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Anil Chandra Jha
Nepal Sanskrit University, Valmeeki Campus, Kathmandu, Nepal

Dinesh Panthi
Nepal Sanskrit University, Valmeeki Campus, Kathmandu, Nepal

Shailendra Kumar Mishra
Institute of Engineering, Tribhuvan University, Kathmandu, Nepal

Corresponding Author:
Anil Chandra Jha
Nepal Sanskrit University, Valmeeki Campus, Kathmandu, Nepal

Hindu mathematics in the early classical period

Anil Chandra Jha, Dinesh Panthi and Shailendra Kumar Mishra

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Abstract
Hindu Mathematics is an ancient system of mathematical knowledge that was developed and practiced during the early classical period, from 500 BC to 500 AD. This period saw the emergence of a number of notable mathematicians and scholars who made significant contributions to the field of mathematics. By the time of this period, Hindu mathematicians had systematized most of the basic procedures of arithmetic, algebra, geometry, and trigonometry that are taught in schools today, as well as many more that are more advanced and of importance in mathematics. This paper aims to provide an overview of some of the key developments in Hindu mathematics during this period, with a particular focus on the work of Jaina, Piṅgala, the Bakṣālī Manuscript, and Āryabhaṭa I.

Keywords: Hindu mathematics, jaina mathematics, piṅgala, the bakṣālī manuscript, and āryabhaṭa I

Introduction

Over the course of more than five thousand years, the Hindu mathematicians made significant contributions and played a crucial role in the development of mathematics. For numerous reasons, the work of Hindu mathematicians is unfortunately still uncovered. Hindu mathematics requires serious consideration and a more trustworthy and scholarly treatment (Eves, 1969, p. 15) [9]. In Hindu mathematics, the Sutra period comes after the Vedic epoch; the Āryabhaṭa epoch is followed by the Hindu mathematical renaissance, and the Bhāskarācārya epoch is followed by the commentary period (Deka, 2005, p. 11) [8]. The purpose of this paper is to provide an overview of some significant Hindu mathematics developments during the early classical period. In fact, the classical period on the basis of the time period can be divided as: the early classical period (500 BCE - 500 AD) and the later classical period (500 AD - 1200 AD). The works of Jaina, Piṅgala, the Bakṣālī Manuscript, and Āryabhaṭa I will be the primary focus of this paper.

Jaina Mathematics

The development of Hindu mathematics can be divided into several periods. The earliest period is the Vedic period. The Vedic period is followed by the Jaina period. During the Jaina period, Jaina mathematicians made significant contributions to the development of mathematics. The religious works of Jainas (c.500-300 B.C.) contain the mathematical works of the Jainas. They are: Sūryaprajñapti, Jambudvipaprajñapti, Candraprajñapti, Sthanāṅga Sūtra, Uttarāyaṅa Sūtra, Bhagavatī Sūtra, and Anuyogadvāra Sūtra. The Tattvārthādhigama Sūtra Bhaṣya of Umāsvāti is another significant book on Jaina mathematics. Gaṇitānuyogī which means "the system of calculations" is the name of religious literature of the Jainas (Puttaswamy, 2012, p. 75) [18]. Measurement (Māna) of Jainas was linked to the idea of numbers. Gaṇitamāna, or counting, is one of the subdivisions of māna. A Jaina treatise known as Trikasāra, divides Gaṇitamāna into three categories: 1) Saṅkhyāta, which means "countable" or "numerable," 2) Asaṁkhyāta, which means "uncountable," and 3) Ananta, which means "infinity." There are also three subclasses for each class: Jaghanya (minimum), Madhyama (medium), and Utkṛsta (maximum). Sthānāṅgasūtra (c.100 B.C.) divides Ananta into six types: i) Akoṭa Ananta, ii) Dvidhā Ananta, iii) Deśa Vistāra Ananta, iv) Svavistāra Ananta, and v) Śahuananta Ananta.

Jainas used the varga (square) of two, for instance first varga $=2^2=4$; second varga $=4^2= 16$; third varga$=16^2 = 256$ and so on to develop large numbers. Deka (2005) [8] writes that the number of human beings in Anuyogadvāra Sūtra (c. 150 B.C., vs. 140-142) is mentioned as 5^{th} varga x 6^{th} varga. In Dhavalā text, the vargita saṁvargita (indices) of n is n^n (Datta, 1930) [5].

The first, second and third vargita saṁvargita found in the same treatise are respectively n^n, $(n^n)^{n^n}$ and $((n^n)^{n^n})^{n^n}$ (Deka, 2005, p. 47) [8].

Jaina mathematicians developed a system of numbers known as the Jain numerals. Jain numerals are a decimal system of numbers that include nine digits and a symbol for zero. Jainas were familiar with the four fundamental arithmetic operations and they used them in the case of rational fractions. A.N. Singh (1929) [21] finds fourteen types of sequences in Trikasāra. Jaina mathematicians made important contributions to the field of geometry. They developed the concept of the "golden ratio" and discovered several geometric properties of triangles and circles. In Sūryaprajñapti, the geometric figures Rhombus, quadrilateral, rectangle, circle, ellipse, and semicircle are mentioned and the ratio of the circumference of the circle to its diameter is taken as $\sqrt{10}$, that is, $\pi = \sqrt{10}$ and in Tattvārthādhigama Sūtra Bhaṣya, the following mensuration formulae are provided (Puttaswamy, 2012, p. 76) [18]:

1. Circumference of a circle = $\sqrt{10}$ diameter.
2. Area of a circle = 1/4 (circumference) (diameter).
3. Chord = $\sqrt{4\,\text{śāra}(diameter - \text{śāra})}$ [Śāra is the height or arrow of the segment]
4. Arc length of a segment = $\sqrt{6(\text{śāra})^2 + (chord)^2}$
5. Śāra = 1/2[diameter- $\sqrt{(diameter)^2 - (chord)^2}$]
6. Diameter = $\frac{(\text{śāra})^2+\left(\frac{1}{4}\right)(chord)^2}{shara}$.

The formula to calculate the area of annulus first occurs in the Tiloyappaṇṇatti (Amma, 1999, p. 168) [1]. The calculation of the area of annulus is another significant contribution to Jaina mathematics (Upadhye & Jain, 1943) [24]. The rule is: subtract twice the breadth of the (desired) annulus from twice the outer diameter and multiply the square of the remainder by the square of the half of the breadth and by ten (Amma, 1999, p. 169) [1]. The accurate area of the (desired) annulus is the square root of the product. Mathematically,

Area of annulus = $\sqrt{10\left(\frac{b}{2}\right)^2(2d-2b)^2}$,

where d is the outer diameter and b, the breadth of the annulus.

Thus Jaina has made significant contributions to various fields, including mathematics. Jaina mathematicians made important contributions to the development of number theory, geometry, and algebra. The contributions of Jain mathematicians have had a significant impact on the development of Hindu mathematics in early classical period. The Jaina numerals and the concept of the "golden ratio" are still used in modern mathematics. The contributions of Jaina mathematicians have enriched the field of mathematics and have helped shape the way we understand mathematics today.

Piṅgala

Piṅgala (c. 300 B.C.) was a great poet and mathematician. He was the first descriptive linguist who worked on mātrāmeru, binary numeral system and arithmetical triangle. According to Plofker (2007) [15], his Chandas Śutra consists of eight chapters containing 315 verses. The conventional method of material analysis of the verses is presented in the first seven chapters, while a novel approach that leads to binary arithmetic is presented in the final chapter (Nooten, 1993 & Vāsiṣṭha, 2004) [14, 25]. He is best known for his work on binary numbers, which are numbers expressed in a base-2 system. His work on binary numbers is particularly noteworthy because it predates the development of computers by thousands of years. His work was based on a system of syllables, with each syllable representing a binary digit. In his Chandas Śutra, Piṅgala uses the zero symbol for metrics (Datta & Singh, 1962, & Plofker, 2008) [7, 16]. According to Hall (2008) [11], Piṅgala is believed to be the first to use binary numbers, using the combination of light (laghu) and heavy (guru) to describe Sanskrit measure. The Varṇa- Meru of Piṅgala (also known as Pascal's triangle) illustrates this:

Table 1: Varṇa- Meru of Piṅgala

1							
1	1						
1	2	1					
1	3	3	1				
1	4	6	4	1			
1	5	10	10	5	1		
1	6	15	20	15	6	1	
1	7	21	35	35	21	7	1

The basic rules for the construction of the above table is explained in Halāyudha's commentary (c.950) on Piṅgala-sūtras (Shastri, 2002) [19]:

$^{n+1}C_r = {}^{n}C_r + {}^{n}C_{r-1}$.

Pascal's triangle is a mathematical concept that is used in a variety of fields, including probability theory, combinatorics, and algebra. While Pascal's triangle is named after the French mathematician Blaise Pascal, who described it in the 17th century, it was actually described by Piṅgala in the Chandaḥ Sāstra. In his work, Piṅgala described the construction of a triangle of numbers that had a similar pattern to Pascal's triangle. The triangle of numbers described by Piṅgala was used to calculate the number of combinations of syllables in poetry.

Thus Piṅgala's contributions to mathematics were significant, particularly in the areas of binary numbers, and Pascal's triangle. His work in these areas laid the foundation for further developments in mathematics, particularly in the areas of computer science and probability theory. Piṅgala's work is a testament to the ingenuity and creativity of ancient Hindu scholars and their contributions to the field of mathematics.

The Bakṣālī Manuscript

The Bakṣālī manuscript is an ancient Hindu mathematical text that dates back to the early classical period. On the date of the manuscript, there is absolutely no information that suggests a satisfactory solution and the presence of handwritings from multiple individuals suggests that it was written much later and is a commentary rather than an original work (Deka, 2005) [8]. Datta (1929) [4] believes that the estimated time of it is the third or fourth century A.D. According to Sridharan (2005) [23], the script used a modified form of prākṛta called Gāthā. In 1881 A.D., the Bakṣālī Manuscript was discovered in a location near Peshawar called Bakṣālī. It was discovered by a farmer during excavations and was given to the Bodleian Library at Oxford in 1902 by A.F.R. Hoernle, where it remains today (Hayashi, 1995) [12]. There were only 70 leaves in the manuscript, and the rest had been lost.

The Bakṣālī manuscript is an important historical document that provides insight into the mathematical knowledge. The manuscript contains a wealth of mathematical knowledge, including advanced techniques for solving algebraic equations, as well as methods for calculating square roots and fractions. The Bakṣālī Manuscript's author dealt with issues such as summation, missing terms, and others of geometric progression and arithmetic progression in a very effective manner, and made some new series from them. Presentation of new terms for novel thoughts, and use of abbreviations for existing terms are very frequent in the text (Deka, 2005) [8].

The manuscript contains a number of mathematical concepts that were not discovered in the West until much later. For example, the manuscript contains a method for calculating square roots that is similar to the algorithm used in modern computers. The following sūtra is provided for computing the square root of a nonsquare number:

akṛte śṛliṣṭakṛtyunāt śeṣacchedo dvisaṁguṇaḥ |
tadvargadala saṁśṛliṣṭahṛti śuddhikṛti kṣayaḥ ||

The meaning of the verse is: Divide a number that has a square root by its approximate root; divide the resulting śeṣa by two; square it (the recently obtained fraction); halve it; divide it by the composite fraction; subtract (amounting to the composite fraction); the outcome is the refined root.

The Bakṣālī manuscript also contains a number of examples of the use of negative numbers in mathematical calculations. This is significant because the use of negative numbers was not introduced in the West until the 17th century, nearly 1,000 years after the Bakṣālī manuscript was written. A cross (+) after the affected number indicates a negative quantity. In terms of being able to recognize and represent a negative number, this is a significant advancement. The author indicates 17 - 8 by 17 8+ (ibid.). In Hindu mathematics, circling a negative number came much later than Bakṣālī work. The manuscript contains a number of mathematical problems and solutions, including problems related to fractions, and algebra. The numerator is placed above the denominator in fractional quantities, and there is no horizontal line between them; the numerator might be bigger than the denominator and contractions yu for yuta, and mū for mūla, are used for addition and square root respectively. In the Bakṣālī work, zero or dots are used to denote the unknown quantity in an algebraic expression (Hayashi, 1995) [12]. In an algebraic context, this practice of using a symbol to indicate an unknown may represent a novel approach.

Thus The Bakṣālī manuscript is an important historical document that provides insight into the mathematical knowledge of ancient Hindus. The manuscript contains a wealth of mathematical knowledge, including advanced techniques for solving algebraic equations, as well as methods for calculating square roots and fractions. One of the most significant contributions of the Bakṣālī manuscript to Hindu mathematics is its use of zero as a number. The Bakṣālī manuscript is an important milestone in the development of mathematics and continues to be studied by mathematicians and historians alike.

Āryabhaṭa I

Among the classical Hindu mathematicians and astronomers, Āryabhaṭa I was the most well-known mathematician and astronomer. This Āryabhaṭa I is different person from his namesake of mathematician and astronomer Āryabhaṭa II, who lived after five centuries. According to Shukla & Sarma (1976) [20], "Āryabhaṭa I was born in 476 A.D. and wrote Āryabhaṭīya in 499 A.D." He is best known for his work on trigonometry, which he developed as a tool for astronomy. He also made significant contributions to the study of algebra and calculus. The Āryabhaṭīya of Āryabhaṭa includes the notable mathematical contributions listed below:

The alphabetical system of numeral notation

Āryabhaṭa I is credited with the development of the concept of place value in the decimal system. He was the first mathematician to use the concept of zero as a placeholder. In the following verse of Gītikā-pāda, the alphabetical system of numeral notation defined by Āryabhaṭa I is much more effective at expressing number concisely (Ganguly, 1926 & Das, 1927) [10]:

vargākṣarāṇi varge avarge avargākṣarāṇi kāt ṅmau yaḥ |
khadvinavake svarā nava varge avarge navantyavarge vā || 2 ||

The meaning of this verse is: The varga should be written in varga places and the avarga in the avarga places. The varga letters ka to ma take respectively the numerical values 1, 2, 3 … 25; the numerical value of the first avarga letter y is equal to ṅ + m = 5 + 25= 30.

The values of the avarga letters y to h, also known as unclassified consonants are: y = 30, r = 40, l = 50, v = 60, ś = 70, ṣ = 80, s = 90, h = 100. The vowels indicate multiplication of powers of one hundred in order. For example, the first vowel a may be considered as equivalent to 100^0, the second vowel i is equivalent to 100^1 (=100 = 10^2) and so on.

The first ten notational places

Āryabhaṭa gives place value up to ten digit using decimal system. He knew decimal system very well. The first ten notational places are given in the following verse of Gaṇita-pāda:

ekaṁ ca daśaṁ ca śataṁ ca sahasraṁ ayutaniyute tathtā prayutam |
koṭayarbudaṁ ca vṛndaṁ sthānāt sthānaṁ daśaguṇaṁ syāt ||2||

The meaning of this verse is: eka (units place), daśa (tens place), śata (hundreds place), sahasra (thousands place), ayuta (ten thousands place), niyuta (hundred thousands place), prayuta (millions place), koṭi (ten millions place), arbuda (hundred millions place) and vṛnda (thousand millions place) are respectively, from place to place, each ten times the previous one (Shukla & Sarma, 1976, p. 33-4) [20].

Approximate Value of π

The circumference of a circle whose diameter is 20,000 is stated in the following verse of Gaṇita-pāda (ibid. p. 45):

caturadhikaṁ śatamaṣṭaguṇaṁ dvāṣaṣṭistathā sahasrāṇām |
ayutadvayaviṣkambhasyāsanno vṛtapariṇāhaḥ ||10||

The meaning of the verse is: Add four to one hundred; multiply by eight, and sixty two thousand is added to the product. This is the approximate circumference of a circle with a diameter of 20,000.
This gives approx. circumference = (100 + 4) 8 + 62, 000 = 62, 832 and the value of π is

$$\pi = \frac{c}{d} = \frac{62{,}832}{20{,}000} = 3.1416$$

This value is accurate to four decimal places (Prasad & Shukla, 1950) [17] and is superior to Ptolemy's value of 3.141666 (Smith, 1958) [22]. Āryabhaṭa I says that the above value of π is approximate which shows that Āryabhaṭa I knew that π is incommensurable.

The table of sine-differences

Trigonometry was known as jyotipatti gaṇita in ancient Hindu astronomy, which means mathematics of generating sines, and it is known as trikoṇamiti in the modern period, which means mensuration of a triangle (Puttaswamy, 2012, p. 108) [18]. Āryabhaṭa I made significant contributions to the field of trigonometry. He developed the trigonometric functions of sine, cosine, and versine. He also introduced the concept of the trigonometric table, which was used to calculate the values of the trigonometric functions for different angles. Āryabhaṭa I calculated Rsine differences in the following verse of Gītikā-pāda:

makhi bhakhi phakhi dhakhi ṇakhi ñakhi
ṅakhi hasjha skaki kiṣga śghaki kidhva |
ghlaki kigra hakya dhaki kica
sga śjha ṅva kla pta pha cha kalārdhajyāḥ || 2 ||

The meaning of this verse is: 225, 224, 222, 219, 215, 210, 205, 199, 191, 183, 174, 164, 154, 143, 131, 119, 106, 93, 79, 65, 51, 37, 22, and 7 are the jyā differences at intervals of 225 minutes of arc in terms of minutes of arc.
This gives 24 values of jyā starts from $3^0 45'$ = 225`. The first value of jyā 225` = R sin225` = 225`. The successive values are obtained by the addition of successive jyā differences given above, namely 225, 224, and so on. He considered the value of R to 3438 (approx.) (Naraharayya, 1923) [13].
The first astronomer and mathematician to provide a table of sine-differences was Āryabhaṭa I. His estimations of the values of sine are precise. Rather than giving direct value of jyā he provides difference of two consecutive jyā.

Results in progression

The concepts of arithmetic progression, their sum, and mean were also well known in those days. Āryabhaṭa I gives the rule to calculate the sum (or partial sum) of a series in A.P. in Gaṇita-pāda.
He gave the formula for number of terms n as

$$n = \frac{1}{2d}\left(\sqrt{8dS + (2a-d)^2} - 2a + d\right)$$

(Shukla & Sarma, 1976, pp. 61-2, & Clark 1930, pp. 35-6) [20, 2]. This shows that he was well-versed in finding the solution to a quadratic equation.
The following results are found out for the first time in the Āryābhaṭīya of Āryabhaṭa (Puttaswamy, 2012, p. 126, & Shukla & Sarma, 1976, p. 64-6) [18, 20]:

$$1^2+2^2+3^2+ \dots + n^2 = \frac{1}{6} n (n+1) (2n+1)$$

$$1^3+2^3+3^3+ \dots +n^3 = [1+2+3+ \dots +n]^2 = \frac{1}{4} n^2(n+1)^2$$

$$1+ (1+2)+(1+2+3)+ \dots \text{ n terms} = \frac{1}{6} n(n+1)(n+2) = \frac{1}{6}\{(n+1)^3-(n+1)\}$$

Linear Indeterminate Equation

Āryabhaṭa I was the first Hindu mathematician to develop the rule for determining general solutions to linear indeterminate equations of the form by =ax + c, where a, b, c are integers and x, y unknowns (Puttaswamy, 2012, p. 128). Āryabhaṭa I describes the entire methods in two verses, namely 32 and 33 of Gaṇita-pāda as follows:

adhikāgrabhāgahāraṁ chindyādūnāgrabhāgahāreṇa |
śeṣaparasparabhaktaṁ matiguṇamagrāntare kṣiptam ||32||
adhauparigunitamantyayugūnāgracchedabhājite śeṣam |
adhikāgracchedaguṇaṁ dvicchedāgramadhikāgrayutam ||33||

The difficulties and ambiguity of translating the preceding two verses stem from the fact that none of the intended actions have been fully and precisely described (Datta & Singh, 1962, p. 131) [7]. Datta (1932) [6] has given the following translation: Divide the divisor for the larger remainder by the divisor for the smaller remainder. After the residue and the divisor that corresponds to the smaller remainder have been mutually divided, the final residue should be multiplied by an optional integer so that the product will be exactly divisible by the last remaining remainder in the event that the number of quotients of the mutual division is even or odd. In a column, sequentially place the quotients of the mutual division one below the other; below them is the quotient that was just obtained, as well as the optional multiplier. The penultimate number is multiplied by the number immediately above it and then added to the number immediately below it. Divide the last number by the divisor that corresponds to the remainder that is smaller; then add the greater remainder after multiplying the residue by the divisor of the greater remainder. The number that corresponds to the two divisors will be the result.
Āryabhaṭa I provides the procedure of solving the aforementioned types of problems, without giving evidence as today. Using present-day notations, we can easily retrace his prescribed procedure and locate the underlying logic (Puttaswamy, 2012, p. 129) [18].
Thus Āryabhaṭa I made significant contributions to the field of mathematics, particularly in algebra, and trigonometry. His works in these areas continue to be studied and used in modern mathematics. Āryabhaṭa I's contributions to mathematics have had a lasting impact and continue to inspire and inform the work of mathematicians around the world.

Conclusion

Since ancient times, Hindu mathematicians have made significant contributions to the field of mathematics. The modern form of mathematics was developed by the Jainas (c. 500-300 B.C.) in response to their religious obligation. Arithmetic, geometry, and astronomy were the three branches of mathematics that they developed. They divided the counting numbers into three categories: Saṅkhyāta, which means "countable," Asaṁkhyāta, which means "uncountable," and Ananta, which means "infinity." They developed a logarithm for the base two and extended the "Śulbasūtras" linear measure (the "rajju concept") to include linear, superficial, and voluminal measurements. Chandas Śutra, written by Piṅgala around 300 B.C., contains a novel

approach, binary arithmetic, in its final chapter. He is credited with using binary numbers for the first time. In 1881 A.D., the Bakṣālī Manuscript, whose date is unknown, was discovered after that. It mostly addresses problems in algebra and arithmetic, with a few issues in geometry and measurement. It includes problems with summation, finding the missing terms, etc. of geometric and arithmetic progression very effectively. The text uses a lot of abbreviations for terms that already exist and introduces new terms for new ideas. The name "Āryabhaṭa I" is the most well-known Hindu mathematician of the classical era (476 A.D.). The first book, the "Āryabhaṭīya of Āryabhaṭa," deals with astronomy and mathematics and demonstrates maturity in mathematical thinking. His development of the decimal system, including the use of zero as a placeholder, and a number of other mathematical concepts, such as how to construct the sine table was a fundamental contribution to the development of mathematics.
Finally, the early classical period in Hindu mathematics was a time of great intellectual ferment, with scholars and mathematicians making significant contributions to the field of mathematics. Jaina, Piṅgala, the Bakṣālī Manuscript, and Āryabhaṭa I are just a few of the notable figures who played a role in the development of Hindu Mathematics during this period. Their work laid the foundation for later developments in mathematics, and their insights continue to be studied and appreciated by scholars and mathematicians today.

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