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Properties of binomial coefficients

Dragan Obradovic and Lakshmi Narayan Mishra

Abstract

Binomial coefficients $\binom{n}{k}$ they have an unusually large number of applications and they are certainly one of the most important combinatorial terms. Their most important property is stated in Theorem o degree of binoma, that is, in the so-called. Binomial formula. Binomial formula and triangle layout binomial coefficients are usually related to Blaise Pascal (Blaise Pascal), who described them in 17. century. However, they were also known to Chinese mathematician ar Jang Huiu (Yang Hui) in 13. persian mathematicians ar Omar Hajam (Omar Khayyám) in the 11. century.

Keywords: Binomial coefficients, multinomial coefficients

Introduction

A binomial theorem is an elementary algebra theorem and describes the coefficients of the degree of a binoma when it is presented in the developed form. The coefficients that occur in binomial development are called binomial coefficients. They are identical to the numbers that appear in the Pascal triangle. These numbers can be calculated using a simple formula that uses a factorial. The same coefficients occur in the combinatorial, where is the expression $x^{n-k}y^k$ equal to the number of different combinations of k-elements selected from a set of n terms.

As a branch of mathematics, combinatorics is taught in more detail in secondary schools. Within this material, we learn binomial coefficients that are indispensable to continue learning about basic combinatorial counting principles (variations, permutations, and combinations). Application of binomials coefficients are therefore widespread and have been shown to have quite a number of properties.

In addition to the binomial coefficients, which belong to a particular type of numbers, there are many different numbers that can be classified into a group by some common properties. So, for example, from the earliest touch with math we learn for even and odd numbers, squares and cubes of numbers are also one special category of numbers. Apart from these simpler ones, there are some more complex types of numbers like Fibbonacci, Catalan, Mersen, harmonic, Bernoulli and other numbers.

In order to continue with the examples that use the idea of a vaporizer, we must first, let's remind ourselves of some important things about binomial coefficients and to recall some identities, which we will prove combinatorial instead provides standard algebraic evidence. Also, to illustrate beauty combinatorial proof, we will do more examples than we need to give text. Definition. Let n be a natural integer and k a nonnegative integer. We define a binomial coefficient $\binom{n}{k}$ as the number of all k-punched subsets of sets $[n]$.

2. Coefficients in the binomial formula

The coefficients in the binomial formula such as $\binom{n}{0}, \binom{n}{1}, \binom{n}{2}, \binom{n}{n}, \binom{n}{n-k}$ are called binomial coefficients. The coefficient reads as 'n' 'over' k ". These coefficients $\binom{n}{k}$ are obtained using the formula for non-repetitive combinations. The formula for combinations b.p. reads.

$\binom{n}{k} \overset{def}{=} \dfrac{n!}{k!(n-k)!}$. It is also: $n! \overset{def}{=} (n-1)\cdot(n-2)\cdot\ldots\cdot 1 \Leftrightarrow$ n·(n-1)!.

The following applies to binomial coefficients:

- $\binom{n}{0} \overset{def}{=} 1 \wedge \binom{n}{n} \overset{def}{=} 1$
- $\binom{n}{k} = \binom{n}{n-k}$, because by the formula for combinations b.p.

$C_k^n = \frac{n!}{k!\cdot(n-k)!}$, a $C_{n-k}^n = \frac{n!}{(n-k)!\cdot k!}$. How is it $\frac{n!}{(n-k)!\cdot k!} = \frac{n!}{k!\cdot(n-k)!}$, because the n!≡n!,

i (n-k)!·k!=k!·(n-k)! by commutation law, we conclude that $\binom{n}{k} = \binom{n}{n-k}$.

- $\binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1}$ That is because it is $\binom{n+1}{k+1} =$
-

$= \frac{(n+1)!}{(k+1)!\cdot(n+1-k-1)!} = \frac{(n+1)!}{(k+1)!\cdot(n-k)!}$, a $\binom{n}{k} + \binom{n}{k+1} =$

$$= \frac{n!}{k!\cdot(n-k)!} + \frac{n!}{(k+1)!\cdot(n-k-1)!} = \frac{n!}{k!\cdot(n-k)\cdot(n-k-1)!} + \frac{n!}{(k+1)\cdot k!\cdot(n-k-1)!} =$$

$$= \frac{n!\cdot(k+1) + n!\cdot(n-k)}{(n-k-1)!\cdot(n-k)\cdot k!\cdot(k+1)} = \frac{n!\cdot(k+1+n-k)}{(n-k)!\cdot(k+1)!} = \frac{n!\cdot(n+1)}{(n-k)!\cdot(k+1)!} = \frac{(n+1)!}{(k+1)!\cdot(n-k)!},$$

which means that $\binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1}$.

• the coefficient in front of the kth member is equal $\binom{n}{k-1}$, $n, k \in N$.

A derivation of the binomial formula

By the binomial formula, we determine the result and the coefficients that lie in front of the summaries, which determine the result of an expression in which a certain bin is multiplied a certain number of times by itself.

Let's start with the formula: $(a+b)^1 = a + b$

Multiplying both sides of this formula by (a + b) we get: $(a+b)^2 = a^2 + 2ab + b^2$

Continuing to multiply both sides of the formula by (a + b), we get a formula in which the result of the expression $(a + b)^n$ is calculated, $n \in N$, where, respectively, the product of n equal factors is calculated, $(a+b)\cdot(a+b)\cdot\ldots\cdot(a+b)$, where the bin (a + b) occurs n times.

Expression (1) $(a+b)\cdot(a+b)\cdot\ldots\cdot(a+b)$ we calculate by:

• we extract a from each factor and obtain $a \cdot a \cdot \ldots \cdot a = a^n$ because in expression (1) a is multiplied n times by itself;

• we extract b from one factor and from all other a, so we get the expression (2) $a^{n-1}b$. There are such opportunities C_1^n that is, n, which means that we get n terms such as expression 2;
• take a number k such that 1 <k <n. From expression (1)

we can extract the k factor on C_k^n ways, and from the rest n- k the number of factors is a.

Then we get the expression $\mathbf{a}^{n-k}\mathbf{b}^k$ and there are such expressions C_k^n, because the term a^{n-k} can be defined at C_{n-k}^n ways, bk on C_k^n find on. How is it $C_k^n = C_{n-k}^n$ if we determine b^k on C_k^n we also determined an-k in the same number of ways. The same is true if we first define the expression a^{n-k};

• from each factor we take the number **b** and get b^n. Hence the formula:

$(a+b)^n = a^n + C_1^n \cdot a^{n-1} \cdot b + \ldots + C_k^n \cdot a^{n-k} \cdot b^k + \ldots + C_{n-1}^n \cdot a \cdot b^{n-1} + b^n$

or formula: $(a+b)^n = \binom{n}{0} \cdot a^n \cdot b^0 + \binom{n}{1} \cdot a^{n-1} \cdot b^1 + \binom{n}{2} \cdot a^{n-2} \cdot b^2 + \ldots + \binom{n}{n-1} \cdot a^1 \cdot b^{n-1} + \binom{n}{n} \cdot a^0 b^n$

called the binomial formula.

In it we notice that a starts from the nth degree and decreases to 0-this one, while b starts from 0-this one and reaches nth degree.
The binomial formula is proved by mathematical induction, which I will not apply in this paper.

3. References

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