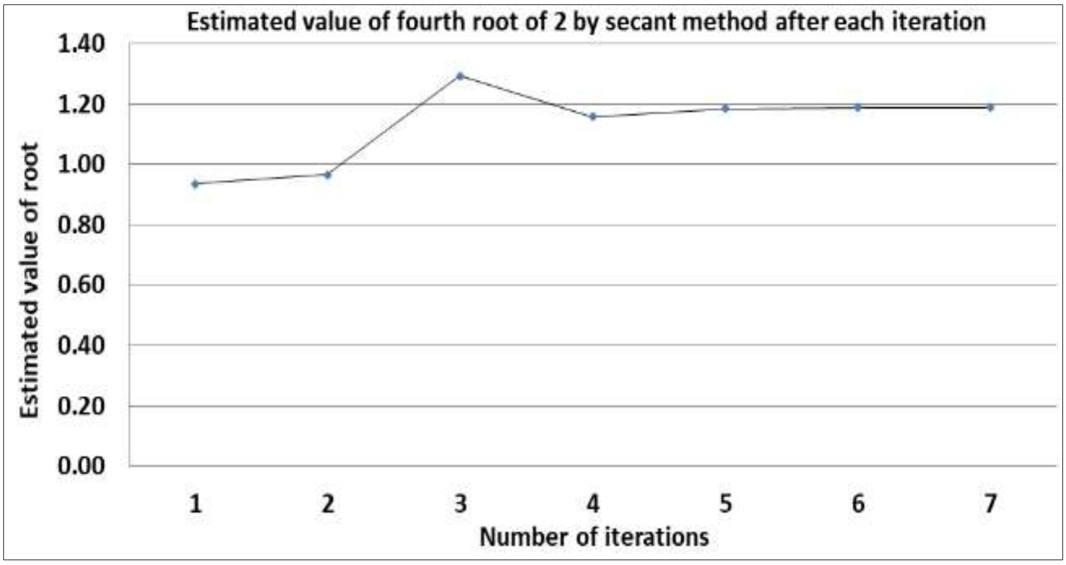
$f(x) = x^4 - 2 = 0$

with two initial guesses, $x_0 = 0.9$ and $x_1=3.0$ by using C++ computer program. No. of iterations, root guessed by secant method in each iteration (x_n) and value of function at $x = x_n$ are shown in Table-2. The value of root and the value of function after each iteration in the calculation of fourth root of equation $f(x) = x^4 - 2 = 0$ by secant method is shown in Graph-3 and Graph-4.

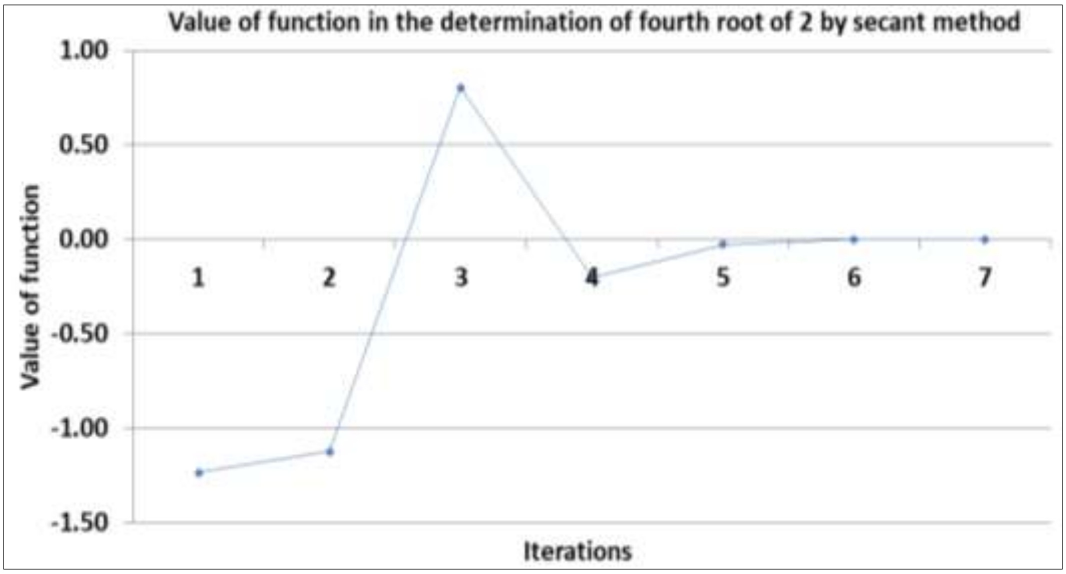
Table 2: No. of iterations, root of equation $f(x) = x^4 - 2 = 0$ guessed by secant method in each iteration (x_n) and value of function at $x = x_n$

No. of iterations	Root guessed by secant method (x_n)	Value of function at $x = x_n$
1.	0.935126364231	-1.235317404931
2.	0.966917514801	-1.125906866625
3.	1.294069528580	0.804338562573
4.	1.157744407654	-0.203402597476
5.	1.185260295868	-0.026419042246
6.	1.189367651939	0.001080178180
7.	1.189206361771	-0.000005067117

Actual value of fourth root of 2	1.189207115003
Calculated value of fourth root of 2 by secant method	1.189206361771
Difference between actual and calculated values of fourth root of 2 by secant method	0.000000753232
Percentage error in the value of fourth root of 2 calculated by secant method	0.000063338985
Numerical rate of convergence of secant method in the calculation of fourth root of 2	6.802721088435



Graph 3: Value of root after each iteration in the determination of root of equation $f(x) = x^4 - 2 = 0$ by secant method



Graph 4: Value of function after each iteration in the determination of roots of equation f(x) = x4 – 2 = 0 by secant method

Consolidated analysis of the fourth roots of numbers from 1 to 30 calculated by Secant method

The value of fourth root, error in the determination of fourth root, percentage error and numerical rate of convergence in the secant method are shown in Table-3(a) and Table-3(b). The actual value of fourth root and the value of fourth root calculated by secant method are shown in Graph-5. Error in the value of fourth root calculated by secant method is given in Graph-6. Percentage error in the values of fourth root calculated by secant method is given in Graph-7. Numerical rate of convergence in the determination of the fourth roots by secant method is given in Graph-8.

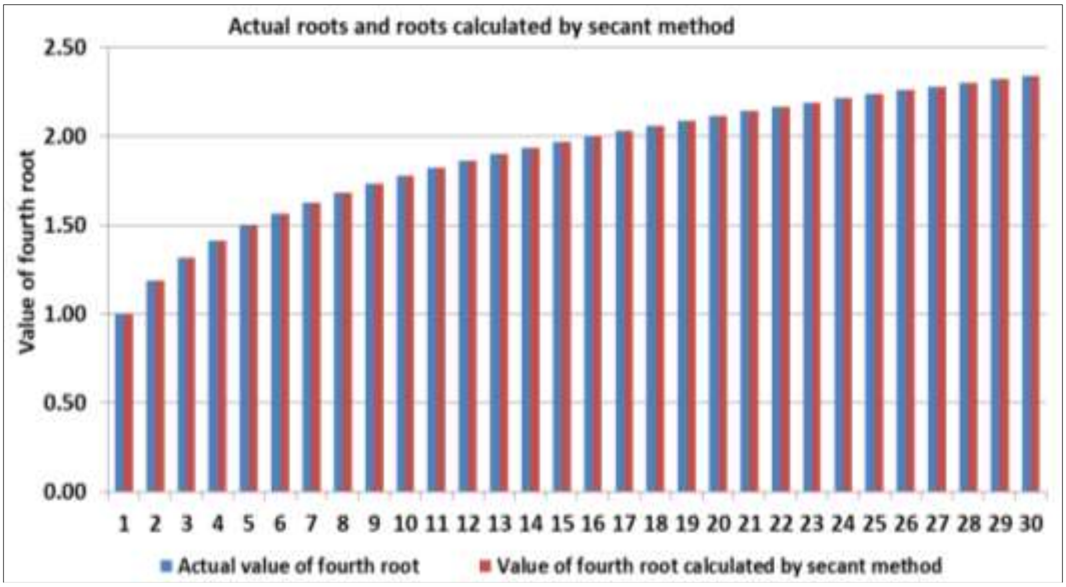
Table 3 a): Actual value of fourth root, value of fourth root calculated by secant method and error in the determination of fourth root by secant method in finding the roots of equations $f(x) = x^4 - n = 0$; n = 1, 2, 30

S. No.	Function	No. of Iterations	Actual value of fourth root	Value of fourth root calculated by secant method	Error in the fourth root calculated by secant method
1.	$f(x)=x^4-1$	6	1.000000000000	1.000000119209	0.000000119209
2.	$f(x)=x^4-2$	7	1.189207115003	1.189206361771	0.000000753232
3.	$f(x)=x^4-3$	8	1.316074012952	1.316074013710	0.000000000758
4.	$f(x)=x^4-4$	8	1.414213562373	1.414213180542	0.000000381831
5.	$f(x)=x^4-5$	9	1.495348781221	1.495348811150	0.000000029929
6.	$f(x)=x^4-6$	9	1.565084580073	1.565084576607	0.000000003466
7.	$f(x)=x^4-7$	9	1.626576561698	1.626576542854	0.000000018844
8.	$f(x)=x^4-8$	9	1.681792830507	1.681792855263	0.000000024756
9.	$f(x)=x^4-9$	9	1.732050807569	1.732050776482	0.000000031087
10.	$f(x)=x^4-10$	9	1.778279410039	1.778279423714	0.000000013675
11.	$f(x)=x^4-11$	9	1.821160286838	1.821160316467	0.000000029629
12.	$f(x)=x^4-12$	9	1.861209718204	1.861209750175	0.000000031971
13.	$f(x)=x^4-13$	9	1.898828922116	1.898828864098	0.000000058018

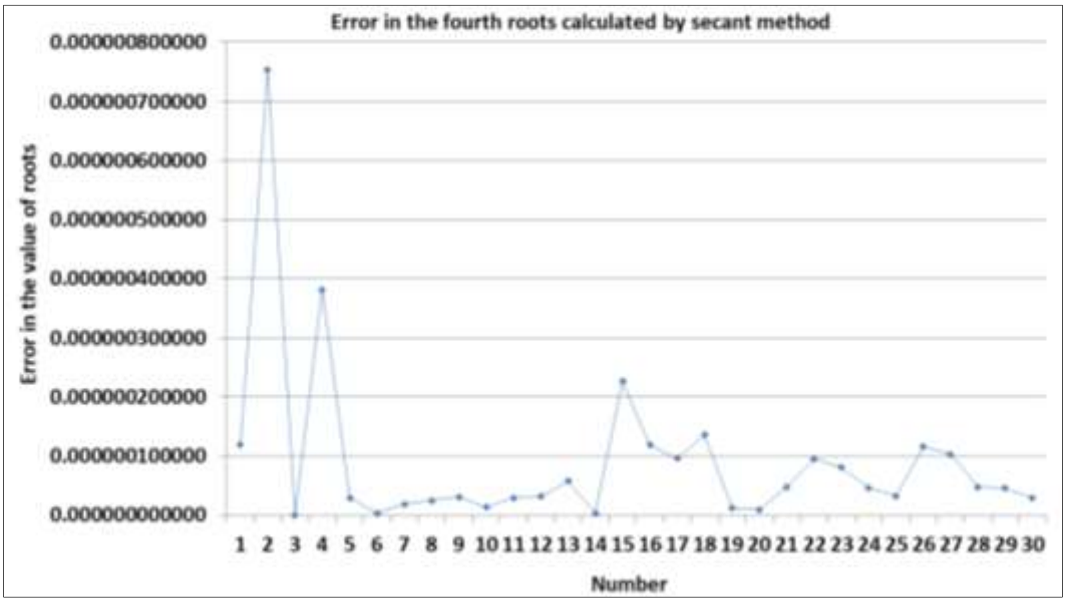
14.	$f(x)=x^4-14$	9	1.934336420268	1.934336423874	0.000000003606
15.	$f(x)=x^4-15$	8	1.967989671265	1.967989444733	0.000000226532
16.	$f(x)=x^4-16$	8	2.000000000000	1.999999880791	0.000000119209
17.	$f(x)=x^4-17$	8	2.030543184869	2.030543088913	0.000000095956
18.	$f(x)=x^4-18$	8	2.059767143907	2.059767007828	0.000000136079
19.	$f(x)=x^4-19$	8	2.087797629930	2.087797641754	0.000000011824
20.	$f(x)=x^4-20$	8	2.114742526881	2.114742517471	0.000000009410
21.	$f(x)=x^4-21$	8	2.140695142928	2.140695095062	0.000000047866
22.	$f(x)=x^4-22$	8	2.165736770668	2.165736675262	0.000000095406
23.	$f(x)=x^4-23$	8	2.189938703095	2.189938783646	0.000000080551
24.	$f(x)=x^4-24$	8	2.213363839401	2.213363885880	0.000000046479
25.	$f(x)=x^4-25$	8	2.236067977500	2.236068010330	0.000000032830
26.	$f(x)=x^4-26$	8	2.258100864353	2.258100748062	0.000000116291
27.	$f(x)=x^4-27$	8	2.279507056955	2.279507160187	0.000000103232
28.	$f(x)=x^4-28$	8	2.300326633791	2.300326585770	0.000000048021
29.	$f(x)=x^4-29$	8	2.320595787106	2.320595741272	0.000000045834
30.	$f(x)=x^4-30$	7	2.340347319321	2.340347290039	0.000000029282
Average value					0.000000091494
Minimum value					0.000000000758
Maximum value					0.000000753232

Table 3 b): Actual value of fourth root, percentage error in the calculation of fourth root and numerical rate of convergence of secant method in the determination of roots of equations $f(x) = x^4 - n = 0$; n = 1, 2, 30

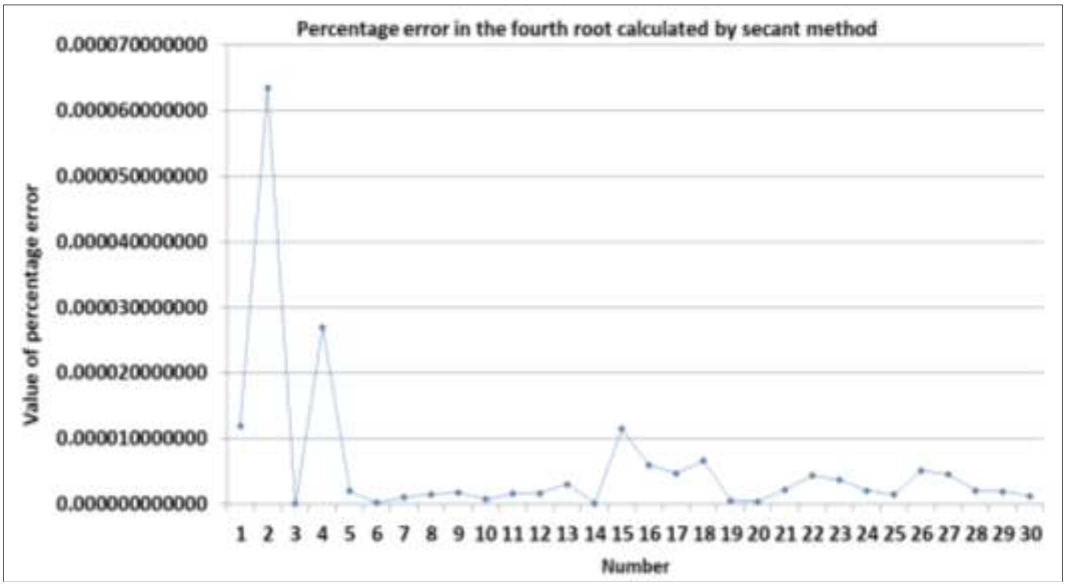
S. No.	Function	No. of Iterations	Actual value of fourth root	Percentage error in the fourth root calculated by secant method	Numerical rate of convergence of by secant method
1.	$f(x)=x^4-1$	6	1.000000000000	0.000011920900	7.936507936508
2.	$f(x)=x^4-2$	7	1.189207115003	0.000063338985	6.802721088435
3.	$f(x)=x^4-3$	8	1.316074012952	0.000000057558	5.952380952381
4.	$f(x)=x^4-4$	8	1.414213562373	0.000026999536	5.952380952381
5.	$f(x)=x^4-5$	9	1.495348781221	0.000002001458	5.291005291005
6.	$f(x)=x^4-6$	9	1.565084580073	0.000000221476	5.291005291005
7.	$f(x)=x^4-7$	9	1.626576561698	0.000001158494	5.291005291005
8.	$f(x)=x^4-8$	9	1.681792830507	0.000001471975	5.291005291005
9.	$f(x)=x^4-9$	9	1.732050807569	0.000001794802	5.291005291005
10.	$f(x)=x^4-10$	9	1.778279410039	0.000000769006	5.291005291005
11.	$f(x)=x^4-11$	9	1.821160286838	0.000001626937	5.291005291005
12.	$f(x)=x^4-12$	9	1.861209718204	0.000001717743	5.291005291005
13.	$f(x)=x^4-13$	9	1.898828922116	0.000003055459	5.291005291005
14.	$f(x)=x^4-14$	9	1.934336420268	0.000000186438	5.291005291005
15.	$f(x)=x^4-15$	8	1.967989671265	0.000011510855	5.952380952381
16.	$f(x)=x^4-16$	8	2.000000000000	0.000005960450	5.952380952381
17.	$f(x)=x^4-17$	8	2.030543184869	0.000004725629	5.952380952381
18.	$f(x)=x^4-18$	8	2.059767143907	0.000006606529	5.952380952381
19.	$f(x)=x^4-19$	8	2.087797629930	0.000000566346	5.952380952381
20.	$f(x)=x^4-20$	8	2.114742526881	0.000000444977	5.952380952381
21.	$f(x)=x^4-21$	8	2.140695142928	0.000002236006	5.952380952381
22.	$f(x)=x^4-22$	8	2.165736770668	0.000004405244	5.952380952381
23.	$f(x)=x^4-23$	8	2.189938703095	0.000003678238	5.952380952381
24.	$f(x)=x^4-24$	8	2.213363839401	0.000002099942	5.952380952381
25.	$f(x)=x^4-25$	8	2.236067977500	0.000001468212	5.952380952381
26.	$f(x)=x^4-26$	8	2.258100864353	0.000005149957	5.952380952381
27.	$f(x)=x^4-27$	8	2.279507056955	0.000004528708	5.952380952381
28.	$f(x)=x^4-28$	8	2.300326633791	0.000002087582	5.952380952381
29.	$f(x)=x^4-29$	8	2.320595787106	0.000001975100	5.952380952381
30.	$f(x)=x^4-30$	7	2.340347319321	0.000001251170	6.802721088435
Average values				0.000005833857	5.854749307130
Minimum values				0.000000057558	5.291005291005
Maximum values				0.000063338985	7.936507936508



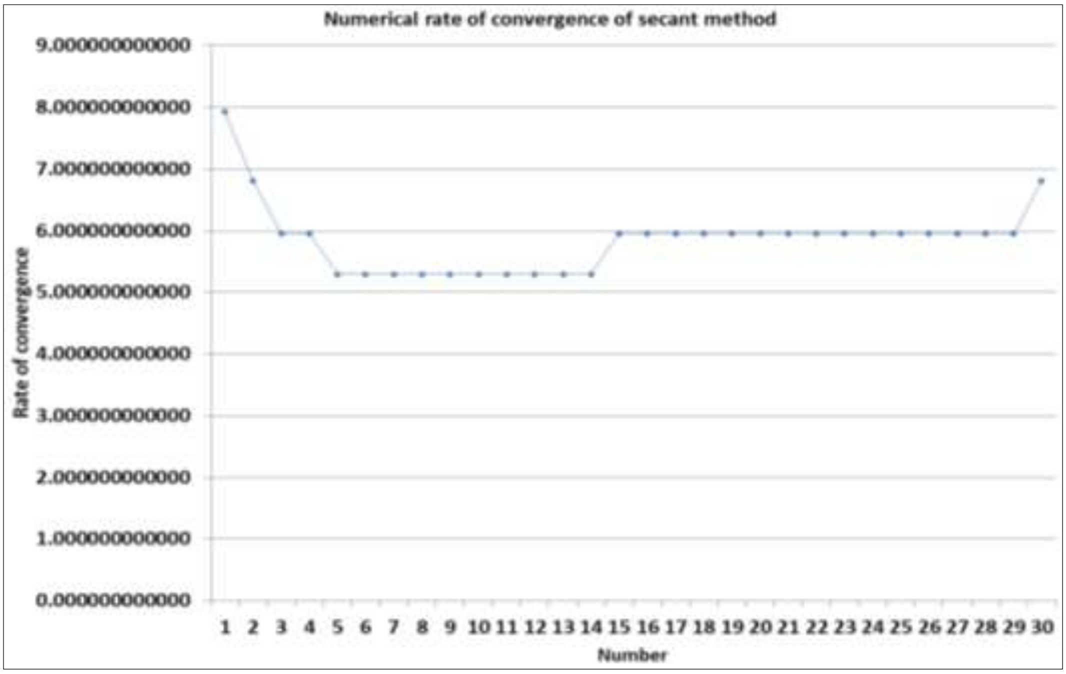
Graph 5: Actual value of fourth root and the value of root calculated by secant method in the equations $f(x) = x^4 - n=0$; n=1, 2, 30



Graph 6: Error in the value of root calculated by secant method in the equations f(x) = x4 – n=0; n=1, 2, 30



Graph 7: Percentage error in the value of root calculated by secant method in the equations f(x) = x4 – n=0; n=1, 2, 30



Graph 8: Numerical rate of convergence in the determination of the fourth root by secant method in the equations f(x) = x4 – n=0; n=1, 2, 30

Conclusion

Fourth roots of the natural numbers from 1 to 30 have been found by secant method and these values have been compared with the actual values. The minimum error 0.000000000758 and minimum percentage error 0.000000057558 have been obtained in the determination of fourth root of 3. The average value in the error is 0.000000091494. The maximum error 0.000000753232 and maximum percentage error 0.000063338985 have been obtained in the determination of fourth roots of 2. The average value of percentage error is 0.000005833857. Minimum, Maximum and average values of the numerical rate of convergence are 5.291005291005, 7.936507936508 and 5.854749307130 respectively.

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