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Fitting of binomial distribution by using recurrence relation method between rainfall and ground water levels: A case study

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Abstract

Present paper deals with the application of 'Distribution Theory' to analyze and predict Rainfall (RF) and Ground Water Levels (GWLs) in Anantapuramu district based on the data collected from January 2007 to December 2016. Through with Binomial Distribution by Using Recurrence Relation Method for the purpose of analysis the district is divided into five zones or Revenue Divisions (RD) namely, 1. Anantapuramu RD 2. Penukonda RD 3. Kadiri RD 4. Kalyandurg RD 5. Dharmavaram RD [5, 6]. We have estimated the Binomial Distribution by Using Recurrence Relation Method values and compared among them by using the data. Further, validation of the fitted distribution identified the best suitable zone. i.e., Residual Analysis or Error Analysis or Residual Sum of Squares (RSS) or Error Sum of Squares (ESS) or least Mean Square Error (MSE) value of the zone and forecast on the Rainfall and Ground Water Levels of this district. We also calculate Critical Difference (C.D) test and conclusions are drawn based on the results obtained [5, 6].

Keywords: Rainfall, ground water level, residual analysis, validation of the distribution, C.D. test

1. Introduction

Earlier we have discussed in the previous paper [1] the method of curve fitting is the best for estimating trend. The nature of the curve that is appropriate for the given data can be satisfactorily decided either by a theoretical understanding of the data or by observing the scatter diagram that is constructed for the given data.

The methods of fitting Straight Line, Second Degree Parabola, Exponential Curve and Power Curves by least squares method was discussed [1].

Linear, Parabolic, Exponential and Power Curve projections generally assume that growth or decline continues without limit. While these trends continue for some time they are not continue forever. There are a number of situations in which there is an asymptote to growth or decline. There are three types of Growth Curves or Models is there, that is:

1. Modified Exponential Model [2, 6]

2. Gompertz Model [3, 5]

3. Logistic Model. These models also discussed earlier papers in 'Time Series Analysis and Forecasting' concept [4].

Now we have discussed 'Distribution Theory' for different distributions we will analyze, especially Binomial Distribution by Using Recurrence Relation Method we can fit in this paper.

The data is collected on Average Rainfall and Average Ground Water Levels are given in the following table-1 for a ready reference [1-6].

2. Statistical analysis

Some of the Preliminary Statistical analysis is done for the data provided in the above table-1, such as yearly averages of Rainfall and Ground water levels are calculated and Karl-Pearson's Correlation Coefficient (r) is calculated between Average Rainfall (X) and Average Ground water levels (Y) Zonal wise [1-3].

To forecast Rainfall and Ground Water Levels through Binomial Distribution by Using Recurrence Relation Method for different zones we can consider given as follows:

The Probability Mass Function of Binomial Distribution is given by

$$P(X = x) = {}^n C_x = p^x q^{n-x} \text{ where } x = 1, 2, 3, \dots, n \quad (2.1)$$

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Table 1: Average rainfall and average ground water levels data from 2007 to 2016

Year	Zone-I		Zone-II		Zone-III		Zone-IV		Zone-V	
	RF (in mm)	GWL	RF (in mm)	GWL	RF (in mm)	GWL	RF (in mm)	GWL	RF (in mm)	GWL
2007	65.60	10.57	58.20	22.58	67.20	14.23	52.00	14.97	60.50	17.03
2008	53.90	9.96	77.90	20.73	65.20	9.27	61.30	10.88	62.70	9.09
2009	45.40	12.17	50.60	17.53	46.30	11.08	57.10	9.58	38.70	10.24
2010	53.90	12.74	71.50	15.02	70.80	12.03	64.60	8.58	56.30	11.79
2011	39.50	12.69	42.30	15.20	48.90	11.48	31.80	8.93	36.60	12.84
2012	43.20	14.98	43.40	20.49	45.30	16.08	40.50	13.76	41.90	13.22
2013	35.00	15.94	52.30	23.03	47.10	18.69	34.80	16.98	38.10	14.30
2014	31.10	15.87	30.30	23.40	27.10	21.16	37.10	18.92	22.80	16.30
2015	44.10	14.90	62.60	26.88	66.30	25.80	46.00	19.26	54.30	17.66
2016	33.50	15.57	33.40	27.27	32.30	15.35	25.70	19.51	30.10	16.15

In general,

Here $p = \text{Probability of success}$,
 $q = \text{probability of failure and } n \text{ is the number of years we will consider}$

The Binomial Distribution we know that

$$\bar{x} = \frac{\sum_{i=1}^n f_i x_i}{N}; \bar{x} = np; p = \hat{p} = \frac{\bar{x}}{n}; q = 1 - p = \hat{q}$$

Substitute these p and q values in the above Binomial Distribution equation we get the Probability Mass Function of a Binomial Distribution has

$$P(X = x) = {}^n_x c = \hat{p}^x \hat{q}^{n-x} \text{ where } x = 1, 2, 3, \dots, \dots (2.2)$$

To find expected frequencies

To find the Expected Frequencies, we use the following Recurrence Relation formula for Binomial Distribution is given by

$$P(x + 1) = \frac{n-x}{x+1} \frac{p}{q} p(x) \tag{2.3}$$

The fitted Binomial Distribution by Using Recurrence Relation Method for Average RF and Average GWLs:

A: For average rainfall

Zone-I

The Probability Mass Function of Binomial Distribution is given by

$$P(X = x) = {}^{10}_x c = (0.50)^x (0.50)^{10-x}$$

Recurrence Relation is given by

$$P(x + 1) = \frac{10-x}{x+1} \frac{(0.50)}{(0.50)} p(x)$$

Zone-II

The Probability Mass Function of Binomial Distribution is given by

$$P(X = x) = {}^{10}_x c = (0.50)^x (0.50)^{10-x}$$

Recurrence Relation is given by

$$P(x + 1) = \frac{10-x}{x+1} \frac{(0.50)}{(0.50)} p(x)$$

Zone-III

The Probability Mass Function of Binomial Distribution is given by

$$P(X = x) = {}^{10}_x c = (0.50)^x (0.50)^{10-x}$$

Recurrence Relation is given by

$$P(x + 1) = \frac{10-x}{x+1} \frac{(0.50)}{(0.50)} p(x)$$

Zone-IV

The Probability Mass Function of Binomial Distribution is given by

$$P(X = x) = {}^{10}_x c = (0.49)^x (0.51)^{10-x}$$

Recurrence Relation is given by

$$P(x + 1) = \frac{10-x}{x+1} \frac{(0.49)}{(0.51)} p(x)$$

Zone-V

The Probability Mass Function of Binomial Distribution is given by

$$P(X = x) = {}^{10}_x c = (0.50)^x (0.50)^{10-x}$$

Recurrence Relation is given by

$$P(x + 1) = \frac{10-x}{x+1} \frac{(0.50)}{(0.50)} p(x)$$

B: For average ground water levels

Zone-I

The Probability Mass Function of Binomial Distribution is given by

$$P(X = x) = {}^{10}_x c = (0.59)^x(0.41)^{10-x}$$

Recurrence Relation is given by

$$P(x + 1) = \frac{10-x}{x+1} \frac{(0.59)}{(0.41)} p(x)$$

Zone-II

The Probability Mass Function of Binomial Distribution is given by

$$P(X = x) = {}^{10}_x c = (0.58)^x(0.42)^{10-x}$$

Recurrence Relation is given by

$$P(x + 1) = \frac{10-x}{x+1} \frac{(0.58)}{(0.42)} p(x)$$

Zone-III

The Probability Mass Function of Binomial Distribution is given by

$$P(X = x) = {}^{10}_x c = (0.62)^x(0.38)^{10-x}$$

Recurrence Relation is given by

$$P(x + 1) = \frac{10-x}{x+1} \frac{(0.62)}{(0.38)} p(x)$$

Zone-IV

The Probability Mass Function of Binomial Distribution is given by

$$P(X = x) = {}^{10}_x c = (0.61)^x(0.39)^{10-x}$$

Recurrence Relation is given by

$$P(x + 1) = \frac{10-x}{x+1} \frac{(0.61)}{(0.39)} p(x)$$

Zone-V

The Probability Mass Function of Binomial Distribution is given by

$$P(X = x) = {}^{10}_x c = (0.58)^x(0.42)^{10-x}$$

Recurrence Relation is given by

$$P(x + 1) = \frac{10-x}{x+1} \frac{(0.58)}{(0.42)} p(x)$$

Where $x = 1, 2, 3 \dots$ Substitute in the above equations we can get the values of $p(1), p(2), p(3) \dots$ multiplying these $p(1), p(2), p(3) \dots$ values by the $N = \sum_{i=1}^n f_i$ we get the required Expected Frequencies, these are denoted by $f(1), f(2), f(3) \dots$

3. Validation of the fitted distribution

Validation of the fitted distribution is necessary to check the suitability of the distribution for the given data this is done by considering X = Years and Y = Average RF or Average GWL given in table 1 and estimated the Average RF (Y) or Average GWL (Y) denoted by \hat{Y} . The estimated Average RF and Average GWLs are given in the following tables.

Table 2: Estimated average RF \hat{Y} for binomial distribution by using recurrence relation method

Year	Zone-I		Zone-II		Zone-III		Zone-IV		Zone-V	
	Actual	Estimates	Actual	Estimates	Actual	Estimates	Actual	Estimates	Actual	Estimates
2007	65.60	4.45	58.20	5.23	67.20	5.17	52.00	4.51	60.50	4.42
2008	53.90	22.26	77.90	26.13	65.20	25.83	61.30	18.04	62.70	22.10
2009	45.40	57.88	50.60	67.93	46.30	67.15	57.10	45.09	38.70	57.46
2010	53.90	102.40	71.50	120.18	70.80	118.80	64.60	76.65	56.30	101.66
2011	39.50	124.66	42.30	146.30	48.90	144.62	31.80	90.18	36.60	123.76
2012	43.20	102.40	43.40	120.18	45.30	118.80	40.50	72.14	41.90	101.66
2013	35.00	57.88	52.30	67.93	47.10	67.15	34.80	40.58	38.10	57.46
2014	31.10	22.26	30.30	26.13	27.10	25.83	37.10	13.53	22.80	22.10
2015	44.10	4.45	62.60	5.23	66.30	5.17	46.00	4.51	54.30	4.42
2016	33.50	0	33.40	0	32.30	0	25.70	0	30.10	0

Table 3: Estimated average GWL \hat{Y} for binomial distribution by using recurrence relation method

Year	Zone-I		Zone-II		Zone-III		Zone-IV		Zone-V	
	Actual	Estimates	Actual	Estimates	Actual	Estimates	Actual	Estimates	Actual	Estimates
2007	10.57	0.27	22.58	0.42	14.23	0.16	14.97	0.14	17.03	0.28
2008	9.96	1.76	20.73	2.55	9.27	1.09	10.88	0.99	9.09	1.66
2009	12.17	6.77	17.53	9.33	11.08	4.66	9.58	4.10	10.24	6.10
2010	12.74	17.06	15.02	22.49	12.03	13.34	8.58	11.17	11.79	14.69
2011	12.69	29.52	15.20	37.33	11.48	26.07	8.93	20.92	12.84	24.40
2012	14.98	35.34	20.49	43.06	16.08	35.38	13.76	27.28	13.22	28.14
2013	15.94	29.11	23.03	33.94	18.69	33.05	16.98	24.32	14.30	22.18
2014	15.87	15.71	23.40	17.61	21.16	20.17	18.92	14.28	16.30	11.51
2015	14.90	5.01	26.88	5.30	25.80	7.29	19.26	4.95	17.66	3.47
2016	15.57	0.68	27.27	0.64	15.35	1.24	19.51	0.71	16.15	0.42

In the above tables 2 and 3 for the validation of the distribution, Residual Analysis or Error Analysis or

Residual Sum of Squares (RSS) or Error Sum of Squares (ESS) or Mean Square Errors (MSE's) are calculated zone

wise by considering

$$\text{Residual sum of squares (RSS)} = \sum_{i=1}^n (y_i - \hat{y}_i)^2 \dots (3.1)$$

Where y_i or o_i represents actual or observed values given in

table 1 and \hat{y}_i or \hat{e}_i is the estimated values through fitted distribution is given in tables 2 and 3. Residual Sum of Squares was calculated and is given in the following table [5, 6].

Table 4: Residual sum of squares values for average RF for binomial distribution by using recurrence relation method

Type of the distribution	Zone-I	Zone-II	Zone-III	Zone-IV	Zone-V
Binomial Distribution Recurrence Relation Method	21301.29	29535.76	27884.79	11796.34	22140.25

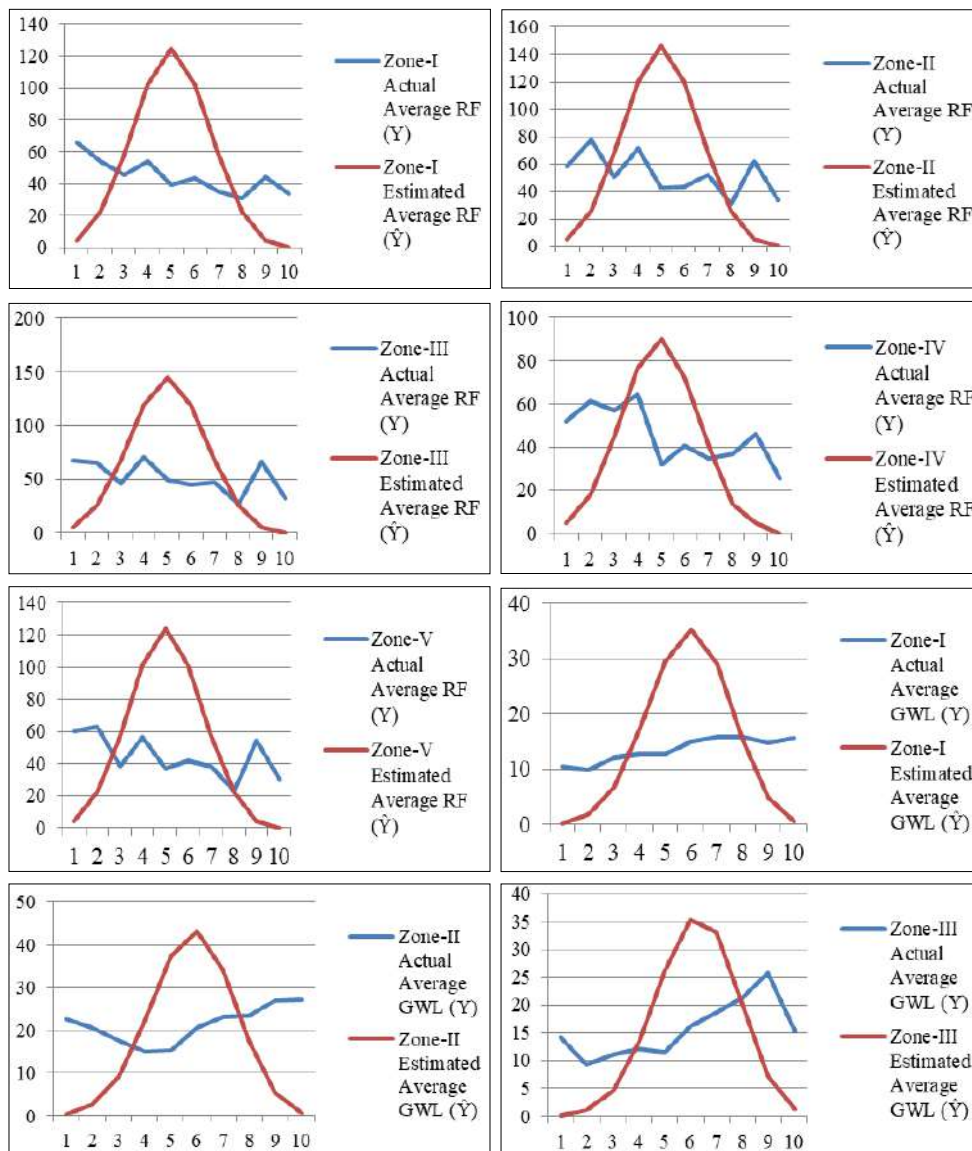
Table 5: Residual sum of squares values for Average GWLs for binomial distribution by using recurrence relation method

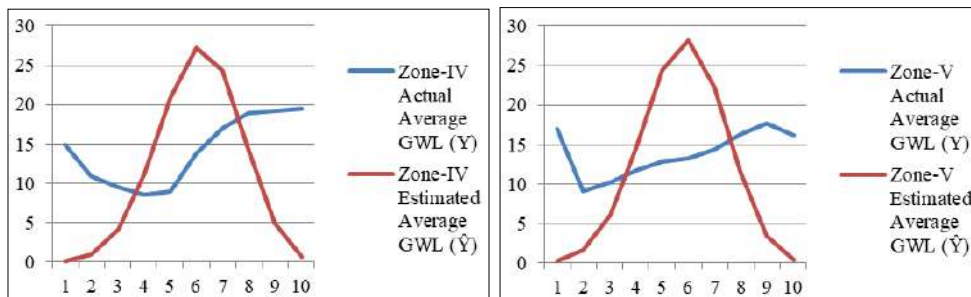
Type of the distribution	Zone-I	Zone-II	Zone-III	Zone-IV	Zone-V
Binomial Distribution Recurrence Relation Method	1411.93	3271.17	1642.07	1314.66	1251.37

Conclusions

By Comparing Residual Sum of Squares values for Average RF and Average GWLs through Binomial Distribution by Using Recurrence Relation Method under consideration, for RF of zone-IV is least and GWLs for zone-V Residual Sum of Squares values is least. Next to zone-IV, zone-I has least

Residual Sum of Square value in RF and GWLs zone-IV is least. Further, the behaviours of RF and GWL through this distribution in different zones are represented in the following Figure-1. Similar conclusions can be drawn from the following graphs also [5, 6].





Note: In the above graphs x-axis represents years in the last decade i.e. from 2007 to 2016. On y-axis average RF measured in mille meters or average GWLs measured in meters ^[1-6].

Fig 1: Behaviour of RF and GWLs actual and estimated values for binomial distribution by using recurrence relation method in zone –I, II, III, IV and V

4. Further statistical analysis

Now, we proceed to analyse the given estimates in tables-2 and 3 using ANOVA two-way classification by considering rows as different years and columns as different zones and the following Null Hypothesis are formed and tested ^[5, 6].

H01: There is no significant difference between different years of average RF in Anantapuramu district ^[1-6].

H02: There is no significant difference between average RF of different zones in Anantapuramu district ^[1-6].

H03: There is no significant difference between different years of average ground water levels in Anantapuramu district ^[1-6].

H04: There is no significant difference between average ground water levels of different zones in Anantapuramu district ^[1-6].

Table 6: Anova two-way table for RF

Source of variation	d.f.	S.S.	M.S.S.	F-cal
Rows (years)	9	98736.11	10970.68	151.281
Columns (Zones)	4	3151.787	787.9467	10.86545
Error	36	2610.668	72.51856	
Total	49	104498.6		

By comparing F-calculated value of Rows (Years) with F-critical value at 5% level of significance we reject the H01 i.e. There is a significant difference between different years of Average RF in Anantapuramu District. Similarly by comparing F-calculated value of Columns (Zones) with F-critical value at 5% level of significance we reject the H02 i.e. there is a significant difference between Average RF of different zones in Anantapuramu District.

Table 8: Year wise aggregate average RF for binomial distribution by using recurrence relation method estimates

Year	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016
Average	4.756	22.872	59.102	103.938	125.904	103.036	58.2	21.97	4.756	0

Table 9: If we can arranged ascending order

Year	2016	2015	2007	2014	2008	2013	2009	2012	2010	2011
Average	0	4.756	4.756	21.97	22.872	58.2	59.102	103.036	103.938	125.904

$$S.E = \sqrt{2 \times Error M.S.S/m}$$

$$= 5.39$$

$$1\% \text{ I.o.f C.D.} = 2.58 \times 5.39 = 13.91$$

Table 7: Anova two-way table for GWLS

Source of variation	d.f.	S.S.	M.S.S.	F-cal
Rows (Years)	9	7259.927	806.6586	95.29301
Columns (Zones)	4	268.5394	67.13484	7.930841
Error	36	304.7412	8.465034	
Total	49	7833.208		

By comparing F-calculated value of Rows (Years) with F-critical value at 5% level of significance we reject the H03 i.e. There is a significant difference between different years of Average GWLs in Anantapuramu District. Similarly by comparing F-calculated value of Columns (Zones) with F-critical value at 5% level of significance we reject the H04 i.e. there is a significant difference between Average GWLs of different zones in Anantapuramu District.

Since F-cal value related to Rows (Years) in RF and GWL is high so there is a necessity for Critical Difference (C.D) Test for sub-grouping various years using the following formula.

$$C.D. = \sqrt{2 \times Error M.S.S/m} \times t_{0.01} \text{ for error d.f. in tables (6) and (7) ...}$$

(4.1)

Where m represents number of replicates in each zone and as well as year ^[5, 6].

5. Critical difference (C.D.) test

Average RF for years

Above notation indicates that 2016-2015-2007, 2014-2008, 2013-2009, 2012-2010 years Average RF come under one category and 2011 year Average RF come under another category because there is no Significant Difference in average RF.

2016 2015 2007 2014 2008 2013 2009 2012 2010 2011

Critical difference (C.D.) test: Average GWL for years

Table 10: Year wise aggregate average GWL for binomial distribution by using recurrence relation method estimates

Year	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016
Average	0.254	1.61	6.192	15.75	27.648	33.84	28.52	15.856	5.204	0.738

Table 11: If we can arranged ascending order

Year	2007	2016	2008	2015	2009	2010	2014	2011	2013	2012
Average	0.254	0.738	1.61	5.204	6.192	15.75	15.856	27.648	28.52	33.84

$$S.E = \sqrt{2 \times Error M.S.S/m}$$

$$= 1.84$$

$$1\% \text{ I.o.f C.D} = 2.58 \times 1.84 = 4.75$$

2007 2016 2008 2015 2009 2010 2014 2011 2013 2012

Above notation indicates that 2007-2016-2008, 2016-2008-2015, 2015-2009, 2010-2014, 2011-2013 years Average GWL come under one category and 2012 year Average GWL come under another category because there is no Significant Difference in average GWL.

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