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Partitioning an even number of the new formulation into all pairs of odd numbers

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Abstract
We present a new algorithm for partitioning an even number of the form $(p_1 + p_2) + (p_2 - p_1)^n$ [1] into pairs of odd numbers. We have also presented a general proof of partitioning the even number of this form into all pairs of odd numbers. Since prime numbers greater than 2 are subsets of odd numbers, it is expected that in these pairs of odd numbers, there exist at least one Goldbach's partition. These results could therefore have remarkable application to the solution to the Strong Goldbach's Conjecture.

Keywords: Goldbach's Conjecture, Goldbach's partition, even numbers, odd numbers, prime numbers, natural numbers

Introduction

Even numbers have many interesting properties, such as the fact that any even number can be expressed as the sum of two prime numbers (according to the Goldbach's conjecture) [2]. Prime numbers greater than 2 are subsets of odd numbers and although odd numbers do not directly play a role in the statement of the Strong Goldbach's Conjecture, as the conjecture is only concerned with even numbers [3], the odd numbers can be indirectly related to the conjecture through the use of parity.

Parity refers to whether a number is odd or even [4]. Every even number can be expressed as the sum of two odd numbers (for example, 8 = 3 + 5) [5]. Similarly, every odd number can be expressed as the sum of an even number and an odd number (for example, 7 = 4 + 3) [6]. Using parity and the fact that every even number can be expressed as the sum of two prime numbers (according to the Strong Goldbach's Conjecture), we can make some conjectures about odd numbers. For example, it is conjectured that every odd number greater than 5 can be expressed as the sum of three prime numbers (for example, 7 = 2 + 2 + 3). This is known as the Weak Goldbach's Conjecture for odd numbers [7]. In general, any even number can be expressed as a sum of two odd numbers [2].

Results and Discussion

The statement of the Strong Goldbach's Conjecture in its simple form gives a relationship between a given even number and two prime numbers as, $2n = p_i + p_j, \forall p_i, p_j \in P$, *and* $i, j, n \in \mathbb{N}$, where P is the set of prime numbers and $\mathbb{N}$ the set of natural numbers. However, the traditional definition of an even number as $2n$ does not directly bring out its relation to the sum of a pair of prime $p_i + p_j$. This definition has been extended to arrive at a new representation of an even number as, $2n = (p_i + p_j) + (p_j - p_i)^n, \forall p_j > p_i$ [1]. The resulting expression $p_i + p_j + (p_j - p_i)^n$ of an even number is formed from the addition of the sum of a pair of prime $(p_i + p_j)$ to the difference of the same pair of prime $(p_j - p_i)^n$, $\forall\, n \in \mathbb{N}$. The even number $p_i + p_j + (p_j - p_i)^n$ is therefore formed from two prime numbers p_i and p_j, and can as a result be easily partitioned. This representation of an even number has been used to develop an algorithm that allows any even number of this form to be partitioned into all pairs of odd numbers as follows.

Let P be the set of all prime numbers, ℕ be the set of all natural numbers and O the set of all odd numbers.

Step 1: Let P_1 and $P_2 \in P$ then $(P_1+P_2)+(P_2-P_1)^n$ is even, $\forall n \in \mathbb{N}$, and $p_2 > p_1$[1].

Step 2: Let d be even and belong to the half-open interval [1, $[\frac{1}{2}((p_1+p_2)+(p_2-p_1)^n]]$.

Step 3: Let z_i *and* $y_i \in 1 \le O \le \frac{1}{2}((P_1+P_2)+(P_2-P_1)^n)$ for $i \in O$ and belonging to the half-open interval [1, $[\frac{1}{2}((p_1+p_2)+(p_2-p_1)^n]]$.

With p_1, p_2 d and z_i, we partition $(p_1+p_2)+(p_2-p_1)^n$ as follows:

Partition 1: $((P_1+P_2)+(P_2-P_1)^n)$ - $(d+z_1)=y_1$

Partition 2: $((p_1+p_2)+(p_2-p_1)^n)$- $(d+z_3)=y_3$

Partition 3: $((p_1+p_2)+(p_2-p_1)^n)$- $(d+z_5)=y_5$

Partition I: $((p_1+p_2)+(p_2-p_1)^n)$- $(d+z_{((\frac{1}{2}((p_1+p_2)+(p_2-p_1)^n)-1)})=y_{((\frac{1}{2}((p_1+p_2)+(p_2-p_1)^n)-1)}$

The set of pairs $(d+z_1,y_1),(d+z_3,y_3),(d+z_5,y_5),\dots,(d+z_{((\frac{1}{2}((p_1+p_2)+(p_2-p_1)^n)-1)},y_{((\frac{1}{2}((p_1+p_2)+(p_2-p_1)^n)-1)})$ of odd numbers are all partitions of the even number $(p_1+p_2)+(p_2-p_1)^n$.

Partitioning any Even Number into all Pairs of Odd Numbers for $1 < n < \infty$

The sum of two odd numbers will always be even [8]. This statement in its basic terms gives a relationship between the sum of two odd numbers and a given even number. It confirms that any even number can always be partitioned into two odd numbers. This relation however does not guarantee partitioning the even number into all pairs of odd numbers and in this study we provide a general proof of partitioning any even number of the form $(P_1+P_2)+(P_2-P_1)^n$ the same set of all pairs of odd numbers whoto the Strong Goldbach's conjecture since prime numbers greater than 2 are subsets of odd numbers. With these results, it is expected that there is at least one pair of prime in these pairs of odd numbers whose sum is the given even number. The following theorem therefore provides a general proof of partitioning any even number of the form $(P_1+P_2)+(P_2-P_1)^n$ into all pairs of odd numbers.

Theorem 1

Let p_1 *and* $p_2 \in P$, where P is the set of all primes, and d be the difference between p_1 *and* p_2 such that $d = p_2 - p_1 > 0$ where $p_2 > p_1$. Let $z_i \in 1 \le O \le \frac{1}{2}((p_1+p_2)+(p_2-p_1)^n)$ be the set of odd numbers for $i \in 1 \le O \le (\frac{1}{2}(p_1+p_2)+(p_2-p_1)^n)$, then any multiple of d in the half-open interval [1, $[\frac{1}{2}((p_1+p_2)+(p_2-p_1)^n]]$ can be used to partition $(p_1+p_2)+(p_2-p_1)^n$ into all pairs of odd numbers.

Proof

Let p_1 *and* $p_2 \in P$, where P is the set of all primes and let d be the difference between p_1 *and* p_2 such that $d > 0$. Then we show that $(p_1+p_2)+(p_2-p_1)^n$ can be partitioned into all pairs of odd numbers. Since p_1 *and* $p_2 \in P$ are odd numbers, they can be represented as $p_1 = 2k_1+1$ and $p_2 = 2k_2+1 \Rightarrow (p_1+p_2)+(p_2-p_1)^n$ can be partitioned as follows for all z_i *and* $y_i \in 1 \le O \le [\frac{1}{2}((p_1+p_2)+(p_2-p_1)^n)]$ and $i \in 1 \le O \le (\frac{1}{2}(p_1+p_2)+(p_2-p_1)^n)$:

For Partition 1: $((P_1+P_2)+(P_2-P_1)^n)$- $(d+z_1)=y_1$ in section 2, we need to prove that for the even number.

$(P_1+P_2)+(P_2-P_1)^n$, both $d+z_1$ and y_1 are odd numbers. Since $(p_1+p_2)+(p_2-p_1)^n$ and d are both even and z_1 is odd, then it follows that partition 1 can be expressed as: $((P_1+P_2)+(P_2-P_1)^n-(d+z_1) \Rightarrow$

$$[((2k_1+1)+(2k_2+1)+(2k_3)^n)]-(2k_3+2k_4+1) \quad ,\forall k_1,k_2,k_3,k_4 \in N$$

$$\Rightarrow ((2(k_1+k_2+(2^{n-1}k_3{}^n) \;\; +1)) \;\; -(2k_3+2k_4+1)$$

Where, $p_1 = 2k_1+1$, $p_2 = 2k_2+1$, $d = 2k_3$ *and* $z_1 = 2k_4+1$.

This further implies that $2(k_1+k_2+(2^{n-1}k_3^n)+1) \in 2n$ and $(2k_3+2k_4+1) \in 2n_1+1$. We are to show that $[2(k_1+k_2+(2^{n-1}k_3^n)+1)]-[(2k_3+2k_4+1)]$ belongs to the set of odd numbers. An odd number subtracted from an even number gives an odd number since, let $2n_1 = 2(k_1+k_2+(2^{n-1}k_3^n)+1)$ and $2n_2+1 = 2k_3+2k_4+1$, then it follows that

$2n_1-(2n_2+1)=2n_1-2n_2-1 \Rightarrow 2(n_1-n_2)\in 2n$ and therefore $2n_3-1\in 2n+1$ for $n_3=n_1-n_2$. This proves that ($(P_1+P_2)+(P_2-P_1)^n$)-$(d+z_1)=y_1\in 2n+1$ and hence, the even number $(P_1+P_2)+(P_2-P_1)^n$ has been partitioned into a pair of odd numbers $d+z_1$ and y_1.

The same argument can be used to show that the even number $(P_1+P_2)+(P_2-P_1)^n$ can been partitioned into pairs of odd numbers for all the other partitions 2, 3, 4, …, and i in the algorithm we have presented for partitioning an even number into all pairs of odd numbers in section 2.These results proves that any even number of the form $(P_1+P_2)+(P_2-P_1)^n$ can be partitioned into all pairs of odd numbers. Further, it is expected in these pairs of odd numbers, there exists at least one Goldbach partition.

In general, it has been proven that any multiple of d in the half-open interval [1, $[\frac{1}{2}((p_1+p_2)+(p_2-p_1)^n]]$ can be used to generate the same set of all pairs of odd numbers whose sum is $(p_1+p_2)+(p_2-p_1)^n$. The same algorithm can be used to partition a given even number of the form $(p_1+p_2)+(p_2-p_1)^n$ using any multiple of d in the range $\frac{1}{2}((p_1+p_2)+(p_2-p_1)^n)<d<(p_1+p_2)+(p_2-p_1)^n$ into pairs of odd numbers belong to the a set or a subset of pairs of odd numbers generated using any multiple of d in the range $1\le d\le \frac{1}{2}((p_1+p_2)+(p_2-p_1)^n)$. The study further proposes the following Corollary that summarizes the partitioning of an even number into pairs of odd numbers using multiples of d in the open interval $[\frac{1}{2}((p_1+p_2)+(p_2-p_1)^n),(p_1+p_2)+(p_2-p_1)^n]$.

Corollary 1

Let p_1 *and* $p_2\in P$, where P is the set of all primes and d be the difference between p_1 *and* p_2 such that $d=p_2-p_1>0$ where $p_2>p_1$ and Let $z_i\in 1\le O\le \frac{1}{2}((p_1+p_2)+(p_2-p_1)^n)$ be the set of odd numbers for $i\in 1\le$ O $\le(\frac{1}{2}(p_1+p_2)+(p_2-p_1)^n)$, then any multiple of d in the range $\frac{1}{2}((p_1+p_2)+(p_2-p_1)^n)<d<(p_1+p_2)+(p_2-p_1)^n$ can be used to generate the same set of pairs of odd numbers or a subset of pairs of odd numbers whose sum is $(p_1+p_2)+(p_2-p_1)^n$. For the set of values of d in the open interval $[\frac{1}{2}((p_1+p_2)+(p_2-p_1)^n),(p_1+p_2)+(p_2-p_1)^n]$, it is expected that as the value of d gets closer to $(p_1+p_2)+(p_2-p_1)^n$,the pairs of odd numbers whose sum is $(p_1+p_2)+(p_2-p_1)^n$ reduces significantly.

Conclusion

The study has proven that an even number of the form $(P_1+P_2)+(P_2-P_1)^n$ can always be partitioned it into all pairs of odd numbers. The strong Goldbach's Conjecture states that every even natural number greater than 2 is the sum of two prime numbers. Since prime numbers greater than 2 are subsets of odd numbers, the results obtained here could be explored as a new method of attack to the solution to the Strong Goldbach's Conjecture.

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