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Study of computation of number of lines from set of collinear and non-collinear points in a given space

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Abstract
Using probability or manual calculation it is possible to compute the total number of lines that can be formed, from a given number of points in a space, considering various possibilities like some points are collinear, some are non-collinear or they form particular shapes. This paper explains a straight forward substitution formula method, which is derived, where the same result is achieved by plugging in the key factors. The direct formula method not only increases the accuracy of the computation but also saves time and presents for an infinite set of possibilities from a given spatial plane of points. The paper further investigates the various other possibilities of points and line, like a given set of points representing a square or rectangle, or other composite shapes.

Keywords: Point line algebra, number of lines from points, Dinival algebra, Dinival algorithm, point line studies, collinear points

1. Introduction

The concept of finding the number of lines from a given set of points can be attributed to probability or projective geometry in general. In this paper, we will discuss on how the work discussed can be mapped to both, and how one can solve the problem through simple formula substitutions. One of the most earlier studies on this was done by Kelly and Moser [1] where they proposed a theorem to find out the number of lines from a given set of points by grouping these points in a certain fashion.

In a latest study, Meng Li *et al.* [2] has observed the correlation between points and lines using O(n) post estimation method. The theorem proposed in this paper can be co-related with our work, thereby improving the efficiency of its operation.

Micha *et al.* [3] in his paper discusses the relationship between points and line in terms of calculating the distance between them. The paper also discusses about binding the tangent pairs of finite set of lines to calculate the number of distinct point line distances. It is to be noted that this work which they have listed can be combined with our work to form a new sub-branch of projective geometry.

The co-relation between point and line in 3 Dimension space is discussed by Brian *et al.* [4]. The basis of this study is applied in identifying the corners of objects using image recognition and processing.

An interesting and relevant paper is discussed, where Roy Mikael [5] studies the effect of nine lines and how it avoids any 3 general points. It is concluded in his paper that when 3 points are considered in a 2D plane, then at any given point at least 1 of constructed line, indirectly through these 3 points, may avoid intersection with them.

In this paper, the approach taken to find relationship between points and line in terms of the number of lines that can be drawn in a plane is explored for the first time through usage of formulae substitution. Various studies are undertaken to find the number of lines from points, where the points may be arranged in collinear, non- collinear or in certain shapes. It is hoped that this paper will kindle further research on this sub-division of algebra which we would like to address as “Dinival Algebra” or “Dinival Algorithm”. The formulae present will be addressed as “Dinival Formula” throughout this paper for ease of understanding. Further studies will also be undertaken to study the point line relationship in 3 Dimensions.

2. Number of points from given collinear and non-collinear points

2.1 Introduction to the formula

For this study, consider a 2D plane with “n” number of points. These points may be arranged in random or any specific way in terms of collinearity.

For this particular analysis, lets assume that 23 points are arranged in a plane, in which there are 4 collinear points (2 sets), 3 collinear points (2 sets), 2 collinear points (2 sets) and 5 non-collinear points as shown in Fig 1.
To calculate the total number of non-repeating lines between these points, we can use trial and error method by calculating manually, by drawing lines, or it can be done using permutation and combination, where we isolate each group, calculate and then add. This paper proposes to cut short time and increase accuracy in computation of this problem by using the Dinival formula as specified in Equation 1. In addition to that such complex computational problems can now be solved with ease using these equations.

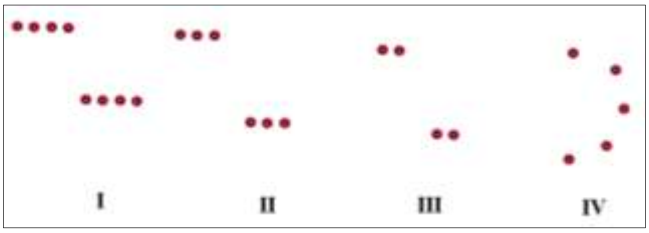

Fig 1: Sample arrangement of the lines in collinear and non-collinear fashion

As shown in Fig 1, there are 4 sets of points, 1st set (or group) g_1 with 4 collinear points c_1. Similarly, second set (g_2) has 3 collinear points and 3rd and 4th set have 2 collinear points, with a total of 9 points.
The total number of lines in the system can be calculated by the Dinival Formula 1

$$N_{G1,\,2,\,3\ldots\,C1,\,2,\,3\ldots} = \{\sum [n_1-c_1+1] + (g_1-1) \sum (c_1-2)\} + \{\sum [n_2-c_2+1] + (g_2-1) \sum (c_2-2)\} + \ldots + (n_2)(n_1) + (n_3)(n_1+n_2) + (n_4)(n_1+n_2+n_3) + \ldots \quad \text{Eq (1)}$$

Where,
N_{GxCy} – Total number of lines in a given set of points.
G_x – Total number of groups or pairs of points
C_y – Total number of collinear points in the specific group
n_1 – Number of points in the 1st set or group
c_1 – Number of collinear points in the 1set or group
g_1 – Number of pairs of line in the 1st set or group

To effectively use this formula and substitute the correct values, step by step procedure is given here. To check the formula, a smaller point space is considered as shown in Fig 2, where the points along with possible lines between them is shown.

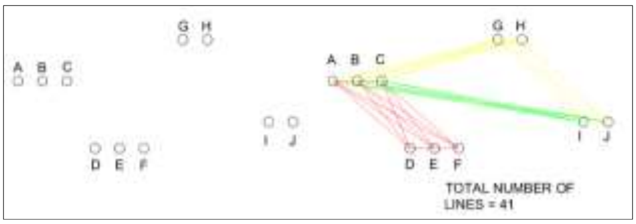

Fig 2: Two sets of points with indication of lines between them

Let each point be labelled from 'A' to 'J' respectively. The system consists of two sets. First set having two groups of three collinear points and the second set having two groups of two collinear points. To find out the number of lines between them, we use both trial and error method, where we draw individual lines from each points and we use Dinival formula 1, where we observe the given set of points and substitute the correct values.

2.2 By Trial and Error method:

We now analyse the number of non-repeating lines by manually drawing lines from each point.

- From point 'A' --- 8 lines
- From point 'B' --- 7 lines
- From point 'C' --- 7 lines
- From point 'D' --- 5 lines
- From point 'E' --- 4 lines
- From point 'F' --- 4 lines
- From point 'G' --- 3 lines
- From point 'H' --- 2 lines
- From point 'I' --- 1 lines
- From point 'J' --- 0 lines

Thus, the total number of non repeating lines that are possible between the given point configuration is 41 (by trial and error method), which is shown in Figure 2. Now, we find out the number of lines possible using the formula. From the given set of points, we can consider the following values.

$n_1 = 6$; $n_2 = 4$; $g_1 = 2$; $g_2 = 2$; $c_1 = 3$; $c_2 = 2$.

Since there are only two sets, the formula from Equation 1 is reduced to the following.

$$N_{g1,\,2,\,c1,\,2,} = \{\sum [n_1-c_1+1] + (g_1-1) \sum (c_1-2)\} + \{\sum [n_2-c_2+1] + (g_2-1) \sum (c_2-2)\} + (n_2)(n_1) \quad \text{Eq (2)}$$

Thus, on substituting the values in the formula, as shown in equation 2, we get total number of lines = 41.
The formula can be checked and validated for other set of points also, say 4 sets of 5 collinear points or other random configuration. The results from the formula will match with the conventional method.

2.3 Current limitations of the formula

The formula currently can be applicable to all sorts of collinear and non-collinear points in space with exception of the following. Current studies are being undertaken to remove these limitations and to make the formula more universal to test any conditions.

a. No three points must be collinear between two sets. As shown in Fig 3, only 2 points can be collinear with each other. If more than 2 are in direct collinearity between 2 sets, then the formula isn't valid.

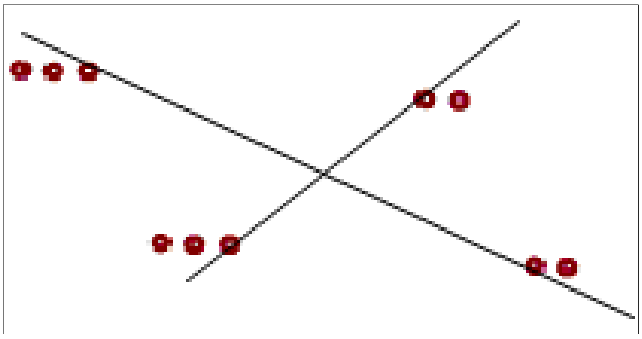

Fig 3: Illustration of 2 points alone being collinear

b. While grouping the points as sets, each set's group must have equal number of collinear points. As shown in Fig 4, all 3 sets have equal number of collinear points within them. In case of unequal number of points, then each group can be considered as individual sets. However, in case the number is not equal, then the Dinival cross formula as mentioned in section V can be used.

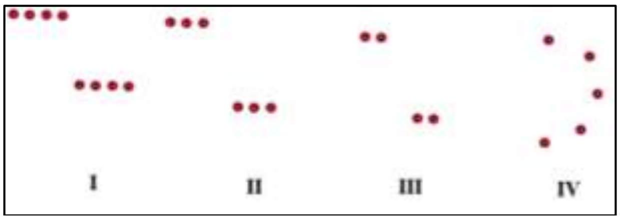

Fig 4: Three sets of points with equal collinear point within the set.

While analysing models with large number of parameters, this process can be substituted in place of conventional methods like *permutation and combinations* and by *trial and error* method.

3. Number of points from an even matrix

3.1 Even Matrix – Dinival Square

To evaluate the options of more than 2 collinear points, we discuss the case of even and uneven matrix. Every matrix can be mentioned as Dinival Square and uneven matrix can be called as Dinival rectangle for representation purpose in this paper.

A Dinival Square is an n*n arrangement of points in the given plane. One important point in this arrangement is the collinear condition. The points are collinear not only in the diagonal elements but within some of the other column element too. For instance, let's take the case of a 5 * 5 matrix as shown in Fig 5.

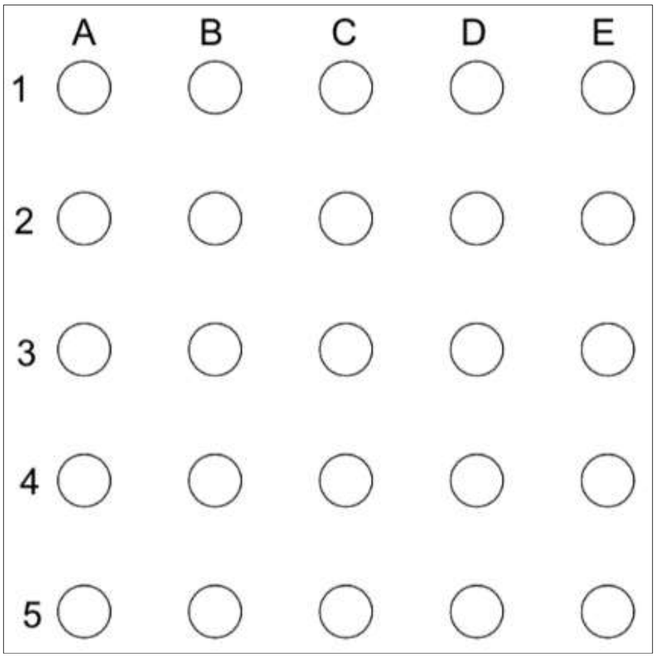

Fig 5: A 5 x 5 square matrix

In this case, the conditions of co linearity and non-repeatability play a very vital part. Let the elements be represented as $a_{i,j}$ where 'i' represents the row elements and 'j' represents the column elements. We know that a_{11}, a_{22}, a_{33}, a_{44} and a_{55} are all collinear with each other. In practical case, a_{11}, a_{23}, a_{35} are collinear too.

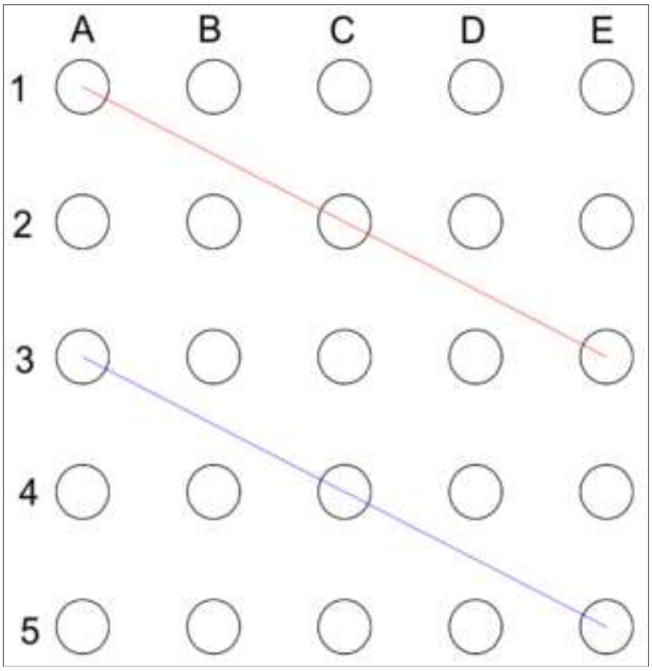

Fig 6: Multiple collinear points in a 5x5 matrix / Dinival square

The red and blue line in Fig.6, indicates that more than 2 points are collinear in this case. As the matrix size increases, so does the number of collinear points in a single line, this making estimation of number of lines through conventional method or permutation and combination, very tough. The devised formula for such a square matrix is given in Equation 3, which can be called as Dinival formula 2.

$$N_{ds} = \sum [8 * (S-3)] + (S-1) \qquad \text{Eq (3)}$$

Where, S is the number of points on one side of the matrix. For the given Dinival Square in Fig 6, the number of possible lines by plotting using trial and error is obtained as 140. This can be verified by manual plotting. By using the formula, we get

$$N_{ds} = \sum [8 * (5-3)] + (5-1)= 140$$

Hence the formula is validated. This formula holds good for S values greater than 4. For values of S less than that, normal plotting method is used, since its less complex. It is to be noted that, an uneven matrix or a Dinival rectangle is work in progress and will be updated in future paper.

4. Study of points on a typical "Star" shape

4.1 A Dinival Star

One of the most stable structures in nature is the star shape as shown in Fig 7, with interconnections between points. Many designs on the architecture side is based on this star shape, including transmission towers, cell phone towers, other electrical interconnections, etc. The star can also be drawn using multiple points, with only condition that no 3 lines are collinear.

The number of lines in any type of these stars (Say 3 points or 4 points etc…) can be determined by Equation 4.

$$N_{st} = \sum (n-3) + ((S/2) - 3) \qquad \text{Eq (4)}$$

Where,

n – Total number of points.

S – Total number of sides in the star.

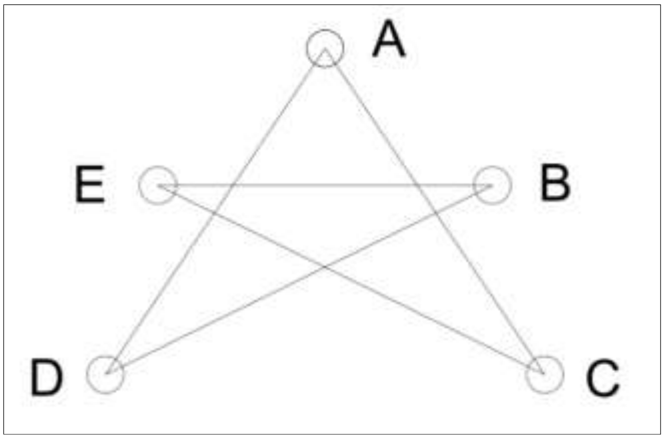

Fig 7: A typical star structure

On further analysis, on introducing external points to the star, which may in a separate set of collinear or non collinear environment, then its combination and computation of the total number of lines can also be calculated, which is shown in Equation 5.

$$N_{st} = \sum (n - 5) + (n - 5) + 3\,e \qquad \text{Eq (5)}$$

Where,
n – Total number of points
e – extra points in the external

Dinival Cross
A Dinival cross can be a "X" shaped figure with multiple points along the line as shown in Fig.8. The number of points per leg can be uneven. The total number of lines that can be drawn can be calculated from Equation 6.

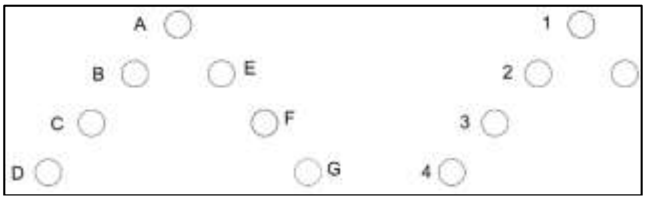

Fig 8: An even and uneven Dinival Cross

$$N_{DC} = \sum (cs + 1) + \sum (cs - 2) + (cel \times cs) \qquad \text{Eq (6)}$$

Where,
N_{DC} – Number of lines in a Dinival Cross
cs – Number of collinear points in the short set
cel – Number of collinear points in the long set

As a continuation to Dinival cross, in case there are many branches to the same line, then one can use Equation 7 for finding out the total number of lines.

$$N_{net} = N_1 + N_2 + N_3 + \ldots N_n - [(br-1) + 3 \times br] \qquad \text{Eq (7)}$$

Where,
N – Total number of lines
N_1 – Number of lines in 1st group
N_2 – Number of lines in 2nd group
br – Number of branches to the line

Dinival Inverted figure
A Dinival Inverted figure is formed by joining the points in a standard Dinival star as shown in Fig 9. The image formed inside by joining the points represented by Dinival star as shown through points 1 to 5. The total number of lines possible from this figure can be found by Equation 8.

$$N_{Dif} = \sum 5 + (n - 10) + [3e + 5e\,(n^{th}\text{ Dinival} - 1)]$$

Where, $n = 10 + 5\,(n^{th}\text{ Dinival} -1)$ Eq (8)

Where,
N_{Dif} – Total number of lines from Dinival inverted figure
n – Number of points
e – Number of extra points.
N^{th} Dinival – Specification of Dinival required.

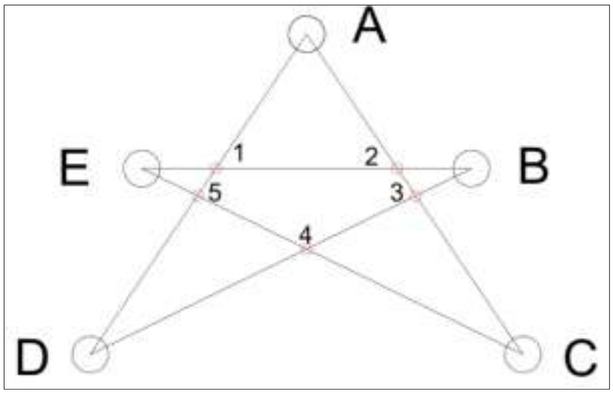

Fig 9: Inverter Dinival Star

For Varied Dinival star configuration
In a case where the star shape is formed by combining various polygons as shown in Fig 10, the study is done to calculate the number of lines in this case, based on external and internal lines formed.

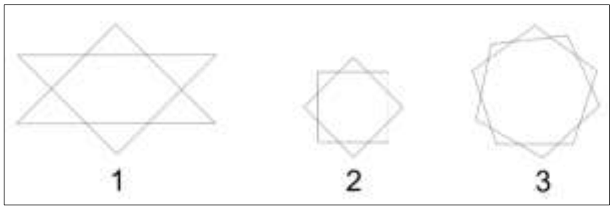

Fig 10: Stars formed by various polygons with 3 sides, 4 sides and 5 sides

In such a case, the number of lines formed is as per Equation 9.

$$N_{LST} = \sum (n - 3) + [\,(S / 2) - 3] \qquad \text{Eq (8)}$$

Where,
N_{LST} – Number of lines in overlapping stars
n – Total number of points
S – Number of sides of the polygon.

5. Results and Discussions
The paper discusses about various co-relation between points and lines, where the main aim was to find out the number of lines possible from a given set of points arranged in various fashion. The ultimate aim is to find a universal equation, where we can substitute the number of points arranged in whatever fashion as discusses in this paper and furthermore, and we find the total number of lines. An extension of this work is to attempt this in 3 Dimensions. This work can be as

base tool in multiple domains which involves the study of multiple points.

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