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Comparison of numerical accuracy of bisection, false position, newton-Raphson's and secant methods

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Abstract
Comparison of numerical accuracy of bisection method, method of false position, Newton-Raphson method and secant method have been made by calculating the fourth roots of numbers 1 to 30 using computer programs developed in C++ programming language. Average percentage error of Bisection method, method of false position, Newton-Raphson method and secant method have been found to be 0.,000003048055, 0.000027500512, 0.000006303776 and 0.000005833857 respectively. These errors indicate that the secant method is better than Bisection method, Newton-Raphson's method and the method of false position. This is analogous to the theoretical interpretation of these methods.

Keywords: Bisection method, method of false position, Newton-Raphson method, secant method, convergence, numerical accuracy, percentage error

Introduction

We have published two research papers entitled "Convergence of bisection method" and "convergence of the method of false position" in the journal "The Scientific Temper" [1, 2]. Our research papers "Convergence of Newton-Raphson method" and "Convergence of Secant method" are in the press [3, 4]. In this research paper we have presented the comparison of the convergences of bisection method, method of false position, Newton-Raphson method and secant method. The Bisection method calls for a repeated halving of subintervals of [a, b] and at each step locating the half containing the root. This procedure is best to use when we only have an interval in which the root is contained [5]. It will also work when there is more than one root in the interval; however, for this problem we assume the root is unique. This method's major drawback is that it's the slowest of the four methods to converge. However, the method always converges to a solution and would be good to use as a starter for one of the other methods.

Similar to the secant method, the false position method also uses a straight line to approximate the function in the local region of interest. The only difference between these two methods is that the secant method keeps the most recent two estimates, while the false position method retains the most recent estimate and the next recent one which has an opposite sign in the function value [6-13].

It is obvious that Newton's method is faster, since it converges more quickly. However, to compare performance, we must consider both cost and speed of convergence. An algorithm that converges quickly but takes a few seconds per iteration may take far more time overall than an algorithm that converges more slowly, but takes only a few milliseconds per iteration. For the purpose of this general analysis, we may assume that the cost of iteration is dominated by the evaluation of the function - this is likely the case in practice. So, the number of function evaluations per iteration is likely a good measure of cost [14-18].

The secant method requires only one function evaluation per iteration, since the value of $f(x_n-1)$ can be stored from the previous iteration. Newton's method requires one function evaluation and one evaluation of the derivative per iteration. It is difficult to estimate the cost of evaluating the derivative in general. In some cases, the derivative may be easy to evaluate, in some cases, it may be much harder to evaluate than the function (if it is possible at all). It seems safe, though, to assume that, in most cases, evaluating the derivative is at least as costly as evaluating the function. Thus, we can estimate that Newton's method takes about two function evaluations per iteration [19, 20].

This disparity in cost means that we can run two iterations of the secant method in the time it will take to run one iteration of Newton's method. So, to compare the performance of the two methods, we must compare the speed of convergence of two iterations of the secant method with one iteration of Newton's method. This is given by [21, 22].

$$\begin{aligned} e_{n+2} &\leq c_f \cdot |e_{n+1}|^{\alpha} \\ &\leq c_f \cdot |c_f e_n^{\alpha}|^{\alpha} \\ &\leq c_f^{\alpha+1} \left| e_n^{\alpha^2} \right| \end{aligned}$$

And, since $\alpha^2 > 2$, we conclude that the secant method has better overall performance than Newton's method.

Materials and Methods

Fourth roots of natural numbers from 1 to 30 have been calculated with the help of Bisection, method of false position, Newton-Raphson and secant methods using the computer programs [1-4].

Result and Discussion

Percentage errors in Bisection method, method of false position, Newton-Raphson method and secant method in the calculation of fourth roots of natural numbers from 1 to 30 is given in Table 1. Average percentage errors in Bisection method, method of false position, Newton-Raphson's method and secant method in increasing order are given in Table 2. Average numerical rate of convergence in bisection method, method of false position, Newton-Raphson method and secant method in increasing order are given in Table 3. Numerical accuracy of root finding method increases as percentage error decreases. Average percentage error in the method of false position is highest which shows that it is the method of least accuracy. Average percentage error in the bisection method is lowest which shows that it is the method of highest accuracy. Average numerical rate of convergence of Newton-Raphson method is lowest and that of secant method is highest.

Table 1: Percentage errors in bisection method, method of false position, Newton-Raphson method and secant method in the calculation of fourth roots of natural numbers from 1 to 30

S. No.	Function	Percentage error in the root calculated by bisection method	Percentage error in the root calculated by the method of false position	Percentage error in the root calculated by Newton-Raphson method	Percentage error in the root calculated by secant method
1.	$f(x)=x^2-1$	0.000000000000	0.000232458100	0.000000000000	0.000011920900
2.	$f(x)=x^2-2$	0.000003193449	0.000123484606	0.000097049224	0.000063338985
3.	$f(x)=x^2-3$	0.000000057558	0.000081463997	0.000000057558	0.000000057558
4.	$f(x)=x^2-4$	0.000001711417	0.000060717003	0.000015147281	0.000026999536
5.	$f(x)=x^2-5$	0.000002001458	0.000037858607	0.000002001458	0.000002001458
6.	$f(x)=x^2-6$	0.000000221476	0.000030688647	0.000022628918	0.000000221476
7.	$f(x)=x^2-7$	0.000001158494	0.000030473867	0.000001158494	0.000001158494
8.	$f(x)=x^2-8$	0.000001471975	0.000026880923	0.000001471975	0.000001471975
9.	$f(x)=x^2-9$	0.000001794802	0.000015559929	0.000001794802	0.000001794802
10.	$f(x)=x^2-10$	0.000000769006	0.000012638280	0.000000769006	0.000000769006
11.	$f(x)=x^2-11$	0.000001626937	0.000018010434	0.000001626937	0.000001626937
12.	$f(x)=x^2-12$	0.000001717743	0.000017497018	0.000001717743	0.000001717743
13.	$f(x)=x^2-13$	0.000009333539	0.000009333539	0.000003055459	0.000003055459
14.	$f(x)=x^2-14$	0.000000186438	0.000012139185	0.000000186438	0.000000186438
15.	$f(x)=x^2-15$	0.000000603945	0.000011510855	0.000000603945	0.000011510855
16.	$f(x)=x^2-16$	0.000000000000	0.000011920950	0.000000000000	0.000005960450
17.	$f(x)=x^2-17$	0.000004725629	0.000004725629	0.000004725629	0.000004725629
18.	$f(x)=x^2-18$	0.000006606529	0.000006606529	0.000004968469	0.000006606529
19.	$f(x)=x^2-19$	0.000000566346	0.000010853248	0.000000566346	0.000000566346
20.	$f(x)=x^2-20$	0.000000444977	0.000011719069	0.000000444977	0.000000444977
21.	$f(x)=x^2-21$	0.000008901451	0.000002236006	0.000002236006	0.000002236006
22.	$f(x)=x^2-22$	0.000004405244	0.000004405244	0.000004405244	0.000004405244
23.	$f(x)=x^2-23$	0.000007208779	0.000007208779	0.000003678238	0.000003678238
24.	$f(x)=x^2-24$	0.000002099942	0.000008671852	0.000002099942	0.000002099942
25.	$f(x)=x^2-25$	0.000009194165	0.000009194165	0.000001468212	0.000001468212
26.	$f(x)=x^4-26$	0.000005408429	0.000005149957	0.000005408429	0.000005149957
27.	$f(x)=x^4-27$	0.000004528708	0.000005930527	0.000004528708	0.000004528708
28.	$f(x)=x^4-28$	0.000008276946	0.000012452154	0.000002087582	0.000002087582
29.	$f(x)=x^4-29$	0.000001975100	0.000001975100	0.000001975100	0.000001975100
30.	$f(x)=x^4-30$	0.000001251170	0.000001251170	0.000001251170	0.000001251170
Average Percentage error		0.000003048055	0.000027500512	0.000006303776	0.000005833857

Table 2: Average percentage errors in Bisection, method of false position, Newton-Raphson and secant methods in increasing order

Method	Average percentage error
Bisection method	0.000003048055
Secant method	0.000005833857
Newton-Raphson method	0.000006303776
Method of false position	0.000027500512

Table 3: Average numerical rate of convergence in Bisection, method of false position, Newton-Raphson and secant methods in increasing order

Method	Average numerical rate of convergence
Newton-Raphson method	1.104698523
Method of false position	1.197514788
Bisection method	1.458082184
Secant method	5.854749307

Conclusion

Fourth roots of natural numbers from 1 to 30 have been calculated using Bisection, method of false position, Newton-Raphson and secant methods with the help of computer programs. Average percentage error of Bisection, method of false position, Newton-Raphson and secant methods have been found to be 0.,000003048055, 0.000027500512, 0.000006303776 and 0.000005833857 respectively. It indicates that the average percentage errors of Bisection, method of false position, Newton-Raphson and secant methods are in the following order.

Bisection method < Secant method < Newton-Raphson method < Method of false position

The accuracy of Bisection, method of false position, Newton-Raphson and secant methods are in the following order.

Bisection method > Secant method > Newton-Raphson method < Method of false position

Average numerical rate of convergence in Bisection, method of false position, Newton-Raphson and secant methods is in the following order.

Newton-Raphson method < Method of false position < Bisection method < Secant method

Above discussions indicate that secant method is better than Bisection method, Newton-Raphson's method and the method of false position.

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